IJRAR.ORG E-ISSN: 2348-1269, P-ISSN: 2349-5138

INTERNATIONAL JOURNAL OF RESEARCH AND ANALYTICAL REVIEWS (IJRAR) | IJRAR.ORG

An International Open Access, Peer-reviewed, Refereed Journal

Sampling inspection plan for exponentially distributed quality characteristic and further improvement

[1]Satrendu Ray

[1]District Consultant Quality monitoring,
[1]Visva-Bharati University,
[1]CMOH Office Bankura, Bolpur, Birbhum

Abstract : In this article, an acceptance sampling inspection plan for variable is discussed when the quality characteristic follows some non-normal distributions, viz., Akash, Sujatha or Amarendra distribution. In this plan, the lot quality is to be measured by the lot mean (μ). The plan parameters have been obtained by the two point approaches: the acceptable quality level (AQL) and the limiting quality level (LQL) or the rejectable quality level(RQL). For the given producer's risk (α) and consumer's risk (β), the plan parameters: sample size (n) and acceptance number (c) are determined for different choices of AQL and LQL. A comparison study has been made in terms of the sample size (n) and some examples are provided to illustrate the performance of the proposed plan. Further modification is done by introducing a new OPPE family distribution, Devya distribution and comparing with Lognormal distribution. Moreover, the sampling inspection plan for Devya is also stated here.

Index Terms - Acceptance Sampling Plan, Statistical Process Control, Non-Normal Distributions, Akash Distribution, Sujatha Distribution, Amarendra Distribution, OPPE Family, Devya Distribution, Lognormal Distribution, AQL, LQL, RQL, Producer's Risk (α), Consumer's Risk (β), Sample Size, Acceptance Number, Quality Inspection

1 Introduction

Product quality is a necessity for the business to thrive in a global market. It is important to differentiate and then select among competitive enterprises. Statistical process control and acceptance sampling plan are used for monitoring the quality of a product. Acceptance sampling inspection plans are very helpful for the industrialist to yield high-quality assurance. An important aspect of acceptance sampling is to make the decision: either to accept or to reject a concerned lot of products based on a quality characteristic which is observed from the given lot. In an acceptance sampling inspection plan, a consumer has to decide whether to accept or reject a submitted lot of products on the basis of a sample taken from the lot. This technique is important in industries as it is often not feasible to carry out 100% inspection of all the products of a lot owing to time, cost and risks regarding product liability, etc. Consumer's risk (β) and producer's risk (α) are the probabilities that a bad lot is accepted and a good lot is rejected by consumer respectively. For more details about these risks, one may refer to the books introduced by Duncan [1] and Stephens [2]. Acceptance sampling inspection plan can be broadly classified into two: acceptance sampling by attributes and that by variables. Variable sampling plan uses accurate measurements of quality characteristic for decision-making rather than classifying it as conforming or non-conforming in attribute sampling plans. Recently, several authors studied the development of acceptance sampling by variables and notable among them are Kantam et al. [3,4]; Balakrishnan et al. [5]; Lio et al. [6]; Rao et al. [7]; Aslam et al. [8,9].

The probability density function of the random variable X in a one parameter polynomial exponential (abbreviation, OPPE) family can be written as :

$$f_X(x,\theta) = h(\theta)p(x)\exp(-\theta x), \theta > 0,\ x > 0, \tag{1.1}$$

where, $h(\theta) = \frac{1}{\sum_{k=0}^{r} a_k \frac{\Gamma(k+1)}{\theta^{k+1}}}$, $p(x) = \sum_{k=0}^{r} a_k x^k$.

The distribution can also be written as :

$$f_X(x,\theta) = h(\theta)\sum_{k=0}^{r} a_k x^k \exp(-\theta x) = \frac{\sum_{k=0}^{r} a_k \frac{\Gamma(k+1)}{\theta^{k+1}} f_{GA}(x; k+1, \theta)}{\sum_{k=0}^{r} a_k \frac{\Gamma(k+1)}{\theta^{k+1}}}, \tag{1.2}$$

where $f_{GA}(x; k+1, \theta)$ is the pdf of a gamma distribution with shape parameter $k+1$ and scale parameter θ, and a_k's are non-negative constants. The distribution is a finite mixture of $r+1$ gamma distributions.

Special cases are :

(a) $r = 0,\ a_0 = 1$ gives the exponential distribution,

(b) $r = 2,\ a_0 = 1,\ a_1 = 0,\ a_2 = 1$ gives the Akash distribution ([10]),

(c) $r = 2,\ a_0 = 1,\ a_1 = 1,\ a_2 = 1$ gives the Sujatha distribution ([11]),

(d) $r = 3,\ a_0 = 1,\ a_1 = 1,\ a_2 = 1,\ a_3 = 1$ gives the Amarendra distribution ([12]),

All the distributions, viz., Akash ([10]), Sujatha ([11]) and Amarendra ([12]) other than the exponential distribution, are the finite mixtures of exponential and gamma distributions, have shape and scale parameters whereas the exponential distribution has only a scale parameter. Shape parameter makes all three distributions (Akash, Sujatha and Amarendra) more flexible in comparison with the exponential distribution. Mixture distributions have good statistical properties and have widespread applications.

Fitting of lifetime data by exponential distribution is more popular in the industry. However, in sampling inspection plan, the number of observations to be inspected is important from cost and time aspect.

In the previous project work ([13]) we have come to know that the exponential fitting is sometimes not worthwhile whereas some other flexible distribution like Lindley, xgamma, Akash, etc, are more appropriate. Furthermore, we have seen that Akash distribution works in a more efficient way in terms of sample size rather than the exponential, Lindley and xgamma distributions. Now, here we are in a hunch for some new distributions, which can be shown superior to Akash distribution.

So we have considered two more distributions viz Sujatha and Amarendra. Under the assumption of these flexible distributions, the industry may be benefited by inspection of a small number of items and hence saving time in the inspection and consequently reduce the cost.

In this article, we have chalked out sampling inspection plan for Akash, Sujatha and Amarendra distributed quality characteristics and find out the optimum choices of sample size 'n' and acceptance number 'c'. Also comparison between Lognormal (2-parameters) and another OPPE family distribution, Devya distribution has been shown. After that sampling inspection plan for Devya distribution is presented. By appropriate fitting of distribution for the quality characteristics, we have ample scope to inspect less number of items/units if originally the characteristics are not exponentially distributed. We have tabulated some optimum choices of 'n' and 'c' that may be useful to the practitioners and quality engineers.

