



Numerical Investigation of MHD Boundary Layer Flow for Power-Law Fluids

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Abstract: This study investigates the laminar boundary layer flow of a non-Newtonian power-law fluid over a continuously moving surface under the influence of a transverse magnetic field. Using a group transformation technique, the governing partial differential equations are reduced to nonlinear ordinary differential equations, enabling a similarity solution approach. The resulting system of equations is solved numerically using MATLAB's ODE solver, the influence of three key parameters the magnetic parameter M , the velocity ratio ϵ , and the power-law index n on the velocity profile $f'(\eta)$ in the boundary layer of a non-Newtonian fluid over a moving surface. Through numerical simulations and graphical analysis, it is observed that increasing the magnetic parameter or power-law index enhances momentum diffusion, leading to faster fluid acceleration near the surface. Similarly, higher wall-to-fluid velocity ratios result in a quicker approach to the asymptotic velocity, indicating stronger surface-driven momentum transfer. The study highlights the interplay of magnetic, rheological, and kinematic factors in shaping boundary layer flow behavior.

Keywords: Group transformation technique, Magnetohydrodynamics, Boundary layer flow, Transverse magnetic field, Graphical representation.

I. INTRODUCTION

Magnetohydrodynamics is the study of the dynamics of electrically conducting fluids in the presence of a magnetic field. Such fluids include plasmas, liquid metals, saltwater, and electrolytes. The importance of MHD lies in its broad application in astrophysics, nuclear fusion, geophysics, metallurgy, and various engineering processes such as MHD generators, cooling systems for nuclear reactors, and the design of magnetic drug targeting in biomedical applications.

One of the most critical aspects of MHD flow behavior occurs near solid surfaces where boundary layers are formed. These boundary layers significantly influence the heat and momentum transfer characteristics of the fluid and are crucial in practical engineering design. The interaction of magnetic fields with conducting fluids modifies the flow pattern and thermal properties due to the Lorentz force, which acts as a drag force opposing the motion of the fluid. This phenomenon is fundamental to understanding and controlling the flow dynamics in applications involving electromagnetic fields.

While classical Newtonian fluids obey a linear stress-strain relationship, many fluids of practical interest do not follow this behavior. Instead, they exhibit non-Newtonian characteristics, where viscosity varies with the shear rate. Examples include blood, ketchup, polymer solutions, paints, and slurries. Among the models developed to characterize non-Newtonian behavior, the power-law model is one of the simplest and most widely used. It describes shear-thinning (pseudo-plastic), Newtonian, and shear-thickening (dilatant) fluids depending on the power-law index.

In recent years, there has been a growing interest in investigating MHD boundary layer flow of power-law fluids because many industrial processes involve both non-Newtonian fluids and magnetic fields. Examples include polymer processing, MHD pumps, magnetic control of molten metals, and cooling systems. The presence of a magnetic field introduces an additional level of complexity in modeling and analyzing such flows, especially when the governing equations are nonlinear and do not admit analytical solutions. Hence, numerical investigation becomes an essential tool for analyzing MHD boundary layer flows in power-law fluids.

Timol, M. G. [1], investigation focuses on the magnetohydrodynamic boundary layer flow of power-law fluids near a suddenly accelerated flat plate. The study applies transformation techniques, including linear and spiral group transformations, to derive similarity solutions for the governing equations. By analyzing the effects of the power-law index and magnetic field strength, the research provides insights into the velocity profiles and flow behavior of non-Newtonian fluids under MHD conditions.

Khan, M., Khan W. A. [2], studied on the magnetohydrodynamic (MHD) boundary layer flow of a power-law nanofluid over a nonlinear stretching sheet. The research examines the effects of a transverse magnetic field on the flow behavior and employs similarity transformations to convert the governing boundary layer equations into a system of ordinary differential equations. A newly proposed boundary condition requiring zero nanoparticle mass flux is incorporated into the analysis, and the resulting equations are solved numerically using the shooting method. The study provides insights into the variations of dimensionless temperature and nanoparticle concentration under different parameter conditions, with numerical results presented in graphical and tabular formats.

Abel M. S. [3], investigates the flow and heat transfer characteristics of a power-law fluid over a stretching sheet with variable thermal conductivity and a non-uniform heat source. The study employs similarity transformations to convert the governing partial differential equations into nonlinear ordinary differential equations. Numerical solutions reveal the effects of thermal conductivity variations and heat source parameters on fluid behavior and heat transfer efficiency.

Chen, C. H. [4], investigates the magnetohydrodynamic flow and heat transfer characteristics of a viscoelastic fluid, incorporating the effects of energy dissipation and an internal heat source. The study derives an analytic solution for the governing equations, providing insights into the influence of various physical parameters on fluid behavior. By applying mathematical techniques, the research enhances the understanding of heat transfer mechanisms in electrically conducting viscoelastic fluids, which are relevant to industrial and engineering applications.

