IJRAR.ORG E-ISSN: 2348-1269, P-ISSN: 2349-5138

INTERNATIONAL JOURNAL OF RESEARCH AND ANALYTICAL REVIEWS (IJRAR) | IJRAR.ORG

An International Open Access, Peer-reviewed, Refereed Journal

Clique on Special Type of Graphs and its Application in Traveling Sales man Problem

Bichitra Kalita,

Department of Computer Application,

Assam Don Bosco University, Guwahati -781017, Assam

Email:bichitra1_kalita@rediffmail.com, bichitra.kalita@dbuniversity.ac.in

Abstract: In this paper, the different sizes of clique of a special type of better connected graphs G (2n+8, 11n+13) for n≥1 have been discussed. The application of maximal clique of size K_m for m ≥5 with simultaneous changes of n≥1 has been studied to solve the traveling salesman problem. A heuristic algorithm has been proposed to find the least cost route when the graph is considered as weighted graph with three different conditions. The theoretical result relating to the existence of edge disjoint Hamiltonian circuit of the better connected graph G (2n+8, 11n+13) for n≥1 has also been discussed. In addition to this, it is shown that this graph is m≥4 better connected graph for simultaneous changes of n≥1 with a theorem.

Key words: Better connected graph**,** Clique, edge disjoint Hamiltonian circuit, TSP, Consecutive Weights**.**

1. INTRODUCTION:

There are many unsolved NP-Complete problems in graph theory. Some are going to be solved and some are yet to get solution even though many research works have been done in different corners of the world. Clique problem and traveling salesman problems are very important NP-Complete problems which are more applicable in real life. Generally, traveling salesman problems are related with weighted graph where weights are attached with the edges of the graph. The clique problems are not necessarily weighted graphs but in some cases, especially when one go to find out the maximum weighted clique , then the graph is considered as weighted graph. A clique is defined as a complete sub-graph of a graph. In addition to this, a clique problem is a problem to find out the maximal clique which is related with a social networks. We know that a social network is nothing but a graph where the vertices represent people and the edges represents mutual acquaintance. It is time consuming to find a maximum clique, when considered the clique, who all know each other in social network. It has also been found that the clique problems are related having many applications in bioinformatics and computational chemistry. It is important to note that bioinformatics has been becoming as an important interdisciplinary field, which contains biology, computer science, information technology. Now a days, the subject Mathematics and statistics have also been used to analyze and interpret the biological data. Some of the clique problems can be cited as Maximal clique a clique with maximum no of vertices, maximum weighted clique in a weighted graph, listing of all maximal cliques and testing whether a graph contains a clique larger than given size. On the other hand, the traveling salesman problems is also an NP-Complete problem, which is most important unsolved problem, related to many real life problems for any Hamiltonian graph.It has been found that the term clique was used by Luce & Perry in 1949 and thereafter by Harary & Ross [3]. Harary & Ross first used algorithm to solve the clique problem, related with the application of clique in the direction of social network. After the works of Harary, many researcher have been studying the clique problem with algorithmic approaches but not in complete theoretical nature during the year 1970. There are clique problems and quite a few of them are maximal clique containing largest possible number of vertices, weighted maximal clique for a weighted graph, listing all the maximal cliques of the graphs etc. Finding triangle of a graph or whether the graph has a triangle is a simple non-trivial clique problem. It has been found that a graph contains a triangle if and only if the adjacency matrix and the square of the adjacency matrix contain non-zero entries in the same cell and this property has been forwarded by Itai and Rodeh [2]. One algorithm has been developed by Tsukiyama, S etal [4] considering two conditions for maximal clique and clique obtained from the graph after deleting one vertex from it. Makino, K etal [7] found another algorithm to list all possible cliques in polynomial time for the families of graph such as chordal graphs, complete graphs, triangle free graphs, interval graphs etc. It has been found that many research has been done and is being done for clique problems by different authors and showed their finding for solution of various clique problems. Kalita, B [17] studied the various structure of non –isomorphic Hamiltonian sub –graph of complete graph K_{2m+2} for m ≥2 of the form H(2m+2,6m) for m ≥2 and it has been found that such structures of non –isomorphic Hamiltonian graph give some chemical structure of metal atom cluster compound of organic chemistry. In addition to it, an algorithm has been studied for traveling sales man problems. In the year 2006 Kalita B explained again the various the structure of complete graph K_{2m+3} for m ≥2 and found another algorithm for traveling sales man problem [16]. Kalita, B. etal [13] Studied the maximal triangle free graph and its application relating to the traveling sales man problem. Recently, Kalita, B. etal [14, 15] have been studied the minimum vertex cover where they studied the application of minimum vertex cover in finding the keyword based text summarization process.