The objective of the article is as follows: firstly, an acceptance sampling inspection plan for variable has been introduced by using the Akash ([10]), Sujatha ([11]), Amarendra ([12]) distributed quality characteristics, where the lot quality is to be measured by the lot mean. The second objective is to compare the proposed acceptance sampling inspection plan for the characteristic following the above distributions. Furthermore, we are in a hunch for an another OPPE family distribution which is more efficient and fit better than Lognormal distribution (2-parameters).

Rest of the article is organised as follows: In section 2, we have described the acceptance sampling inspection plan for variable and lot sentencing criteria. Also, we have formulated the operating characteristic (OC) function, producer's risk and consumer's risk of the plan. In section 3, we have given the estimation procedure for the plan parameters: sample size (n) and acceptance number (c) using two point approach, the acceptance quality level (AQL) and limiting quality level (LQL). In section 4, the application of the proposed sampling inspection plan has been presented through two real life examples and their concluding remarks. Furthermore, in sections 5 and 6, we have introduced a new OPPE family distribution through a real life example, which fits better than Lognormal distribution (2-parameters). Finally, the overall conclusions are given in section 7.

2 Acceptance sampling inspection plan for variable

Let Y be the item quality measure under any distributional assumption and lot quality is measured by the lot mean (μ). Now, the acceptance sampling inspection plan for variable is as follows:

Inspect a random sample of size n from the lot with quality characteristic following a specified probability distribution with unknown mean μ. We consider that larger the value of μ better is the quality of the lot. Measure the quality characteristic Y_i, the lifetime of the i^{th} unit and calculate the sample mean $\bar{Y}$, where,

$$\bar{Y} = \frac{\sum_{i=1}^{n} Y_i}{n}.$$

The acceptance sampling inspection plan is based on the selection of c, the acceptance number, such that

- Accept the lot if $\bar{Y} \geq c$, otherwise reject.

Generally, in acceptance sampling inspection plan under any distributional assumption, we have to estimate the plan parameters n, the sample size, and c, the acceptance value of the quality. The OC function of the plan is

$$\begin{aligned} L(\mu) &= P(\bar{Y} \geq c|\mu) \\ &= P\left(\sum_{i=1}^{n} Y_i \geq nc|\mu\right) \\ &= 1 - P\left(\sum_{i=1}^{n} Y_i < nc|\mu\right). \end{aligned} \tag{2.3}$$

In an acceptance sampling plan, a submitted lot may be either accepted or rejected on the basis of sampled items from the lot. The probability of rejecting a good lot by the customer under the plan is called the producer's risk and is denoted by α; while the probability of accepting a bad lot by the customer under the plan is called the consumer's risk which is denoted by β. The producer's risk (α) and the consumer's risk (β) for the plan are given as

$$\begin{aligned} \alpha &= P(\text{Rejecting a lot}|\mu = \mu_0) \\ &= P(\bar{Y} < c|\mu = \mu_0) \qquad (2.4) \\ &= P\left(\sum_{i=1}^{n} Y_i < nc|\mu = \mu_0\right), \qquad (2.5) \end{aligned}$$

and

$$\begin{aligned} \beta &= P(\text{Accepting a lot}|\mu = \mu_1) \\ &= P(\bar{Y} > c|\mu = \mu_1) \qquad (2.6) \\ &= P\left(\sum_{i=1}^{n} Y_i > nc|\mu = \mu_1\right), \qquad (2.7) \end{aligned}$$

where μ_0 and μ_1 are AQL and LQL respectively.

3 Estimation procedure of the plan parameters

The consumer often establishes a sampling plan for a continuous supply of raw materials with reference to an AQL. AQL represents the minimum level of quality for the supplier's process that the consumer would consider to be acceptable at a process

average. Note that AQL is a property of the supplier's manufacturing process, not a property of the sampling plan. The consumer will often design the sampling procedure so that the OC curve gives a high probability of acceptance at AQL. The consumer will also be interested in the other end of the OC curve i.e., in the protection that is obtained for individual lots of poor quality. In such a situation, the consumer may establish a LQL. LQL is the minimum level of quality that the consumer is willing to accept in an individual lot. It is to be noted that, LQL is also not a characteristic of the sampling plan, but is a level of lot quality, specified by the consumer and it is possible to design sampling plan that gives specified probability of acceptance at LQL. Subsequently, the plan parameters are determined so as to minimize the sample size n such that α is satisfied at AQL(μ_0) and β is satisfied at LQL(μ_1). Thus, the following non-linear optimization problem can be considered:
Minimize n, subject to

$$L(\mu_0) \geq 1 - \alpha \tag{3.8}$$

and

$$L(\mu_1) \leq \beta. \tag{3.9}$$

Suppose, Y be the early mentioned quality characteristic distributions with scale parameter θ respectively. The PDF, OC function, producer's risk, and consumer's risk for the considered models of the plan, are given in the sequel.

3.1 Akash distribution

Shanker (10) introduced a finite mixture of exponential (θ) and gamma $(3, \theta)$ distributions with mixing proportion $\pi_1 = \frac{\theta^2}{2+\theta^2}$ and $\pi_2 = 1 - \pi_1 = \frac{2}{2+\theta^2}$ to obtain a probability distribution, named as Akash distribution. The PDF of Akash distribution with parameter θ is given by

$$\begin{aligned} f(y;\theta) &= \frac{\theta^3}{(2+\theta^2)}\left(1+y^2\right)e^{-\theta y} \quad ; y > 0,\ \theta > 0 \\ &= 0 \qquad\qquad\qquad\qquad\qquad ;\text{otherwise.} \end{aligned} \tag{3.10}$$

The mathematical, structural and survival properties are investigated and found that in many cases, the Akash distribution is more flexible than the exponential distribution. The population mean of the Akash distribution is $\mu = \frac{(\theta^2+6)}{\theta(\theta^2+2)}$.

Theorem: If $Y_1,\ Y_2,\ ...,\ Y_n$ be a random sample of size n from the Akash distribution with parameter θ, then the pdf of $Y = \sum_{i=1}^{n} Y_i$ is

$$g(y;n,\theta) = \sum_{k=0}^{n} P'_{(k,n)}(\theta).f_{(GA)}(y;3n-2k,\theta),$$

where,

$$P'_{(k,n)} = \binom{n}{k}\frac{2^{n-k}\theta^{2k}}{(2+\theta^2)^n},$$

and

$$f_{(GA)}(y;3n-2k,\theta) = \frac{\theta^{(3n-2k)}}{\Gamma(3n-2k)}y^{(3n-2k)-1}e^{-\theta y},$$

is the PDF of gamma distribution with shape and scale parameters $(3n-2k)$ and θ respectively.

