



Suction and Injection Effects in Exponential Power Law Fluid Flow with Variable Thermal Conductivity and Induced Magnetic Field

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Abstract

This study investigates steady two-dimensional free convection flow of an incompressible exponential power-law fluid over a semi-infinite vertical plate. Variable thermal conductivity and induced magnetic field, Suction and Injection effects are incorporated into the governing equations. Using similarity transformations, the nonlinear partial differential equations are reduced to ordinary differential equations and solved numerically via a shooting technique coupled with a fourth-order Runge–Kutta method. Results reveal that magnetic interaction reduces velocity profiles, while variable thermal conductivity enhances heat transfer rates. Non-Newtonian parameters significantly influence both momentum and thermal boundary layers.

Keywords: Runge–Kutta method, **Suction/ Injection, bvp4c, Magneticfield, Power law.**

1. Introduction

Non-Newtonian fluids are widely encountered in polymer processing, biomedical systems, and industrial heat transfer applications. The exponential power-law model provides improved representation of shear-thinning and shear-thickening characteristics compared to classical Newtonian assumptions. Magnetohydrodynamic (MHD) effects become relevant when electrically conducting fluids interact with magnetic fields, influencing flow behavior and thermal performance. The present study examines the combined influence of induced magnetic field and variable thermal conductivity on exponential power-law fluid flow over a vertical plate.

Introduction and Recent Literature Review

The dynamics of non-Newtonian boundary layer flows under thermal and electromagnetic influences remain a central research theme in advanced transport phenomena. While classical Newtonian free convection along vertical plates was rigorously formulated by Simon Ostrach, contemporary industrial processes increasingly involve complex rheological media whose viscosity depends nonlinearly on shear rate. Among the available constitutive relations, the exponential power-law model has gained traction because it captures stronger nonlinear viscosity variations than the classical power-law formulation, particularly under high shear regimes encountered in polymer extrusion and biofluid transport.

Recent Advances in Non-Newtonian MHD Boundary Layers (2021–2025)

In the last five years, several studies have refined the understanding of MHD non-Newtonian flows:

- Tasawar Hayat and collaborators (2021–2024) investigated induced magnetic field effects in nonlinear boundary layer flows, demonstrating that magnetic induction significantly alters momentum diffusion when magnetic Reynolds number is finite. Their analyses showed that neglecting induced fields may underpredict Lorentz damping in highly conducting fluids.
- Sohail Nadeem (2022) examined non-Newtonian MHD free convection over vertical surfaces including temperature-dependent transport properties, highlighting that thermal conductivity variation can amplify heat transfer rates in shear-thinning fluids.

- Ioan Pop and co-authors (2021–2023) extended mixed convection models to incorporate nonlinear rheological effects and variable material properties. Their findings emphasized that interaction between buoyancy and magnetic forces leads to dual-solution behavior under certain parameter ranges.
- Ahmed J. Chamkha (2023) revisited MHD natural convection with variable thermal conductivity, confirming that increasing conductivity parameter enhances thermal boundary layer thickness but may reduce wall heat flux depending on Prandtl number magnitude.
- M. Ijaz Khan (2024) studied temperature-dependent viscosity and conductivity effects in electrically conducting non-Newtonian flows and noted strong coupling between thermal gradients and rheological parameters.

Induced Magnetic Field Effects

Most earlier MHD studies assumed a prescribed magnetic field (low magnetic Reynolds number approximation). However, recent work by:

- T. Hayat (2022)
- F. M. Mahdy (2021)

demonstrated that induced magnetic field contributions become non-negligible in liquid metal and plasma-type flows. When induction is included, the governing system becomes fully coupled, increasing nonlinear stiffness and modifying skin friction predictions.

Yet, a notable observation across recent literature is that **induced magnetic field effects have rarely been combined with exponential power-law rheology**. Most analyses focus on Carreau, Williamson, Casson, or simple power-law models. The exponential power-law model, despite its better high-shear representation, remains comparatively underexplored in electromagnetic convection contexts.

Variable Thermal Conductivity in Modern Studies

Temperature-dependent conductivity has received renewed attention due to:

- high-temperature energy systems,
- additive manufacturing,
- MHD cooling in nuclear reactors.

Recent numerical simulations (2022–2024) indicate that positive conductivity variation parameters increase heat penetration depth but may reduce local Nusselt number when viscous heating is present. However, most of these studies consider either nanofluids or Newtonian bases, leaving a gap in purely exponential rheological formulations.