In this article, we are going to study theoretical explanation for the existence of edge disjoint Hamiltonian circuit of a special type of better connected graph whose structure is G(2n+8,11n+13) for n≥1. The different structure of cliques have been studied for this graph. We study the process of solution of traveling sales man problem that is to find the least cost route when this graph is considered as weighted graph with a heuristic algorithm with three different conditions. Finally a theorem has been put forwarded that the graph G(2n+8,11n+13) is a m≥4 better connected graph with simultaneous changes of n≥1.

We now considered the following definition with some notations.

2. DEFINITIONS:

2. 1 Clique of a Graph: Let G (V, E) be a connected graph of V vertices and E edges. The clique of the graph G (V, E) is a complete sub-graph of the graph G (V, E). If V →∞ and E→∞, then the graph is called infinite graph. A graph is called simple if there is no self-loop and parallel edges.

2.2. Weighted Graph: A connected graph G (V, E) is called weighted graph if the weights W_i for i ≥1 are attached with all edges of the graph.

2.3. Neighborhood: Let G (V, E) be a connected graph. The neighborhood of a vertex A of the graph is a set of vertices incident with that vertex. It is denoted by N (A), where A € V.

2.4. Better connected graph: A connected graph G (V,E) is called a better connected graph if the graph remains connected after the deletion of maximum degree vertices of the graph one after another until the graph becomes a tree or a cycle.

2.5. m-Better connected graph: A better connected graph is called m-better connected graph if after deletion of m maximum degree vertices of the graph, the graph remains connected. In other ways we can say that m-better connected graph is a graph which contains m reduced sub-graph of the graph.

2.6. Reduced Graph: A Graph G(V,E) is called reduced sub-graph of a graph H(V,E) if it is obtained from the graph H(V,E) deleting a vertex from the graph H.

3. NEW STRUCTURE OF GRAPH: A new structure of better connected graph G (2n+8, 11n+13) for n≥1 is considered here and some properties of this graph has been discussed. Considering the following properties, we discuss the existence of edge disjoint Hamiltonian circuit and a heuristic algorithm with three different conditions to find the least cost route of traveling salesman problems when the graph is weighted graph and the weights are taken as consecutive even numbers 2n for n ≥1.

3.1. Property: The clique K_m of the better connected graph G (2n+8,11n+13) for m ≥5 with simultaneous changes of n≥1 is associated with the maximum degree vertex.

3.2. Property: The better connected graph G (2n+8,11n+13) for n≥1 always contains the cycle of various length such that C_3, C4, C5, C6, C7,……………………………………… C_l for l ≥3 for each m≥5 with simultaneous changes of n ≥1

3.3. Property: The degree of the vertices of the graph G (2n+8,11n+13) for n≥1 lie between 4 and 3n+4 for n ≥1 that is $4 \le d(V_i) \le 3n+4\}$,where $d(V_i)$ are the degrees of the vertices of the graph.

3.4. Property: This graph is not a triangle free graph.

3.5. Property: This graph is a simple graph but Hamiltonian.

3.6. Property: The structure of final reduced graph obtained from the graph G(2n+8,11n+13) for n≥1 with simultaneous changes of m≥4 is a circuit or a path according to even and odd values of n≥1.

3.7. Property: The graph G(2n+8,11n+13) for n ≥1 is a non-planar graph.

4. THEOREM FOR EDGE DISJOINT HAMILTONIAN CIRCUIT:

4.1.Theorem: A simple better connected graph G(2n+8,11n+13) having vertices of minimum degree 4 and one vertex of maximum degree 3n+4 for n ≥1 and degrees of other vertices lie between 4 and 3n+4 for n ≥1 has two edge disjoint Hamiltonian circuits and there exist maximal clique of size K_m for m ≥ 5 with simultaneous changes of n≥1.