The OC function under the assumption that the quality characteristic follows the Akash distribution is given by

$$L(\theta) \ = \ 1 - \sum_{k=0}^{n}\binom{n}{k}\frac{2^{n-k}\theta^{2k}}{(2+\theta^2)^n}\Gamma(\theta nc, 3n-2k). \tag{3.11}$$

The producer's risk (α) and the consumer's risk (β) are determined from OC function and the plan parameters n and c are calculated following non-linear optimisation technique, stated above.

3.2 Sujatha distribution

Shanker (11) introduced a finite three component mixture of an exponential distribution with scale parameter θ, a gamma distribution with shape parameter 2 and scale parameter θ, and a gamma distribution with shape parameter 3 and scale parameter θ with their mixing proportions $\frac{\theta^2}{\theta^2+\theta+2}$, $\frac{\theta}{\theta^2+\theta+2}$ and $\frac{2}{\theta^2+\theta+2}$, respectively. Therefore the PDF of a new one-parameter lifetime distribution, Sujatha distribution can be introduced as :

$$f(y,\theta) = \frac{\theta^3}{\theta^2+\theta+2}(1+y+y^2)e^{-\theta y}; y>0, \theta>0 \tag{3.12}$$

The population mean of the Sujatha distribution is $\mu = \frac{\theta^2+2\theta+6}{\theta(\theta^2+\theta+2)}$

Theorem: If Y_1, Y_2, ..., Y_n be a random sample of size n from the Sujatha distribution with parameter θ, then the PDF of $Y = \sum_{i=1}^{n} Y_i$ is

$$g(y;n,\theta) = \sum_{k_1}\sum_{k_2}^{n}\sum_{k_3} P''_{(k_i,n)}(\theta).f_{(GA)}(y;3n-2k_1-k_2,\theta),$$

with $\sum_{i=1}^{3} k_i = n$ where,

$$P''_{(k_i,n)} = \frac{n!}{k_1!k_2!k_3!} \times 2^{k_3} \times \frac{\theta^{(n+k_1-k_3)}}{(\theta^2+\theta+2)^n},$$

and

$$f_{(GA)}(y;(3n-2k_1-k_2),\theta) = \frac{\theta^{(3n-2k_1-k_2)}}{\Gamma(3n-2k_1-k_2)} y^{[(3n-2k_1-k_2)-1]} e^{-\theta y},$$

is the PDF of gamma distribution with shape and scale parameters $(3n-2k_1-k_2)$ and θ respectively.

The OC function under the assumption that the quality characteristic follows the Sujatha distribution is given by

$$L(\theta) \;=\; 1 - \sum_{k_1}\sum_{k_2}^{n}\sum_{k_3} \frac{n!}{k_1!k_2!k_3!} \times 2^{k_3} \times \frac{\theta^{(n+k_1-k_3)}}{(\theta^2+\theta+2)^n} \times \Gamma(\theta nc,(3n-2k_1-k_2)) \quad (3.13)$$

with $\sum_{i=1}^{3} k_i = n$

The producer's risk (α) and the consumer's risk (β) are determined from OC function and the plan parameters n and c are calculated following non-linear optimisation technique, stated above.

3.3 Amarendra distribution

Shanker (12) introduced a distribution by considering a four component mixture of exponential (θ), a gamma (2,θ), a gamma (3,θ) and a gamma (4,θ) with their mixing proportions $\frac{\theta^3}{\theta^3+\theta^2+2\theta+6}$, $\frac{\theta^2}{\theta^3+\theta^2+2\theta+6}$, $\frac{2\theta}{\theta^3+\theta^2+2\theta+6}$ and $\frac{6}{\theta^3+\theta^2+2\theta+6}$ respectively. Therefore the PDF of a new one-parameter lifetime distribution, Amarendra distribution can be introduced as :

$$f(y,\theta) = \frac{\theta^4}{\theta^3+\theta^2+2\theta+6}(1+y+y^2+y^3)e^{-\theta y}; y>0, \theta>0 \quad (3.14)$$

The population mean of the Amarendra distribution is $\mu = \frac{\theta^3+2\theta^2+6\theta+24}{\theta(\theta^3+\theta^2+2\theta+6)}$

Theorem: If $Y_1,\ Y_2,\ ...,\ Y_n$ be a random sample of size n from the Amarendra distribution with parameter θ, then the PDF of $Y = \sum_{i=1}^{n} Y_i$ is

$$g(y;n,\theta) = \sum_{k_1}\sum_{k_2}^{n}\sum_{k_3}\sum_{k_4} P^{'''}_{(k_i,n)}(\theta).f_{(GA)}(y;(3n-2k_1-k_2+k_4),\theta),$$

with $\sum_{i=1}^{4} k_i = n$ where,

$$P^{'''}_{(k_i,n)} = \frac{n!}{k_1!k_2!k_3!k_4!} \times 2^{k_3} \times 6^{k_4} \times \frac{\theta^{(n+2k_1+k_2-k_4)}}{(\theta^3+\theta^2+2\theta+6)^n},$$

and

$$f_{(GA)}(y;(3n-2k_1-k_2+k_4),\theta) = \frac{\theta^{(3n-2k_1-k_2+k_4)}}{\Gamma(3n-2k_1-k_2+k_4)} y^{[(3n-2k_1-k_2+k_4)-1]} e^{-\theta y},$$

is the PDF of gamma distribution with shape and scale parameters $(3n-2k_1-k_2+k_4)$ and θ respectively.

The OC function under the assumption that the quality characteristic follows the Amarendra distribution is given by

$$\begin{aligned} L(\theta) &= 1 - \sum_{k_1}\sum_{k_2}^{n}\sum_{k_3}\sum_{k_4} \frac{n!}{k_1!k_2!k_3!k_4!} \times 2^{k_3} \\ &\times \ 6^{k_4} \times \frac{\theta^{(n+2k_1+k_2-k_4)}}{(\theta^3+\theta^2+2\theta+6)^n} \times \Gamma(\theta nc, (3n-2k_1-k_2+k_4)) \end{aligned} \tag{3.15}$$

with $\sum_{i=1}^{4} k_i = n$

The producer's risk (α) and the consumer's risk (β) are determined from OC function and the plan parameters n and c are calculated following non-linear optimisation technique, stated above.

We have presented plan parameter values (n, c) in Table 1-5. For two limits of μ (sample mean) i.e μ_0 (AQL) and μ_1 (LQL), we get θ_0 and θ_1 and for these specific values of θ_0 and θ_1 at different level of α and β, we obtain the values of plan parameters (n, c).

Here the objective is to minimize the risk from producer's side of rejecting a good lot or from the consumer's side of accepting a bad lot. For example in table 5, for $\mu_0 = 70$, $\mu_1 = 35$, the (n, c) values are (8, 50.86); (10, 50.23) and (10, 50.37) for Amarendra, Sujatha and Akash distributed quality characteristics respectively, whereas in table 1, the corresponding (n, c) values are (12, 48.54); (16, 48.49); (16, 48.64) respectively.