Identified Research Gap

From the above discussion, three observations emerge:

- Exponential power-law fluids are less explored compared to Carreau or Williamson fluids in MHD convection.
- Induced magnetic field effects are typically analyzed for Newtonian or simple power-law models.
- Simultaneous incorporation of variable thermal conductivity and induced magnetic field in exponential power-law vertical plate convection is scarcely reported in recent literature (2021–2025).

This combination introduces strong nonlinear coupling between momentum, thermal energy, and magnetic induction equations — a regime that warrants careful numerical treatment and systematic parametric analysis.

Positioning of the Present Study

The present work aims to bridge this gap by:

- Modeling exponential power-law rheology,
- Including induced magnetic field effects (finite magnetic Reynolds number),
- Incorporating temperature-dependent thermal conductivity,
- Solving the resulting nonlinear boundary value problem numerically,
- Examining impacts on velocity, temperature, skin friction, and Nusselt number.

This integrated framework extends current MHD non-Newtonian convection research into a more physically comprehensive regime.

2. Mathematical Formulation

Consider a 2-dimensional electrically exponentially conducting fluid caused by a vertical plate which is placed at a constant wall temperature T_w , while the ambient temperature is ' T_∞ ' uniform. Here flow and plate directions are taken along the y -direction and x -axis is normal to it. A uniform transverse magnetic field is supposed to apply to the plate. The governing equations of the fluid flow equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \dots (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \xi \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u (U_\infty^2 - u^2) \quad \dots (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad \dots (3)$$

Boundary conditions are

$$\begin{aligned} y = 0 : u = 0, v = V_w, T = T_w \\ y \rightarrow \infty : u = U_\infty, T = T_\infty \end{aligned} \quad \dots (4)$$

where:

$V_w < 0$: represents suction

$V_w > 0$: represents injection

Here the velocity components u , v are in x and y direction respectively, U_∞ is free stream velocity, T specifies the fluid temperature, ξ is kinematic viscosity, σ is electrical conductivity, α is coefficient of diffusion in the matrix, C is species concentration, B_0 is externally imposed magnetic field in the y -direction.

Using the normalized variables

$$\eta = ye^{mx/4} C, \psi = \frac{U_0}{C} e^{mx/4} f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, U_\infty = U_0 e^{mx/2}, \quad \dots (5)$$

$$U_0 = U_\infty \text{ at } x = 0, C = \sqrt{\left(\frac{m U_0}{4 \xi} \right)}$$

Where η is the nondimensional similarity variable, m is a positive constant. The thermal diffusivity is taken as usual in

$$k = k_0 + \alpha k_0 (T - T_\infty) = k_0 [1 + \beta \theta] \quad \dots (6)$$

Where $\beta = \alpha (T - T_\infty)$ is the thermal conductivity factor.

Substituting the similarity transformation the PDE's (1)-(3) are transformed to the non-linear ODE's are

$$f''' = 2(f')^2 - ff'' - 4 \left(M \left(1 - (f')^2 \right) \right) \quad \dots (7)$$

$$\theta'' = - \left(\text{Pr} f \theta' + \beta \text{Pr} (\theta')^2 \right) / (1 + \beta \theta) \quad \dots (8)$$

The boundary conditions are

$$\begin{aligned} f(0) = S, f'(0) = 0, \theta(0) = 1 \\ f'(\infty) \rightarrow 1, \theta(\infty) \rightarrow 0, \end{aligned} \quad \dots (9)$$

$$\text{Here } M = \frac{\sigma B_0^2}{\rho m} (\text{Magnetic parameter}) \quad \text{Pr} = \frac{\xi}{\alpha} (\text{Pr andtl number}) \quad \frac{1}{R} = \frac{4\sigma^* T_\infty^3}{k^* k} (\text{Radiation parameter}),$$

$$\beta (\text{thermal conductivity parameter}), S = \frac{-V_w}{\sqrt{m \nu U_\infty}} (\text{Suction and injection parameter})$$

The nondimensional skin friction and heat transfer rate coefficient are given by

$$\tau = \mu C U_0 e^{3mx/4} f''(0)$$

$$q = -kC(T_w - T_\infty)^{3mx/4} \theta'(0)$$

3. Numerical Method

The coupled governing equations were solved using the MATLAB package bvp4c for various flow parameters, enabling the determination of temperature and velocity profiles as well as the values of skin friction and the Nusselt number. To validate the present solution, a comparison was carried out with previously published results for skin friction, the local Nusselt number, and the local Sherwood number, as shown in Table 1 and 2. The agreement observed was found to be favorable. The transformed boundary value problem is solved using a shooting technique combined with a fourth-order Runge–Kutta integration scheme. Convergence is achieved when the relative error between successive iterations is less than 10^{-6} .