Proof: We first forwards the proof of existence of two edge disjoint Hamiltonian circuit of the simple graph G (2n+8, 11n+13) having minimum degree four and one vertex of maximum degree 3n+4 for n ≥1, and all other vertices lie between 4 and 3n+4 for n ≥1. There after we study the existence of maximal Clique [a clique of having maximum number of vertices] Let V_1, V_2, V_3, V_4, V_5, V_6, ……………………………………………V_{2n+8} for n ≥1 are vertices and E_1, E_2,, E_3, E_4,, E_5, E_6, E_7, ……………E_{11n+13} for n ≥1 are the edges of the graph. Since the minimum degree is 4 and maximum degree is 3n+4 for n≥1 and hence the degrees of all other vertices lie between 4 and 3n+4 for n ≥1. Hence by property 3.5 this graph is a simple Hamiltonian graph and hence there is at least one Hamiltonian circuit. If possible we suppose that this graph has three edge disjoint Hamiltonian circuits. Since one Hamiltonian circuit covers the edges E_1, E_2,, E_3, E_4,, E_5, E_6, E_7, E8, E9, E10 for n =1 connecting every vertices exactly once where starting and ending vertex are same . When n=1, the graph of Figure-1 give ten vertices and 24 edges, where the vertices are A,B,C,D,E,F,G,H,I and J

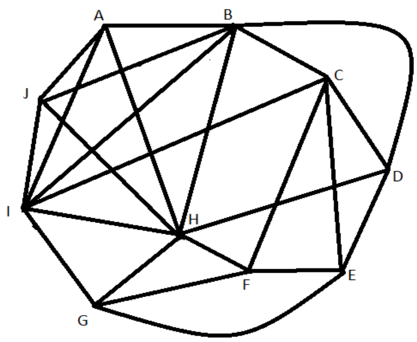

Figure-1 Graph of ten vertices and 24 edges

and the graph of Figure-1 has minimum degree 4 and maximum degree 7 [3n+4=7 when n=1] Again as the graph has 2n+8 number of vertices for n ≥1 and 11n+13 edges for n ≥1 and according to our supposition there are three Hamiltonian circuits. Therefore the minimum degree of the vertices of the graph must be at least 6, which contradict that the minimum degree of the graph is four. Hence there are two Hamiltonian circuits, when n=1. We know that the degree of the vertices of the graph lies between 4 and 3n+4 for n≥1and hence for all values of n≥1 there is at least one vertex of degree four which imply that there must have only two edge edge-disjoint Hamiltonian circuits.

We now prove that the graph G (2n+8, 11n+13) has a maximal clique of size K_m for m ≥5 with simultaneous changes of n ≥1. Now to have a clique of size K_m for m ≥5 in graph G (2n+8,11n+13),we must have at least one circuit of length C_5 C_6, C_7,…………………………………………C_m, connecting five vertices, sex vertices, seven vertices,…………..for m ≥5 with simultaneous changes of n ≥1, and the cycle of length C_3 and C_4 are not considerable here as our circuit starts from C_5 only because of the fact that we are going to find the clique of size K_5 when m=5. From the graph of Figure-1, when n=1 and m=5, we must consider the cycle C_5 connecting five vertices which contains the maximum degree vertex by the property 3.1 It is interesting to note that there may have other cycles of same length ≥5 in the graph G (2n+8, 11n+13) for n≥1 and hence in the process of selection of the cycle of length ≥5 from the graph to obtain the our required clique size K_m for m≥5.Let us find the following cycle of length =5 for n=1 and m=5 from figure-1 and they are from i to v.

i. A→B→H→I→J→A,
ii. B→C→F→H→J→B,
iii. A→H→G→I→J→A,
iv. D→E→F→G→H→D,
v. B→C→F→G→H→B.

Here we see that all above cycles from i to v touches the maximum degree vertex H. Hence we have to choose such a cycle out of these cycles from i to v which will give the graph of size K_5. It is observed that the cycles from ii to v do not give the structure of graph of size K_5 except the cycle i above even though they are cycles of length 5 touching the maximum degree vertex H. Therefore, considering the cycle C_5 of length five as stated in cycle i which connect the vertices A, B, H, I and J and it is seen that each vertices are connected to each other, which is nothing but the complete graph of degree four. That is there is a sub-graph K_5 of the graph G (2n+8, 11n+13) for n=1.Thus we see that the theorem is true for n=1. We now assume that the theorem is true for n=k≥1, and for m=l≥ 5. Hence our original graph takes the form as G (2k+8,11k+13). Now if we put n=k+1 and m=l+1,then the graph G becomes G(2(k+1)+8,11(k+1)+13) which implies G(2k+10,11k+24) and m=l+1. As we consider the value of n ≥1 and simultaneous changes of m≥5, Hence k+1≥1 implies k≥0 for simultaneous changes of m=l+1≥5 implies l≥4, which is true and this complete the proof.