Therefore, proper model selection of the quality characteristics plays an important role in serving cost and time in use of sampling inspection plan for variable. If the quality characteristic data fits the Amarendra or the Sujatha distribution better than the Akash distribution, then it will have a great advantage to us for using these instead of the Akash distribution (we have already seen that the Akash distribution is more efficient than exponential (generally done in industry), referred in the previous project work). The advantages are in the sense that an organization will be benifitted in terms of cost and time.

Table: 1
Values of n and c in Amarendra, Akash, Sujatha Distribution
Alpha = 0.01; beta= 0.01

μ=50		Amarendra				Akash				Sujatha			
μ_0	μ_1	θ_0	θ_1	n	c	θ_0	θ_1	n	c	θ_0	θ_1	n	c
70	30	0.05686	0.13171	8	44.32	0.04283	0.09967	11	44.77	0.04253	0.09814	11	44.60
70	35	0.05686	0.11311	12	48.54	0.04283	0.08550	16	48.64	0.04253	0.08436	16	48.49
70	40	0.05686	0.09911	18	52.15	0.04283	0.07486	24	52.23	0.04253	0.07397	24	52.11
60	40	0.06628	0.09911	34	48.61	0.04996	0.07486	45	48.63	0.04955	0.07397	46	48.66
60	35	0.06628	0.11311	20	45.41	0.04996	0.08551	26	45.31	0.04955	0.08436	26	45.20
60	30	0.06628	0.13171	12	41.59	0.04996	0.09967	16	41.68	0.04955	0.09814	16	41.5
μ=72													
90	50	0.04428	0.07944	16	65.81	0.03332	0.05993	22	66.24	0.03313	0.05935	22	66.13
92	50	0.04331	0.07944	15	66.55	0.03256	0.05993	20	66.70	0.03242	0.05935	20	66.50
92	52	0.04331	0.07641	17	67.97	0.03256	0.05763	23	68.27	0.03242	0.05710	23	68.08
94	52	0.04240	0.07641	16	68.74	0.03191	0.05763	21	68.63	0.03173	0.05710	22	69.07
μ=14													
16.5	10.5	0.23664	0.36520	29	13.05	0.17989	0.27860	38	13.06	0.17531	0.26889	40	13.07
16.75	10.5	0.23320	0.36520	27	13.13	0.17725	0.27860	35	13.12	0.17279	0.26889	37	13.14
17.5	10.75	0.22348	0.35716	25	13.59	0.16980	0.27241	33	13.61	0.16568	0.26306	34	13.58
17.75	10.75	0.22041	0.35716	24	13.71	0.16746	0.27241	31	13.69	0.16344	0.26306	32	13.66

Table: 2
Values of n and c in Amarendra, Akash, Sujatha Distribution
Alpha = 0.02; beta= 0.01

μ=50		Amarendra				Akash				Sujatha			
μ_0	μ_1	θ_0	θ_1	n	c	θ_0	θ_1	n	c	θ_0	θ_1	n	c
70	30	0.05686	0.13171	7	45.45	0.04283	0.09967	9	45.16	0.04253	0.09814	10	46.13
70	35	0.05686	0.11311	11	49.96	0.04283	0.08550	14	49.62	0.04253	0.08436	14	49.48
70	40	0.05686	0.09911	16	53.13	0.04283	0.07486	21	53.08	0.04253	0.07397	21	52.97
60	40	0.06628	0.09911	30	49.23	0.04996	0.07486	39	49.15	0.04955	0.07397	40	49.20
60	35	0.06628	0.11311	17	45.93	0.04996	0.08551	23	46.09	0.04955	0.08436	23	45.99
60	30	0.06628	0.13171	11	42.81	0.04996	0.09967	14	42.53	0.04955	0.09814	14	42.39
μ=72													
94	54	0.04239	0.07359	16	71.40	0.03190	0.05550	21	71.30	0.03173	0.05501	22	71.68
94	56	0.04239	0.07098	18	72.61	0.03190	0.05351	24	72.67	0.03173	0.05306	25	72.97
96	56	0.04152	0.07098	17	73.54	0.03124	0.05351	23	73.77	0.03108	0.05306	23	73.66
μ=14													
16	10	0.24381	0.38242	24	12.74	0.18539	0.29183	31	12.72	0.18056	0.28137	33	12.74
18	10	0.21743	0.38242	15	13.44	0.16517	0.29183	20	13.49	0.16125	0.28137	21	13.49
20	12	0.19618	0.32166	20	15.59	0.14890	0.24514	26	15.58	0.14566	0.23729	27	15.56

Table: 3
Values of n and c in Amarendra, Akash, Sujatha Distribution
Alpha = 0.05; beta= 0.01

μ=50		Amarendra				Akash				Sujatha			
μ_0	μ_1	θ_0	θ_1	n	c	θ_0	θ_1	n	c	θ_0	θ_1	n	c
70	30	0.05686	0.13171	6	48.15	0.04283	0.09967	8	48.25	0.04253	0.09814	8	48.10
70	35	0.05686	0.11311	9	51.88	0.04283	0.08550	11	51.22	0.04253	0.08436	11	51.09
70	40	0.05686	0.09911	13	54.75	0.04283	0.07486	17	54.68	0.04253	0.07397	17	54.58
60	40	0.06628	0.09911	24	50.23	0.04996	0.07486	32	50.28	0.04955	0.07397	33	50.35
60	35	0.06628	0.11311	14	47.37	0.04996	0.08551	18	47.21	0.04955	0.08436	19	47.44
60	30	0.06628	0.13171	9	44.45	0.04996	0.09967	11	43.89	0.04955	0.09814	11	43.77
μ=72													
96	56	0.04152	0.07098	14	75.84	0.03124	0.05352	18	75.55	0.03108	0.05306	18	75.46
96	58	0.04152	0.06855	16	77.08	0.03124	0.05167	21	77.00	0.03108	0.05125	21	76.90
98	58	0.04068	0.06855	15	78.08	0.03060	0.05167	19	77.67	0.03045	0.05125	19	77.55
98	60	0.04068	0.09911	5	64.82	0.03060	0.04996	22	79.03	0.03045	0.04956	22	78.93
100	60	0.03986	0.09911	5	66.16	0.02999	0.04996	20	79.75	0.02984	0.04956	20	79.66
100	62	0.03986	0.06416	18	81.38	0.02999	0.04835	23	81.05	0.02984	0.04797	23	80.96
μ=14													
16	10	0.24381	0.38242	19	13.02	0.18539	0.29183	25	13.04	0.18056	0.28137	26	13.02
16.5	10	0.23664	0.38242	17	13.26	0.17989	0.29183	22	13.25	0.17531	0.28137	23	13.25
16.5	10.5	0.23664	0.36520	21	13.57	0.17989	0.27860	27	13.56	0.17531	0.26889	28	13.54
16.5	11	0.23663	0.34944	26	13.86	0.17989	0.26649	33	13.83	0.17531	0.25746	35	13.84
17	11.5	0.22987	0.33498	28	14.37	0.17470	0.25538	36	14.36	0.17036	0.24696	38	14.37
17.5	11.5	0.22348	0.33498	24	14.59	0.16980	0.25538	31	14.58	0.16568	0.24696	33	14.61