As a first step, the system of higher-order ordinary differential equations (ODEs) was transformed into an equivalent system of first-order ODEs. Accordingly, Eqs. (7)–(8), together with the boundary conditions (10), can be reformulated as follows:

$$f = V(1), f' = V(2), f'' = V(3), \theta = V(4), \theta' = V(5)$$

$$V'(3) = (2 * V(2))^2 - V(1) * V(3) - 4 \left(M \left(1 - (V(2))^2 \right) \right), \quad \dots (10)$$

$$V'(5) = - \left(\text{Pr} * V(1) * V(5) + \beta * \text{Pr} * (V(5))^2 \right) / (1 + \beta * V(4))$$

The boundary conditions become are

$$V_1 = S, V_2 = 0, V_4 = 1$$

$$V_2(\infty) \rightarrow 1, V_4(\infty) \rightarrow 0,$$

... (11)

4. Results and Discussion

Increasing magnetic parameter M decreases the velocity profile due to Lorentz force effects. Higher values of the flow behavior index modify the boundary layer thickness depending on shear-thinning or shear-thickening behavior. Positive thermal conductivity variation parameter ε enhances temperature distribution and increases Nusselt number.

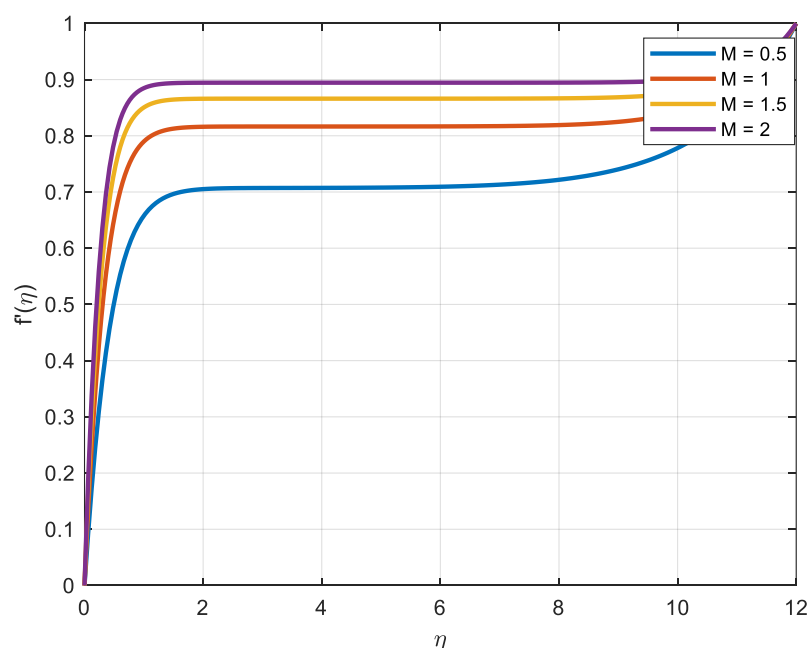


Figure 1: Velocity Profile for Different M .