4.2 Theorem: The better connected graph G (2n+8, 11n+13) for n≥1 is m-better connected graph for m ≥4 with simultaneous changes of n≥1.

Proof: It is known that the graph G (2n+8, 11n+13) for n≥1 has a vertex of maximum degree 3n+4 for n≥1 and the degrees of other vertices lie between 4 and 3n+4. We proceed to prove that this graph is m-better connected graph [having m reduced sub-graph] deleting the maximum degree vertex of the graph. The maximum degree of the graph G (2n+8, 11n+13) for n≥1 is 3n+4 for n ≥1. Now after deleting the vertex of maximum degree 3n+4 from the graph, we have the reduced sub- graph H (2n+7, 8n+9) of the graph G (2n+8, 11n+13) for n≥1 and the graph H (2n+7, 8n+9) is again a connected graph. Again deleting the maximum degree vertex of the graph H (2n+7, 8n+9) for n≥1, we have a connected reduced sub-graph say K (2n+6,8n+4) for ≥1. Continuing the same process, we have a connected reduced graph L(2n+5,8n) for n ≥1 from the graph K(2n+6,8n+4) for ≥1 after deleting the maximum vertex of K (2n+6,8n+4) for ≥1.Continuing the process of deletion of vertices of maximum degree of the reduced graphs we ultimately have the reduced sub-graph as a path of length 5,7,9 …………………connecting the vertices 6,8,10 ……….. for the values of n=1,3,5,………………….and a circuit of length 7,9,11 …………….connecting the vertices 7,9,11,………………………..for the values of n=2,4,6………………………….. We see that for n=1, there are four reduced sub-graph [m=4] where the last reduced graph is a path of length 5 with 6 vertices Figure-2 obtained from Figure-1 and for n=2, there are five reduced graph [m=5] where the last reduced graph is a circuit of length 7 connecting 7 vertices Figure-3, obtained from the graph G (2n+8, 11n+13) for n=2.

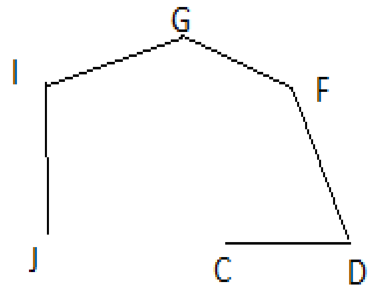

Figure-2 reduced graph of length5

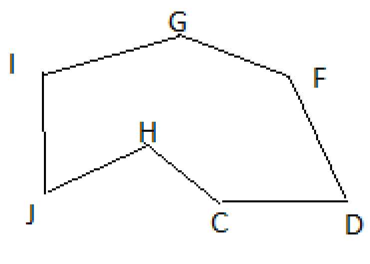

Figure-3 reduced graph a circuit of length 7

Hence for n=1, there are 4 reduced graph and for n=2, there are 5 reduced graph of the graph G(2n+8,11n+13). Hence for n=1 and n=2, theorem is true. We suppose that the theorem is true for n=k for simultaneous changes of m=q. Hence n=k+1 implies k >0 and m= q+1 implies q >3, when k+1≥1 and q+1≥4, which is true. Therefore applying the induction method we have there are m≥4 better connected graph for simultaneous changes of n≥1 and which complete the proof.

Remarks1: We see from the theorem 4.1 that the better connected graph G (2n+8, 11n+13) for n≥1 is a Hamiltonian graph having two edge disjoint Hamiltonian circuits. We now consider that the better connected graph G (2n+8, 11n+13) for n≥1 is a Weighted Hamiltonian graph and the weights of the edges are considered as consecutive even numbers starting from 2, which are randomly considered for the edges of the graph or follows a certain pattern discussed later with three conditions in the heuristic algorithm. Let the weights be denoted by W2, W4, W6,………..W_{2n} for n≥1, which indicates that all the weights are consecutive even numbers attached randomly with the edges of G, and consecutive smallest and consecutive greatest even weights are attached with the maximal clique obtained from the graph G(2n+8,11n+13) for n≥1in two conditions. Here we consider the weights are non-repeated. We now develop an algorithm for the weighted graph G (2n+8, 11n+13) for n≥1.