Table: 4
Values of n and c in Amarendra, Akash, Sujatha Distribution
Alpha = 0.02; beta= 0.02

μ=50		Amarendra				Akash				Sujatha			
μ_0	μ_1	θ_0	θ_1	n	c	θ_0	θ_1	n	c	θ_0	θ_1	n	c
70	30	0.05686	0.13171	7	45.45	0.04283	0.09967	9	45.16	0.04253	0.09814	9	44.99
70	35	0.05686	0.11311	10	49.08	0.04283	0.08550	12	48.15	0.04253	0.08436	13	48.78
70	40	0.05686	0.09911	14	52.06	0.04283	0.07486	19	52.28	0.04253	0.07397	19	52.16
60	40	0.06628	0.09911	27	48.67	0.04996	0.07486	35	48.58	0.04955	0.07397	36	48.64
60	35	0.06628	0.11311	15	45.09	0.04996	0.08551	20	45.16	0.04955	0.08436	21	45.38
60	30	0.06628	0.13171	10	42.05	0.04996	0.09967	12	41.26	0.04955	0.09814	13	41.79

Table: 5
Values of n and c in Amarendra, Akash, Sujatha Distribution
Alpha = 0.05; beta= 0.02

μ=50		Amarendra				Akash				Sujatha			
μ_0	μ_1	θ_0	θ_1	n	c	θ_0	θ_1	n	c	θ_0	θ_1	n	c
70	30	0.05686	0.13171	5	46.27	0.04283	0.09967	7	46.88	0.04253	0.09814	7	46.73
70	35	0.05686	0.11311	8	50.86	0.04283	0.08550	10	50.37	0.04253	0.08436	10	50.23
70	40	0.05686	0.09911	11	53.50	0.04283	0.07486	15	53.75	0.04253	0.07397	15	53.64
60	40	0.06628	0.09911	21	49.59	0.04996	0.07486	28	49.64	0.04955	0.07397	29	49.73
60	35	0.06628	0.11311	12	46.41	0.04996	0.08551	16	46.48	0.04955	0.08436	16	46.38
60	30	0.06628	0.13171	8	43.58	0.04996	0.09967	10	43.16	0.04955	0.09814	10	43.03

4 Example

In this section, two examples with real life data set is given to illustrate the proposed acceptance-sampling plans.

(i) The data given arisen in tests on endurance of deep groove ball bearings ([14], p.228). The data are the number of million revolutions before failure for each of the 23 ball bearings in life test and they are: 17.88, 28.92, 33.00, 41.52, 42.12, 45.60, 48.80, 51.84, 51.96, 54.12, 55.56, 67.80, 68.44, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04, 173.40.

(ii) This data set is the strength data of glass of the aircraft window reported by Fuller et al.,28 : 18.83 ,20.8, 21.657, 23.03, 23.23, 24.05, 24.321, 25.5, 25.52, 25.8, 26.69, 26.77, 26.78, 27.05, 27.67, 29.9, 31.11, 33.2, 33.73, 33.76, 33.89, 34.76, 35.75, 35.91, 36.98, 37.08, 37.09, 39.58, 44.045, 45.29, 45.381.

At first we have checked whether the considered data set is well fitted with the Amarendra, the Sujatha or the Akash distribution by goodness-of-fit test. For this purpose we have used the Akaike Information Criterion (AIC = -2log(likelihood) + $2k$, k is the number of parameters) to verify the goodness-of-fit. The model with the lowest value of AIC could be chosen as the best model to fit the data. Since, the distribution considered involve only one parameter, the best model is to be chosen on the basis of negative-log-likelihood. From table 6, we find the Amarendra distribution is the best fit to the data as the negative-log-likelihood is the least. The histogram and fitted distributions for example (i) and (ii) have been shown in Figure 1 and Figure 2.

Now using Table 3, we summarize the plan for these data set in Table 7. Though the decision is same for all distributional assumptions, the sample size is the least for the Amarendra distribution (that is better fit in this case) than others. OC curve for different distributions have been presented in Figure 3 and Figure 4, they are as expected, identical.

Table 6 : Comparision among the Amarendra,Akash & Sujatha

*******Example(i)*******		
Distribution	**Estimate of θ**	**Negative Log-likelihood**
Amarendra	0.0551094	113.0342
Akash	0.0415156	113.5291
Sujatha	0.0410332	113.5881
*******Example(ii)*******		
Amarendra	0.2811250	103.9212
Akash	0.2140000	103.9819
Sujatha	0.2077852	104.0897

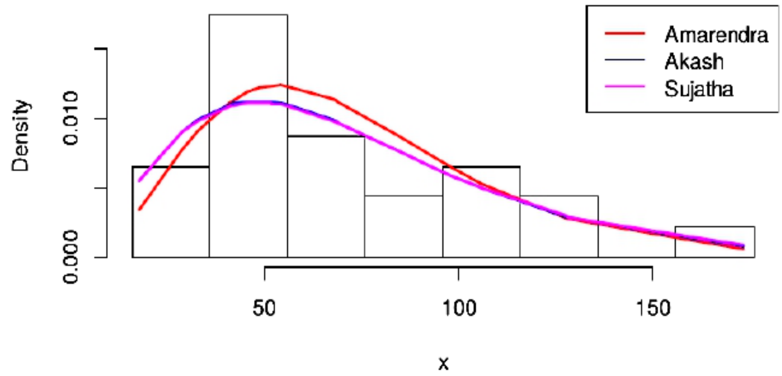

Figure 1: Histogram and fitted distributions for Example(i)

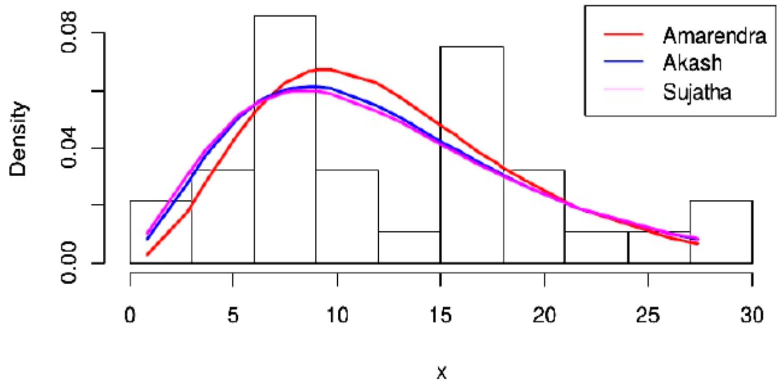

Figure 2: Histogram and fitted distributions for Example(ii)