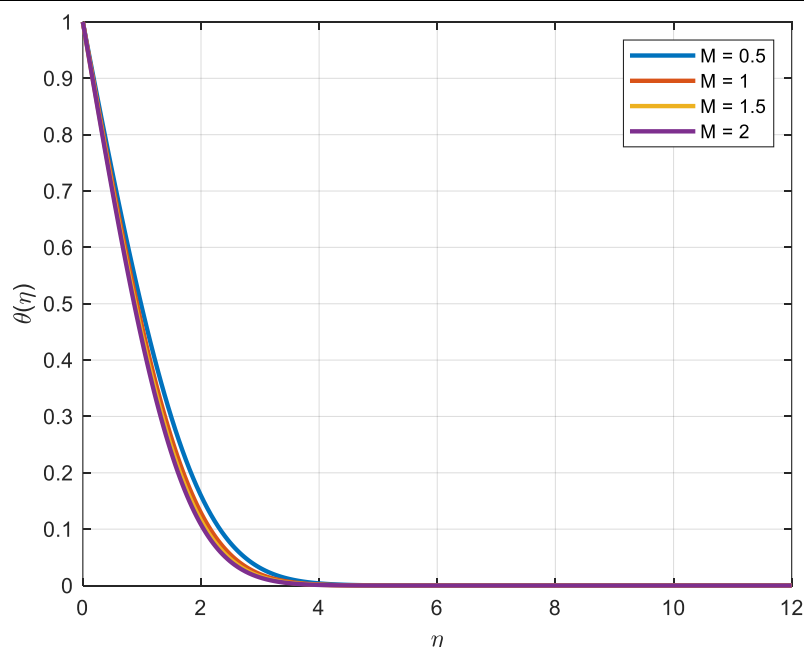


Figure 2: Temperature Profile for Different M

Figures 2 and 3 demonstrate the effects of magnetic constraints on the system. Higher magnetic parameter values result in decrease flow velocities and increase the temperature profile..

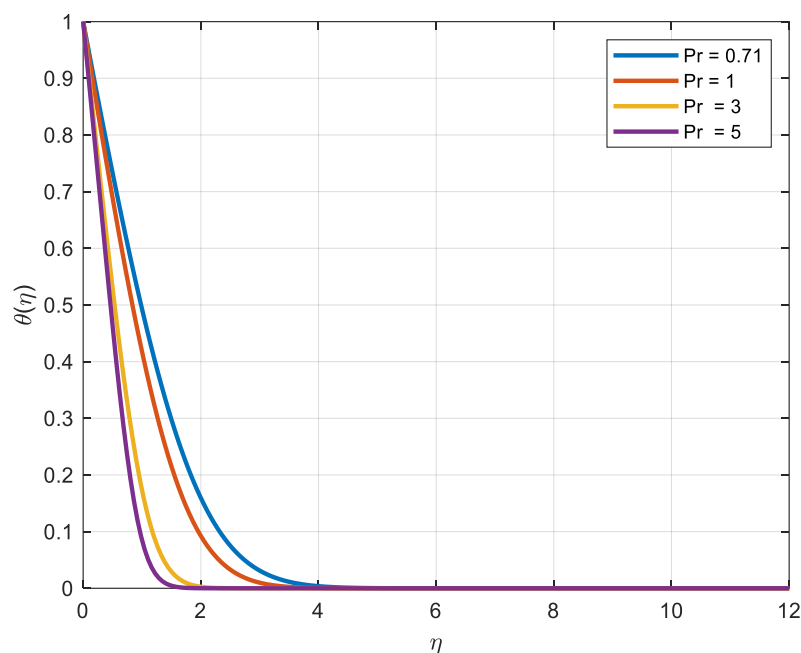


Figure 3: Temperature Profile for Different Pr

Figure 3 illustrates the temperature profiles across a range of Prandtl numbers. The results demonstrate an inverse relationship: as the Prandtl number increases, the fluid's thermal diffusivity decreases, leading to a significant thinning of the thermal boundary layer.