Remark 2: The better connected graph G (2n+8, 11n+13) for n≥1 contains clique of size K_3, K_4 and maximum clique of size Km for m ≥5.

5. HEURISTIC ALGORITHM FOR THREE DIFFERENT CASES OF TRAVELING SALESMAN PROBLEM:

INPUT: Let G (2n+8, 11n+13) for n≥1 be better connected weighted graph.

OUTPUT: Find the least cost route of G (2n+8, 11n+13) for n≥1.

Step1: Consider the non-repeated consecutive weights W2, W4, W6,………..W_{2n} for n≥1 which are attached with the edges of the weighted graph G (2n+8, 11n+13) .

Step2: Find the Clique K_m for m ≥5 of the graph G (2n+8, 11n+13) with simultaneous changes of n≥1.

Step3: If the smallest consecutive weights are attached with the clique K_m for m ≥5, then go to step4, otherwise go to step12.

Step4: Obtain a regular sub graph RG of degree m-1 for m ≥5 from the weighted graph G (2n+8, 11n+13) for n≥1.

Step5: Make a cost matrix of the regular sub graph as stated in Step4.

Step6: Obtained the row sum or column sum of the weights of the cost matrix of step 5.

Step7: Consider the initial vertex say A where it lies with minimum row sum or column sum and find the edge A→B. [That is the minimum weight from vertex A to B or B to A]

Step8: Find the neighborhoods of the vertices A and B.

[As the vertices of the regular graph RG [step4] are of degree m-1 for m ≥5 hence the neighborhoods of each vertex of RG will be a set of m-1 vertices for m ≥5, which are incident with the vertex].

Step9: Let neighborhoods of A be N(A) and where N(A)={ n+3 number of vertices for n ≥1 for simultaneous changes of m ≥5}. Similarly the neighborhoods of B is N(B) and N(B)={ n+3 number of vertices for n≥1 for simultaneous changes of m ≥5 [For n=1 and m=5 neighborhoods of each vertex will be four, that is there are four incident vertices with A or B]

Step10: Consider the smallest weighted edge which are incident with the vertex A and B

[Say N(A)={C,B,I,J} and N(B)={C,A,D,J} . Then B→A→J is a path of smallest weights this path is considerable path].

Step11: Continue the process till the selection of all vertices of the graph are considered which form Hamiltonian circuit.

Step12: Stop.

Step13: If the Greatest Consecutive weights attached with the clique K_m for m ≥5, then go to step14 otherwise go to step 21

Step14: Go to step4.

Step15: Go to step5.

Step16: Go to step6.

Step17: Take the average of row or column sum called AVRS or AVCS.

Step 18: Take the floor of AVRS or AVCS.

Step 19: Consider the vertices say A and B where the floor of AVRS or AVCS lies.

Step 20: Continue the steps from step8 to step 11

Step 21: Go to step 12.

Step22: If the weights are randomly attached with the edges of the graph then go to step23

Step23: Make a cost matrix of the regular sub graph as stated in Step5.

Step24. Find the neighborhood of each vertices of the regular sub graph of degree four (step5)

Step25. Take the intersection of neighborhood obtained from step24 [Pair wise]

Step26: The intersection always give a set having only two vertices, V_i and V_j

Step27: Find the minimum weight [mw] incident with V_i and V_J

Step28: Calculate the minimum weight [mw] of all vertices, using step24 and step25 and Step26.

Step29: Construct a sub-graph starting from the vertex V_i or V_j with the minimum weighted Edges obtained from step27

Step30: The sub-graph obtained from step29 will contain the 2n+4 number of minimum weighted edges for n≥1 for simultaneous changes of m≥5.

Step31: Form the Hamiltonian circuit with 2n+4 number of minimum weighted edges for n≥1.

Step32: The Hamiltonian circuit obtained from step31 is the least cos route.

Step 33: Go to step12.

6. EXPERIMENTAL RESULTS (CONDITION 1): We consider the weighted graph for n=1 and the smallest consecutive weights are attached to the edges of the maximal clique K_5 of the graph and they are 2, 4, 6……………………20. The following figure-4 is a weighted graph for m=5 and n=1 and the regular sub-graph of degree four is shown in figure-5. The table1 shows the cost matrix of the regular sub graph of degree 4.