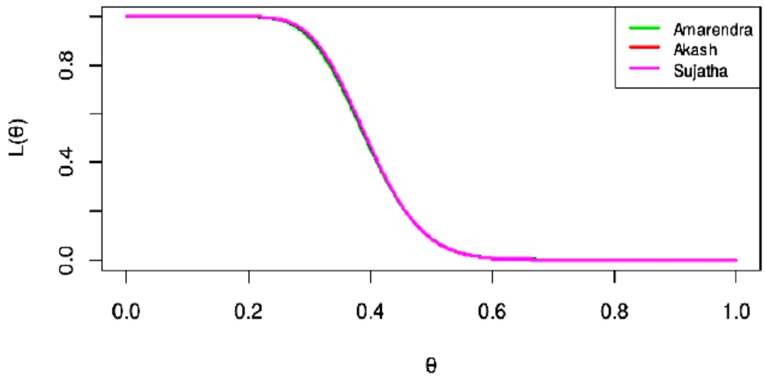

Figure 3: Operating characteristic curve for the plan based on Example(i)

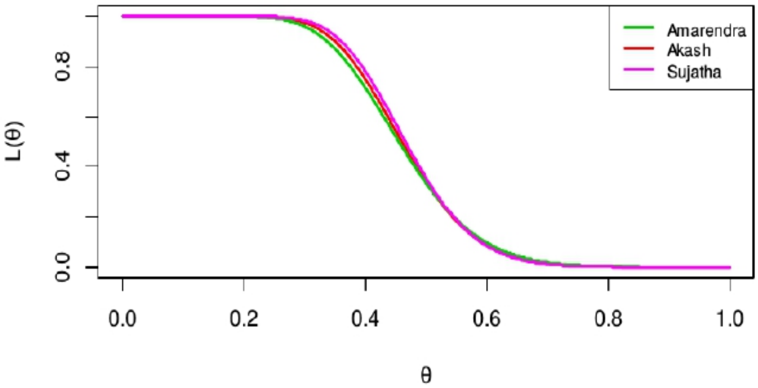

Figure 4: Operating characteristic curve for the plan based on Example(ii)

Table 7 : Data analysis at level $\alpha = 0.05$ and $\beta = 0.01$

*******Example(i)*******				
Distribution	**n**	**c**	$\overline{Y}$	**Decision**
Amarendra	7	67.42	36.83	Reject the lot
Akash	10	68.36	41.58	Reject the lot
Sujatha	10	68.24	41.58	Reject the lot
*******Example(ii)*******				
Amarendra	8	10.79	5.68	Reject the lot
Akash	10	10.71	6.27	Reject the lot
Sujatha	11	10.79	6.58	Reject the lot

4.1 Remarks

Based on the points discussed till now, it can be stated that the Amarendra distribution fits better (least AIC value) than Akash and Sujatha distribution in view of acceptance sampling inspection plan, Amarendra distribution works very effectively than aforesaid distributions in terms of sample size (n). Also the steepness of the OC curves for the distributions are as expected.

5 Introduction of a new OPPE family distribution and comparison with Lognormal distribution

Now from (1.2) we get,

(e) $r = 4,\ a_0 = 1,\ a_1 = 1,\ a_2 = 1,\ a_3 = 1,\ a_4 = 1$ gives the Devya distribution ([15]),

This is also a finite mixture of exponential and gamma distributions.

5.1 Devya distribution

Shanker (15) introduced a five component mixtures of exponential (θ) distribution, a gamma (2, θ) distribution, a gamma (3, θ) distribution, a gamma (4, θ) distribution and a gamma (5, θ) distribution with their mixing proportions $\frac{\theta^4}{\theta^4+\theta^3+2\theta^2+6\theta+24}$, $\frac{\theta^3}{\theta^4+\theta^3+2\theta^2+6\theta+24}$, $\frac{2\theta^2}{\theta^4+\theta^3+2\theta^2+6\theta+24}$, $\frac{6\theta}{\theta^4+\theta^3+2\theta^2+6\theta+24}$ and $\frac{24}{\theta^4+\theta^3+2\theta^2+6\theta+24}$ respectively. Therefore, the PDF of the new one-parameter lifetime distribution, Devya distribution can be given by :

$$f(y,\theta) = \frac{\theta^5}{\theta^4+\theta^3+2\theta^2+6\theta+24}(1+y+y^2+y^3+Y^4)e^{-\theta y}; y>0, \theta>0 \qquad (5.16)$$

The population mean of the Devya distribution is $\mu = \frac{\theta^4+2\theta^3+6\theta^2+24\theta+120}{\theta(\theta^4+\theta^3+2\theta^2+6\theta+24)}$

Theorem: If $Y_1,\ Y_2,\ ...,\ Y_n$ be a random sample of size n from the Devya distribution with parameter θ, then the PDF of $Y = \sum_{i=1}^{n} Y_i$ is

$$g(y;n,\theta) = \sum_{k_1}\sum_{k_2}^{n}\sum_{k_3}\sum_{k_4}\sum_{k_5} P''''_{(k_i,n)}(\theta).f_{(GA)}(y;(3n-2k_1-k_2+k_4+2k_5),\theta),$$

with $\sum_{i=1}^{5} k_i = n$ where,

$$P''''_{(k_i,n)} = \frac{n!}{k_1!k_2!k_3!k_4!k_5!} \times 2^{k_3} \times 6^{k_4} \times 24^{k_5} \times \frac{\theta^{(n+3k_1+2k_2+k_3-k_5)}}{(\theta^4+\theta^3+2\theta^2+6\theta+24)^n},$$

and

$$f_{(GA)}(y;(3n-2k_1-k_2+k_4+2k_5),\theta) = \frac{\theta^{(3n-2k_1-k_2+k_4+2k_5)}}{\Gamma(3n-2k_1-k_2+k_4+2k_5)} y^{[(3n-2k_1-k_2+k_4+2k_5)-1]}e^{-\theta y},$$

is the PDF of gamma distribution with shape and scale parameters $(3n-2k_1-k_2+k_4+2k_5)$ and θ respectively.