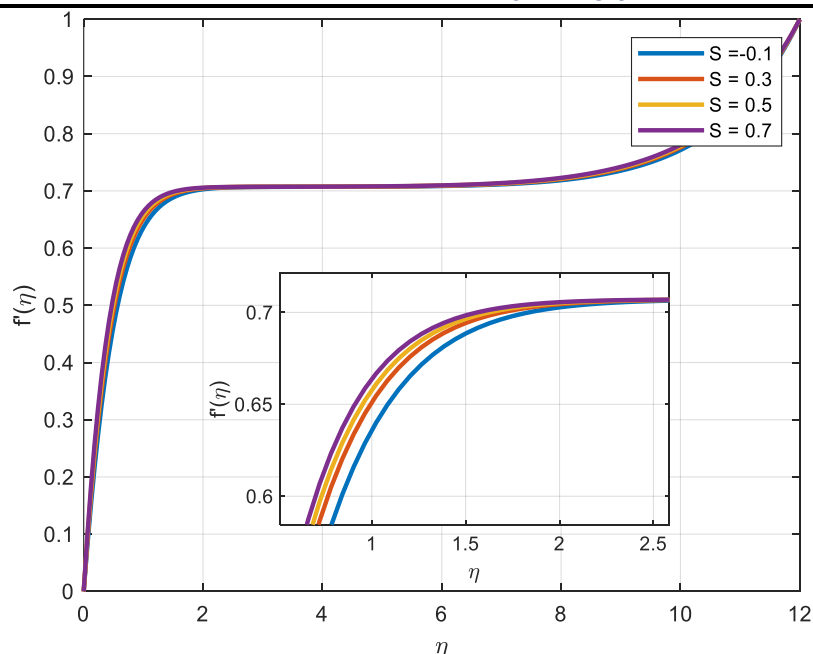


Figure 4: Velocity Profile for Different S

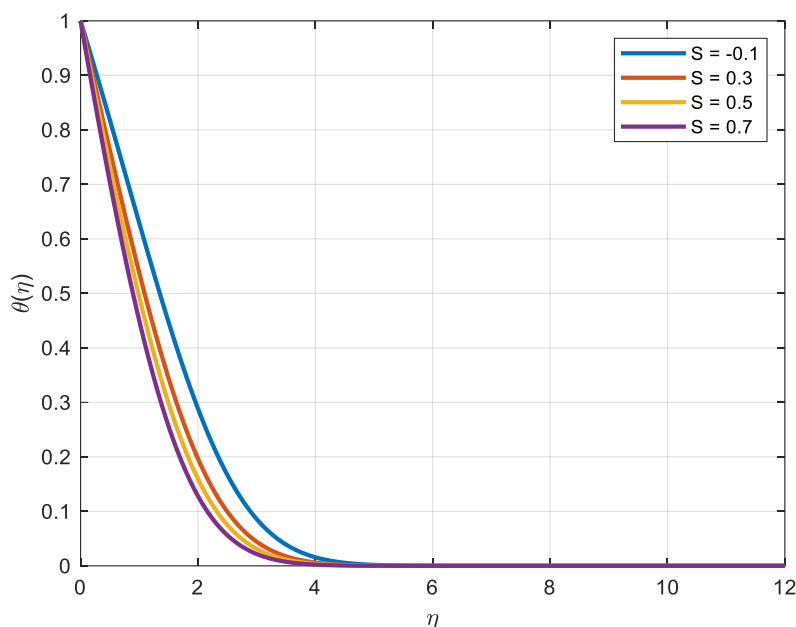


Figure 5: Temperature Profile for Different S.

Figures 4 and 5 illustrate the influence of the suction/injection parameter (S) on the velocity and temperature distributions. It is observed that increasing the suction strength (S) leads to a marked reduction in both velocity and temperature profiles, as the heated fluid is drawn closer to the porous surface, thereby narrowing the boundary layer thickness

Table 1: Effect of Suction Parameter S on Skin friction and heat transfer for fixed $M = 1, Pr = 0.71, \beta = 0.7$

S	$f''(0)$	$-\theta'(0)$
0.0	2.10993	0.39199
0.5	2.48172	0.45631
1.0	2.94785	0.52940
1.5	3.50861	0.61275
2.0	4.16429	0.70682

Observation:

- Skin friction increases with suction.
- Nusselt number increases significantly with suction.
- Boundary layer thickness decreases as S increases.

Table 2: Combined Effect of M and S for fixed $Pr = 0.71, \beta = 0.7$

M	S	$f''(0)$	$-\theta'(0)$
1	0	2.10993	0.39199
1	1	2.94785	0.52940
3	0	4.38131	0.43250
3	1	5.61277	0.60184

Observation:

- Skin friction increases with both magnetic parameter M and suction parameter S .
- Suction significantly reduces momentum and thermal boundary layer thickness.
- Heat transfer rate $-\theta'(0)$ increases as suction parameter increases.
- The combined effect of magnetic field and suction enhances overall thermal performance.

5. Conclusions

The combined influence of induced magnetic field and variable thermal conductivity on exponential power-law fluid flow has been analyzed. A numerical investigation of exponential power-law MHD flow over a semi-infinite vertical plate with induced magnetic field, variable thermal conductivity, and uniform suction has been presented. Major findings:

- Prandtl number does not affect velocity profile.
- Magnetic parameter enhances velocity and skin friction.
- Thermal conductivity parameter increases temperature and heat transfer rate.
- Suction significantly reduces boundary layer thickness.
- Heat transfer rate increases substantially with suction.

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