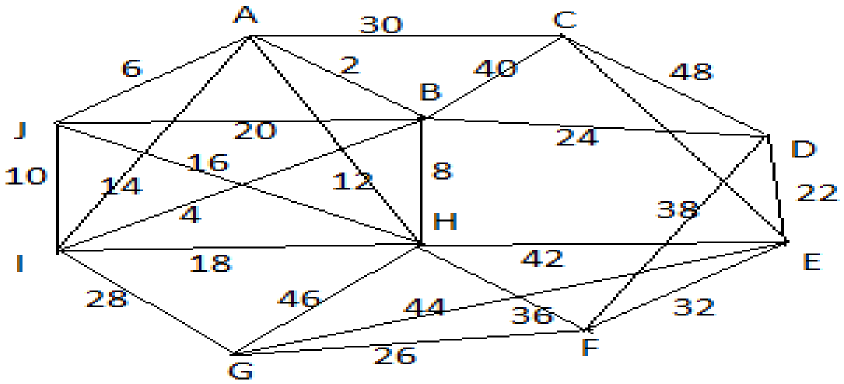

Figure-4 weighted graph for m=5 and n=1

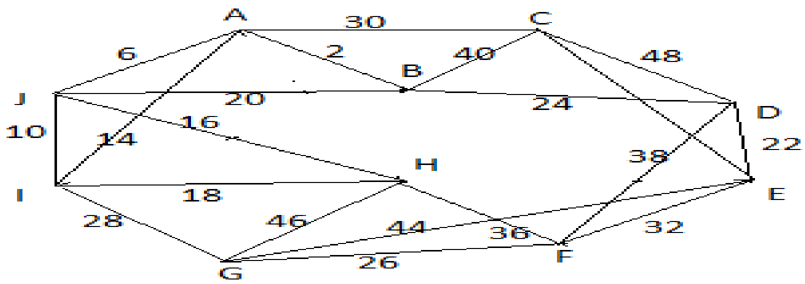

Figure-5 regular sub graph of degree four.

Table-1: Cost matrix of the regular sub graph of degree four:

	A	B	C	D	E	F	G	H	I	J	Row sum
A	&	2	30	&	&	&	&	&	14	6	52
B	2	&	40	24	&	&	&	&	&	20	86
C	30	40	&	48	34	&	&	&	&	&	152
D	&	24	48	&	22	38	&	&	&	&	132
E	&	&	34	22	&	32	44	&	&	&	132
F	&	&	&	38	32	&	26	36	&	&	132
G	&	&	&	&	44	26	&	46	28	&	144
H	&	&	&	&	&	36	46	&	18	16	116
I	14	&	&	&	&	&	28	18	&	10	70
J	6	20	&	&	&	&	&	16	10	&	52
Column sum	52	86	152	132	132	132	144	116	70	52	

Applying the step7 of the algorithm, it is found that the minimum weight of row sum /column sum is 52 hence we consider the initial vertex A or B and A or J as there are two minimum weights and A→B or B→A is of weight 2. Considering the other steps [step8 to step12] of the algorithm we have the minimum weighted Hamiltonian circuit B→A→J→I→H→F→G→E→C→D→B with weight 248.

CONDITION2: The following graph of Figure-6 is a weighted graph for m=5 and n=1 where the greatest consecutive weights from 30 to 48 are attached with the edges of the maximal clique K_5 and table-2 is a cost matrix of the regular graph of degree four obtained from the graph of figure-6.

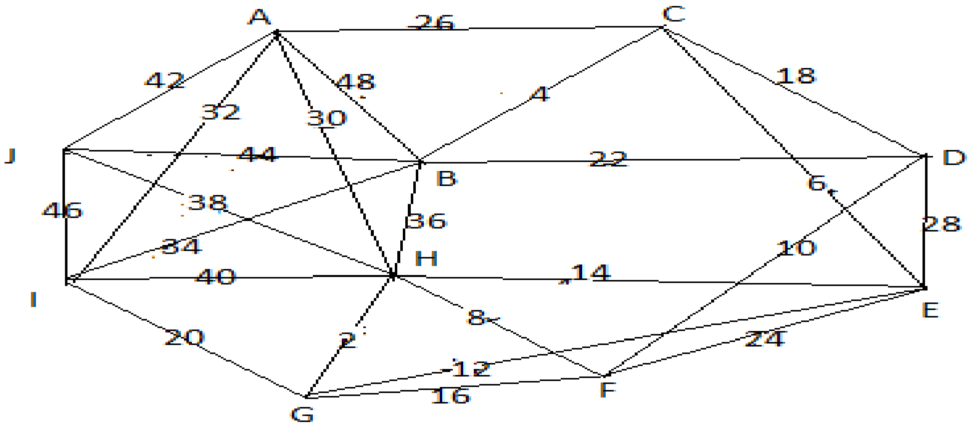

Figure-6

Table-2: Cost matrix of the regular sub graph of degree four:

	A	B	C	D	E	F	G	H	I	J	Rowsum
A	&	48	26	&	&	&	&	&	32	42	148
B	48	&	4	22	&	&	&	&	&	44	118
C	26	4	&	18	6	&	&	&	&	&	54
D	&	22	18	&	28	10	&	&	&	&	78
E	&	&	6	28	&	24	12	&	&	&	70
F	&	&	&	10	24	&	16	8	&	&	58
G	&	&	&	&	12	16	&	2	20	&	50
H	&	&	&	&	&	8	2	&	40	38	88
I	32	&	&	&	&	&	20	40	&	46	138
J	42	44	&	&	&	&	&	38	46	&	170
Colmsum	148	76	54	78	70	58	50	88	138	170	

Now using the steps from 13 to 21 we have the least cost route as E→F→H→G→I→A→J→B→C→D→E with cost 222. [floor of AVRS is 24 but ceiling is 25]

CONDITION3: The graph of Figure-7 is a weighted graph for m=5 and n=1 where the weights 2, 4,……………48 are attached with the edges of the graph randomly and the cost matrix of the regular sub-graph of degree four obtained from the graph of figure-7 is shown in table-3.

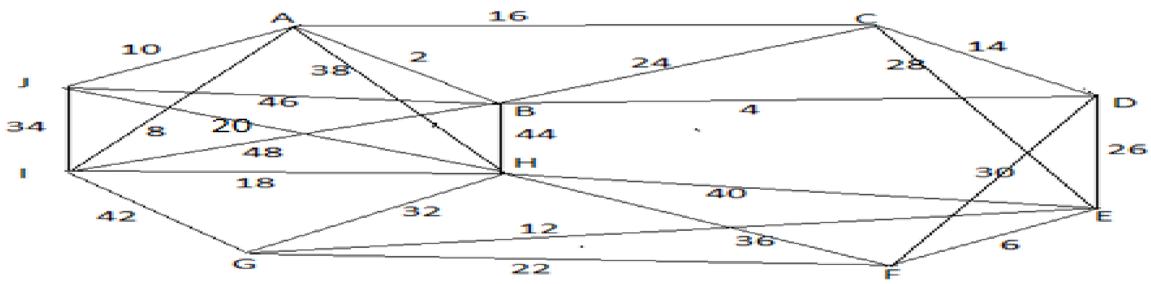

Figure-7

Table-3: Cost matrix of regular sub graph of degree four:

	A	B	C	D	E	F	G	H	I	J
A	&	2	16	&	&	&	&	&	8	10
B	2	&	24	4	&	&	&	&	&	46
C	16	24	&	14	28	&	&	&	&	&
D	&	4	14	&	26	30	&	&	&	&
E	&	&	28	26	&	6	12	&	&	&

F	&	&	&	30	6	&	22	36	&	&
G	&	&	&	&	12	22	&	32	42	&
H	&	&	&	&	&	36	32	&	18	20
I	8	&	&	&	&	&	42	18	&	34
J	10	46	&	&	&	&	&	20	34	&

Using the steps from 22 to 33, we have the least cost route is as A→B→D→C→E→F→G→H→I→J→A with 170.

Note1: It is interesting to note that one can consider the cost matrix of regular sub –graph of degree four when the smallest/ greatest consecutive weights [Table-1 and Table-2] may differ according to consideration of another pattern if exist.

Note2: This algorithmic procedure will give the least cost route when we add any constant weight to every edges of the graph or subtract any constant weight from the weight of the edges of the graph.

Note3: The weights of the edges of the better connected graph G (2n+8, 11n+13) for n≥1are of the form 2,4,6……..which are denoted by W2, W4, W6,………..W_{2n} for n≥1. Hence n=1 gives the consecutive weights which lies between 2 and 48, n=2 gives the consecutive weights which lies between 2 and 70, n=3 gives the consecutive weights which lies between 2 and 92 etc.