The OC function under the assumption that the quality characteristic follows the Devya distribution is given by

$$L(\theta) = 1 - \sum_{k_1}\sum_{k_2}^{n}\sum_{k_3}\sum_{k_4}\sum_{k_5} \frac{n!}{k_1!k_2!k_3!k_4!k_5!} \times 2^{k_3} \times 6^{k_4} \times 24^{k_5}$$
$$\times \frac{\theta^{(n+3k_1+2k_2+k_3-k_5)}}{(\theta^4+\theta^3+2\theta^2+6\theta+24)^n} \times \Gamma(\theta nc,(3n-2k_1-k_2+k_4+2k_5)) \qquad (5.17)$$

with $\sum_{i=1}^{5} k_i = n$

The producer's risk (α) and the consumer's risk (β) are determined from OC function and the plan parameters n and c are calculated following non-linear optimisation technique, stated above.

We have presented plan parameter values (n, c) in Table 8-12. For two limits of μ (sample mean) i.e μ_0 (AQL) and μ_1 (LQL), we get θ_0 and θ_1 and for these specific values of θ_0 and θ_1 at different level of α and β, we obtain the values of plan parameters (n, c).
Here, our objective is to show Devya distribution fits better than Lognormal distribution and also to minimize the risk from producer's side of rejecting a good lot or from the consumer's side of accepting a bad lot.

Table: 8
Values of n and c in Devya Distribution
Alpha = 0.01; beta= 0.01

μ=50		Devya			
μ_0	μ_1	θ_0	θ_1	n	c
70	30	0.07116	0.16518	7	45.35
70	35	0.07116	0.14177	10	48.97
70	40	0.07116	0.12418	15	52.51
60	40	0.08297	0.12418	27	48.59
60	35	0.08297	0.14177	16	45.44
60	30	0.08297	0.16518	10	41.96
μ=68					
90	50	0.05539	0.09948	13	66.02
92	50	0.05419	0.09948	12	66.57
92	52	0.05419	0.09567	14	68.30
94	52	0.05305	0.09567	13	68.94

Table: 9
Values of n and c in Devya Distribution
Alpha = 0.02; beta= 0.01

μ=50		Devya			
μ_0	μ_1	θ_0	θ_1	n	c
70	30	0.07116	0.16518	6	46.23
70	35	0.07116	0.14177	9	50.20
70	40	0.07116	0.12418	13	53.28
60	40	0.08297	0.12418	24	49.24
60	35	0.08297	0.14177	14	46.14
60	30	0.08297	0.16518	9	43.01
μ=68					
94	54	0.05304	0.09215	13	71.58
94	56	0.05304	0.08887	15	73.02
96	56	0.05194	0.08887	14	73.87

Table: 10
Values of n and c in Devya Distribution
Alpha = 0.05; beta= 0.01

μ=50		Devya			
μ_0	μ_1	θ_0	θ_1	n	c
70	30	0.07116	0.16518	5	48.59
70	35	0.07116	0.14177	7	51.67
70	40	0.07116	0.12418	10	54.49
60	40	0.08297	0.12418	20	50.44
60	35	0.08297	0.14177	11	47.28
60	30	0.08297	0.16518	7	44.28
μ=68					
96	56	0.05194	0.08887	11	75.69
96	58	0.05194	0.08582	13	77.24
98	58	0.05089	0.08582	12	78.10
98	60	0.05089	0.08297	13	78.84
100	60	0.04987	0.08297	12	79.70
100	62	0.04987	0.08031	14	81.14

Table: 11
Values of n and c in Devya Distribution
Alpha = 0.02; beta= 0.02

μ=50		Devya			
μ_0	μ_1	θ_0	θ_1	n	c
70	30	0.07116	0.16518	5	44.23
70	35	0.07116	0.14177	8	49.11
70	40	0.07116	0.12418	11	51.94
60	40	0.08297	0.12418	21	48.54
60	35	0.08297	0.14177	12	45.12
60	30	0.08297	0.16518	8	42.08

Table: 12
Values of n and c in Devya Distribution
Alpha = 0.05; beta= 0.02

μ=50		Devya			
μ_0	μ_1	θ_0	θ_1	n	c
70	30	0.07116	0.16518	4	46.30
70	35	0.07116	0.14177	6	50.31
70	40	0.07116	0.12418	9	53.70
60	40	0.08297	0.12418	17	49.66
60	35	0.08297	0.14177	10	46.70
60	30	0.08297	0.16518	6	43.11

6 Example

In this section, an example with real life data set is given to illustrate the proposed acceptance-sampling plans.
The data is given by Birnbaum and Saunders16 on the fatigue life of 6061 T6 aluminum coupons cut parallel to the direction of rolling and oscillated at 18 cycles per second.

The data set consists of 101 observations with maximum stress per cycle 31,000 psi. The data are presented below (after subtracting 65) 5, 25, 31, 32, 34, 35, 38, 39, 39, 40, 42, 43, 43,43, 44, 44, 47, 47, 48, 49, 49, 49, 51, 54, 55, 55,55, 56, 56, 56, 58, 59, 59, 59, 59, 59, 63, 63, 64,64, 65, 65, 65, 66, 66, 66, 66, 66, 67, 67, 67, 68,69, 69, 69, 69, 71, 71, 72, 73, 73, 73, 74, 74, 76,76, 77, 77, 77, 77, 77, 77, 79, 79, 80, 81, 83, 83,84, 86, 86, 87, 90, 91, 92, 92, 92, 92, 93, 94, 97,98, 98, 99, 101, 103, 105, 109, 136, 147.

At first we have checked whether the considered data set is well fitted with the Devya or the Lognormal distribution by goodness-of-fit test. For this purpose we have used the Akaike Information Criterion (AIC = -2log(likelihood) + $2k$, k is the number of parameters) to verify the goodness-of-fit. The model with the lowest value of AIC could be chosen as the best model to fit the data. From table 13, we find the Devya distribution is the best fit to the data as the negative-log-likelihood is the least. The histogram, fitted distributions and OC curve have been shown in Figure 5 and Figure 6.