Dash G.C. [5], studied focuses on the boundary layer stagnation-point flow past a stretching or shrinking sheet in a power-law fluid. Using numerical methods, the research examines the effects of stretching and shrinking parameters on velocity and temperature distributions. The findings provide insights into fluid behavior under varying flow conditions, contributing to advancements in computational fluid dynamics.

Mahantesh B. [6], investigates the heat transfer characteristics in magnetohydrodynamic flow of a power-law fluid over a stretching surface with a non-uniform heat source or sink. The study employs similarity transformations to simplify the governing equations and utilizes numerical methods to analyze the effects of various physical parameters on fluid flow and thermal behavior. The findings provide valuable insights into the influence of heat source variations on temperature distribution and energy transfer efficiency in electrically conducting non-Newtonian fluids.

Gireesha B. J. [7], studied examines the magnetohydrodynamic flow of a power-law fluid over a stretching sheet, incorporating the effects of a non-uniform heat source or sink. The research employs similarity transformations to simplify the governing equations and utilizes numerical methods to analyze the influence of various physical parameters on fluid flow and thermal behavior. The findings provide valuable insights into how heat source variations impact temperature distribution and energy transfer efficiency in electrically conducting non-Newtonian fluids.

Mishra, S. R. [8], studied the magnetohydrodynamic flow of a power-law fluid over a stretching sheet embedded in a porous medium, considering variable thermal conductivity. Using similarity transformations, the governing equations are converted into nonlinear ordinary differential equations, which are then solved numerically via the Runge-Kutta method with a shooting technique. The investigation highlights the effects of thermal conductivity variations and magnetic field strength on fluid behavior and heat transfer efficiency.

Siddiqui, A. M. [9], examined the magnetohydrodynamic boundary layer slip flow and heat transfer of a power-law fluid over a flat plate with slip boundary conditions. Using similarity transformations, the governing nonlinear partial differential equations are converted into a system of ordinary differential equations and solved numerically. The research highlights the effects of slip parameters, magnetic field strength, and Prandtl number on velocity and temperature profiles, providing valuable insights into fluid behavior in electrically conducting non-Newtonian flows.

Patel M., Timol M.G. [10], investigated the magnetohydrodynamic boundary layer flow past a continuously moving flat surface. The research employs similarity transformations to simplify the governing equations and investigates the effects of various physical parameters, such as the magnetic field strength and fluid properties, on the flow behavior. Numerical methods are used to solve the transformed equations, providing insights into velocity and temperature distributions. The findings contribute to the understanding of electrically conducting fluid dynamics and have applications in engineering and industrial processes.

Pakdemirli M. [11], study focuses on the similarity analysis of three-dimensional boundary layer equations for a class of non-Newtonian fluids, where the stress is an arbitrary function of strain rates.

The research explores scaling and spiral group transformations to determine conditions under which similarity solutions exist. The findings highlight that for an arbitrary stress function, only specific wedge flow configurations lead to similarity solutions, while more restricted forms allow solutions for arbitrary wedge angles. The study also presents similarity equations for power-law and Newtonian fluids as special cases.

Kumari and Nath [12], investigated the magnetohydrodynamic boundary layer flow of a non-Newtonian power-law fluid over a continuously moving surface with a parallel free stream. The governing partial differential equations for the non-similar flow are solved numerically using an implicit finite-difference scheme. The research highlights the effects of the magnetic parameter, power-law index, and Prandtl number on skin friction and heat transfer coefficients. Baag [13], examined the magnetohydrodynamic flow of a viscoelastic fluid through a porous medium between infinite parallel plates, incorporating time-dependent suction effects. The research provides a theoretical analysis using asymptotic methods to explore the influence of viscoelastic properties, magnetic fields, and fluctuating suction on fluid behavior. Seshadri R.S. and Na T.Y., [14] worked Group Invariance in Engineering Boundary Value Problems explores the application of transformation groups to simplify complex differential equations in engineering and physics. The study presents methods for obtaining similarity transformations, reducing nonlinear equations to more manageable forms, and deriving self-similar solutions. By leveraging group invariance principles, the book provides valuable insights into solving boundary value problems efficiently.

Timol M.G. [15], study explores the magnetohydrodynamic boundary layer flow of non-Newtonian power-law fluids over a continuously moving surface, incorporating the effects of a transverse magnetic field. Using similarity transformations, the governing partial differential equations are converted into nonlinear ordinary differential equations through a deductive group theoretic method. The research provides insights into the influence of fluid properties and magnetic field strength on velocity and temperature distributions, contributing to advancements in electrically conducting fluid dynamics.

Motivated by this work, we extend the investigation by incorporating additional physical effects such as variable thermal conductivity, non-uniform heat sources, and porous media interactions. By employing numerical techniques, including MATLAB's ODE solver, we aim to analyze the impact of these factors on flow behavior and heat transfer efficiency. Our study seeks to refine existing models and provide deeper insights into the complex interactions governing MHD boundary layer flows in non-Newtonian fluids.

II Mathematical Problem

Considering the steady two-dimensional laminar boundary layer flow of a power-law fluid an electrically conducting, viscous, and incompressible fluid over a continuously moving surface with constant velocity and the governing equations are formulated based on the approach used in Timol M.G. [15].