Notes: The algorithm is also applicable if the odd consecutive number is applied to every edges with same pattern .That is 1, 3, 5, 7………………. Consecutive odd numbers are considered with three conditions discussed in the algorithm.

CONCLUSION: The use of heuristic algorithm for the weighted better connected graph G (2n+8, 11n+13) for n≥1 gives a direction of solution of particular type of traveling salesman problem. The finding of edge disjoint Hamiltonian circuit of this graph with the help of a theoretical explanation is an important discussion. Finally the m-better connected for m ≥4 is found from another theoretical result for simultaneous changes of the values of n≥1.

References

[1]. Battiti, R. Protasi, M. “Reactive local search for the maximum clique problem”[2001], Algorithmica , 29(4), 610-637, doi:10.1007/s004530010074.

[2]. Itai, A; Rodeh,m; “ Finding a minimum circuit in a graph”[1978],SIAM journal on computing 7(4),413-423, doi: 10.1137/0207033.

[3]. Harary, F; Ross, I.C “ A procedure for clique detection using the group matrix [1957], Sociometry 20(3):205-215, doi:10.2307/2785673.

[4]. Tsukiyama, S; Ide,M; , Ariyoshi, I; Shirakawa, I; “ A new algorithm for generating all the maximal independent sets” [1977], SIAM journal on computing 6(3): 505-517, doi: 10.1137/0206036.

[5]. Lisenbrand, F; Grandoni, F; “ On the complexity of fixed parameter clique and dominating set”[2004]Theoretical computer Sc, 326(1-3) 57-67, doi:10.1016/j.tcs.2004.05.009.

[6]. Grosso, A; Locatelli, M; Dellacroce,F; “Combining swaps and node weights in an adaptive greedy approach for the maximal clique problem” [2004] Journal of Heuristics 10(2), 135-152, doi: 10.1023 /B:HEUR.0000026264.51747.7F

[7]. Makino, K; Uno, T; “ New algorithms for enumerating all maximal clique” [2004] Algorithm Theory, SWAT, 2004.

[8]. Katayama, K; Hamamoto,A; Narihisa, H; “ An effective local search for the maximum clique problem”[2005], Information processing letters, 95(5), 503-511, doi:10.1016/jipl.2005.05.010

[9]. Chem,Jianer; Huang,Xiuzhen; Kanj,Iyad.A; Xia,Ge; “ strong computational lower bounds via parameterized complexicity” [2006] , J.comput.syst.sci, 72(8): 1346-1367, doi: 10.1016/j.jcss.2006.04.007

[10]. Konc,J; Janezic,D; “An improved branch and bound algorithm for the maximum clique problem”[2007], Communications in Mathematical and Computer chemistry” 58(3) 569-590

[11]. Magniez,Frederic; Santha, Miklos; Szegedy, Mario “ Quantum algorithms for the triangle problem” [2007] SIAM Journal on computing 37(2) 413-424, doi:10.1137/050643684

[12]. Cazals,F; Karande,C; “A note on the problem of reporting maximal cliques” [2008] Theoretical Computer Sc. 407(1) 564-568, doi: 10.1016/j.tcs.2008.05.010

[13]. Kalita, B; etal “Maximal triangle Free Graph and Traveling Salesman Problem” [2014] International journal of Computer Application,vol.85, No.17,2014,pp 22-30.

[14]. Kalita, B; etal "Minimum Vertex Cover of Circulant Graph having Hamiltonian Path and its Application" [2016] Indian Journal of Science and Technology, Vol.32, pp1-7 DOI.10.17485/IJST/2016/09137/98001, OCT/2016.

[15]. Kalita, B; "Application of Minimum Vertex Cover for Keyword based Text Summarization Process" [2017] International Journal of Computational Intelligence Research Vol.13, No.1 pp113-125.

[16]. Kalita, B; "Sub Ghraph of Complete graph" International conference of foundation of computer science (FCS'06), PP71-77, Las Vegas 2006.

[17]. Kalita,B; " Non –ismorphic Hamiltonian Sub-graph of complete graph and their application" Proc. 92nd Indian science congress, Ahmedabad, 2005.

[18]. Kalita,B etal " Regular planar sub-graphs of complete graph and their application" International journal of applied Engineering Research, SSN0973-4562, Vol5 No.3(2010) pp377-386.