Table 13: Comparison between Devya & Lognormal Distribution

Distribution	Estimate of θ	Negative log-likelihood
Lognormal	Mu_hat=4.1579 Sigma_hat=0.4112	468.7945
Devya	0.072906	462.1305

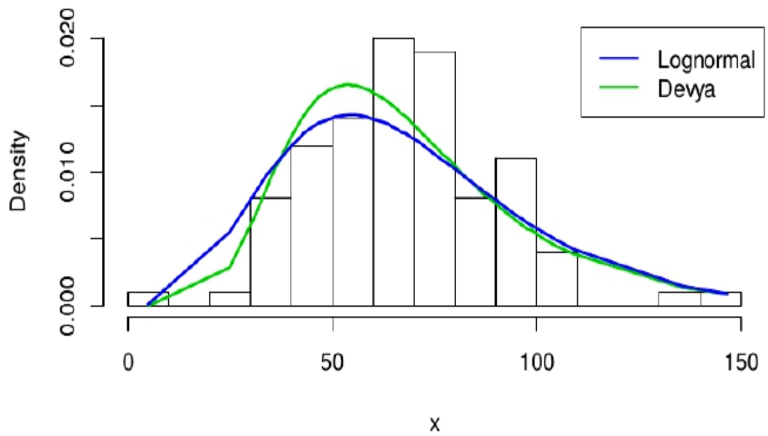

Figure 5: Histogram and fitted distributions for given Example

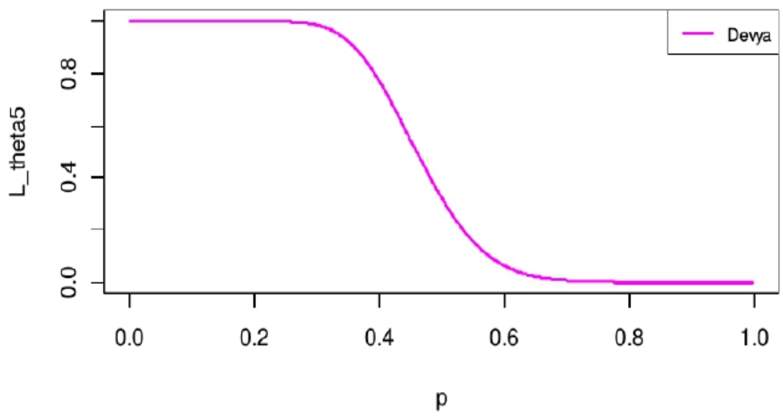

Figure 6: Operating characteristics curve for the plan based on above example

Table: 14
Data analysis for Devya Distribution
Alpha = 0.05; beta= 0.01

n	c	$\overline{Y}$	Decision
11	75.69	32.73	Reject the lot
13	77.24	34.31	Reject the lot
12	78.10	33.58	Reject the lot

6.1 Remarks

Here we have found that Devya distribution (OPPE family) fits better (least AIC value) than Lognormal distribution (2-parameters). The OC curve for Devya distribution is as expected, steeper. This is also evident to say that Divya distribution works better than Amarendra in view of sampling inspection plan.

7 Concluding Remarks

In this article, we have discussed the acceptance sampling inspection plan for variable for the Amarendra, Sujatha and Akash distributed quality characteristics. Estimation of the plan parameters are based on two- point OC curve approach, the acceptable quality level (AQL) and the limiting quality level (LQL). In comparison study, we have observed that the acceptance sampling inspection plan for the Amarendra distribution works in a more efficient way in terms of sample size rather than the Akash and the Sujatha distributions. In other words, plan for the Amarendra distribution required smaller sample size to draw a verdict about acceptance or rejection of the lot based on the acceptance sampling rather than the Akash and the Sujatha distributions. We have also found that, once again Akash distribution works in a profound way than Sujatha distribution as in this case most of the time we have seen Akash distribution needs smaller (or sometimes equal) sample size compared to Sujatha distribution, for drawing a judgement about acceptance or rejection of the lot based on the acceptance sampling plan. Tables of plan parameters have been obtained through computational technique at various values of $[AQL(\mu_0),\ \alpha]$, $[LQL(\mu_1),\ \beta]$. Also, the sampling plan has been explained through real-life examples.

Moreover, we have found that most of the real-life data sets fit good with OPPE family distributions rather than two parameters distributions (e.g Lognormal distribution). We expect that the industrial engineers can implement effectively the acceptance sampling approach to non-normal quality data.

***Acknowledgement*:**

The satisfaction and euphoria on the successful completion of any task would be incomplete without mentioning the people who made it possible, whose constant guidance and encouragement crowned out effort with success. I would like to express my heartfelt gratitude to my supervisor Prof. Dr. Sudhansu S. Maiti for his immense support, guidance, valuable suggestions, and encouragement throughout in the preparation of this project. Special thanks to my elder sister Dr. Poulami Roy for his kind co-operation during this journey. I am grateful to the Department of Statistics, Visva-Bharati for giving me the opportunity to execute this project, which is an integral part of the curriculum in M. Sc programme at the Visva- Bharati University, Santiniketan. Many Thanks to those who directly or indirectly helped me in my project work by giving their generous contribution towards enriching the quality of the work. This acknowledgement would not be complete without expressing my sincere gratitude to my parents and my sister for their love, patience, encouragement and understanding which are the source of my motivation and inspiration throughout my work.

References

[1] Duncan, A. J. (1986). Quality Control and Industrial Statistics. Homewood, USA: Richard D Irwin.

[2] Stephens, K. S. (2001). The Handbook of Applied Acceptance Sampling: Plans, Procedures and Principles. Milwaukee, WI: ASQ Quality Press.

[3] Kantam, R. R. L., Rosaiah, K., Rao, G. S. (2001). Acceptance sampling based on life tests: Log-logistic model. Journal of applied statistics, 28(1), 121-128.

[4] Kantam, R. R. L., Srinivasa Rao, G., Sriram, G. (2006). An economic reliability test plan: log-logistic distribution. Journal of Applied Statistics, 33, 291-296.

[5] Balakrishnan, N., Leiva, V., Lopez, J. (2007). Acceptance sampling plans from truncated life tests based on the generalized BirnbaumSaunders distribution. Communications in Statistics -Simulation and Computation, 36(3), 643-656.

[6] Lio, Y. L., Tsai, T. R., Wu, S. J. (2010). Acceptance sampling plans from truncated life tests based on the Burr type XII percentiles. Journal of the Chinese institute of Industrial Engineers, 27(4), 270-280.

[7] Rao, G. S. (2013). Acceptance sampling plans for percentiles based on the Marshall-Olkin extended Lomax distribution. Journal of Statistics and Economics, 11, 83-96.

[8] Aslam, M., Kundu, D., Ahmad, M. (2010). Time truncated acceptance sampling plans for generalized exponential distribution. Journal of Applied Statistics, 37(4), 555-566.

[9] Aslam, M., Azam, M., Jun, C. H. (2017). A new sampling plan under the exponential distribution. Communications in Statistics -Theory and Methods, 46(2), 644-652.

[10] Shanker, R. (2015). Akash distribution and Its Applications, International Journal of Probability and Statistics, 4(3), 65-75.

[11] Shanker, R. (2016 b). Sujatha distribution and Its Applications, Statistics in Transition, 17 (3), 391-410.

[12] Shanker, R. (2016 b). Amarendra distribution and Its Applications, American Journal of Mathematics and Statistics, 6 (1), 44-56.