ON COMMUTATIVITY FOR CERTAIN NON ASSOCIATIVE PRIMITIVE RINGS WITH $x(xy)^2 – (xy)^2x$ € Z(R)

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***ABSTRACT**:*

Johnson, Outcalt and Yaqub [3] proved that if a non- associative ring R satisfy the identity $x^2 y^2 = y^2 x^2$ for all x,y in R, then R is commutative. The generalization of this result proved by R. D. Giri and others [1] stoctes that if R is a non-associative primitive ring satisfies the identity $x^2 y^2 - y^2 x^2 \in z(R)$, where z(R) denotes the center, then R is commutative. A modification of Johnson's identity viz, $x^2 y^2 - y^2 x^2$ for all x,y in R, for a non - associative ring R which has no element of additive order 2, is commutative was proved by R. N. Gupta [2], R. D. Giri and others [1] generalized Gupta's result by taking $x(xy)^2 -(xy)^2 x \in z(R)$.

KEYWORDS: *Center, Commutativity, Primitive Ring.*

INTRODUCTION:

The study of associative and non- associative rings has evoked great interest and importance. The results on associative and non-associative rings in which one does assume some identities in the center have been scattered throughout the literature. Many sufficient conditions are well known under which a given ring becomes commutative.Notable among them are some given by Jacobson, Kaplansky and Herstein. Many Mathematicians of recent years studied commutativity of certain rings with keen interest. Among these mathematicians Herstein, Bell, Johnsen, Outcalt, Yaqub, Quadri and Abu-khuram are the ones whose contributions to this field are outstanding.

PRELIMINARIES:

Non - Associative Ring:

If R is an abelian group with respect to addition and with respect to multiplication R is distributive over addition on the left as well as on the right.

For every elements x, y,z of R

$(x+y)z =xa+yz, z(x+y) = zx+zy$

Commutator:

For every x, y in a ring R satisfying $[x,y]=xy-yx$ then $[x,y]$ is called a commutator.

Commutative Ring:

For every x, y in a ring R if xy = yx then R is called a commutative ring. Non commutative ring is split from the commutative ring, i.e., R is not commutative with respect to multiplication. i.e., we cannot take xy = yx for every x,y in R as an axiom.

Primitive Ring:

A ring R is defined as primitive in case it possesses a regular maximal right ideal, which contains no two-sided ideal of the ring other than the zero ideal.

Torsion-Free ring:

If R is m-torsion free ring, then mx=0 implies x=0 for positive integer m and x is in R.

Center :

In a ring R, the center denoted by Z(R) is the set of all elements $x \in R$ Such that xy=yx for all $X \in R$, It is important to note that this definition does not depend on the associative of multiplication and in fact, we shall have occasion to deal with derivation of non-associative algebras.

MAIN RESULTS:

THEOREM 1 :

If R is a non-associative primitive ring with unity 1 satisfying $x^2 y^2 - y^2 x^2 \in z(R)$ for all x,y in Rand it is of char≠2 then R is commutative.

PROOF:-

By hypothesis,

$$x^2 y^2 - y^2 x^2 \in z(R) \qquad (1.1)$$

Replacing x by x+1 in 1.1 and using 1.1, we get,

$(x+1)y^2 - y^2(x+1)^2 \in z(R)$

That is, $xy^2 - y^2x \in z(R)$ (1.2)

Now replacing y by y+1 in 1.2 and using 1.2,

we obtain, $2xy - 2yx \in z(R)$

That is, $2(xy - yx) \in z(R)$

Since R is of char ≠2, $xy-yx \in z(R)$.

Therefore xy- yx is a two sided ideal of R. We know that if R is a primitive ring then R has a maximal right ideal which contains no non-zero ideal of R.

Consequently we obtain, (xy - yx)R=(0)

Which further yields xy yx=0 due to primitivity of R. This completes the proof of the theorem.

THEOREM 2:-

If R is a non-associative primitive ring with unity 1 such that $x(xy)^2 - (xy)^2x \in z(R)$ and it is of char 4 then R is commutative.

PROOF:

Given that $x(xy)^2 - (xy)^2x \in z(R)$ (2.1)

Replacing x by x+1 in 2.1 and using 2.1 we obtain

$x[(xy)y+y(xy)+y^2]-[(xy)y + y(xy)+ y^2]x \in z(R)$ (2.2)

Now replacing y by y+1 in 2.2 and using 2.2 we get,

$x(3xy + yx + 2y) - (3xy + yx + 2y)x \in z(R)$ (2.3)

We replace x by x+1 in 2.3 and using 2.3we obtain,

$4xy-4yx \in Z(R)$

That is, $4(xy-yx) \in z(R)$

Since R is of char $\neq 4$ $xy - yx \in z(R)$

Now using the same argument as in the last paragraph of the proof of the theorem 2.1.

We concludes that is commutative.

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