We have

$$\psi_y \psi_{xy} - \psi_x \psi_{yy} = \gamma (|\psi_{yy}|^{n-1} \psi_{yy}) - S_0 \psi_y \quad (1)$$

Where $S_0 = \frac{\sigma B_0^2}{\rho U_\infty}$ is magnetic field parameter.

With boundary conditions

$$y = 0, \quad \psi_y = U_w, \quad \psi_x = 0 \quad (2)$$

$$y = \infty, \quad \psi_y = U_\infty \quad (3)$$

III Generalized group theoretic method

By using linear group transformation,

$$\underline{x}^* = \wp^{J_1} x, \quad \underline{y}^* = \wp^{J_2} y, \quad \underline{\psi}^* = \wp^{J_3} \psi, \quad \underline{S_0}^* = \wp^{J_4} S_0 \quad (4)$$

Where J_1, J_2, J_3, J_4 and \wp are constants

For the dependent and independent variables. From equation (4) one obtains

$$\left(\frac{x^*}{x}\right)^{\frac{1}{J_1}} = \left(\frac{y^*}{y}\right)^{\frac{1}{J_2}} = \left(\frac{\psi^*}{\psi}\right)^{\frac{1}{J_3}} = \left(\frac{S_0^*}{S_0}\right)^{\frac{1}{J_4}} = \wp \quad (5)$$

IV Establishing Ordinary Differential Equations

Consider the following transformation Seshadri R.S. and Na T.Y., [14],

$$\psi = ax^\alpha f(\eta) \quad \text{and} \quad \eta = b \frac{y}{x^\beta} \quad (6)$$

Where $\alpha = \frac{J_3}{J_1}$, $\beta = \frac{J_2}{J_1}$, a and b are real numbers and similarity variable η .

Using (6) the similarity variables in terms of equations (1) results in the following ordinary differential equations as

$$(|f''(\eta)|^{n-1}f'''(\eta)) - S_0f'(\eta) + \frac{1}{n+1} f(\eta)f''(\eta) = 0 \tag{7}$$

Where $S_0 = \frac{\sigma B_0^2}{\rho U_\infty}$ is magnetic field parameter, σ electrical conductivity of the fluid, B_0 strength of the magnetic field, U_∞ free-stream velocity, ρ fluid density.

With boundary conditions

$$\eta = 0, \quad f(\eta) = 0, \quad f'(\eta) = \frac{U_w}{U_\infty} = V, \quad U_w \text{ velocity of the plate} \tag{8}$$

$$\eta = \infty, \quad f'(\eta) = 1 \tag{9}$$

For Newtonian fluid put $n = 1$ in equation (7) we get,

$$f'''(\eta) - S_0f'(\eta) + \frac{1}{2}f(\eta)f''(\eta) = 0 \tag{10}$$

With boundary conditions

$$\eta = 0, \quad f(\eta) = 0, \quad f'(\eta) = V \quad \text{Velocity Parameter} \tag{11}$$

$$\eta = \infty, \quad f'(\eta) = 1 \tag{12}$$

Interpretation of the Velocity Parameter V , When $V = 0$ i) The plate is stationary (not moving).
ii) Only the fluid is moving. When $V < 0$, the plate and the fluid are moving in opposite directions.

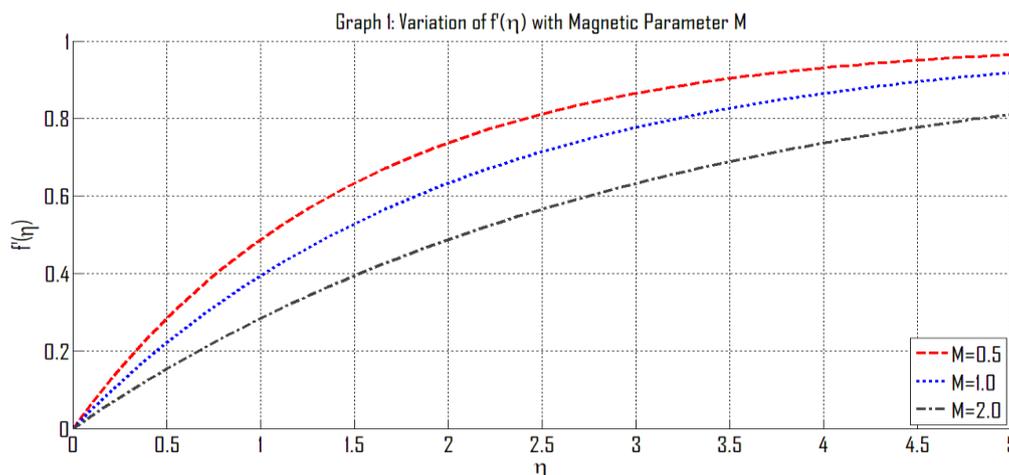
When $V > 0$, the plate and fluid are moving in the same direction. When $V = 1$, the plate and the fluid move at the same velocity.

When $V \in (0,1)$, then the plate moves slower than the fluid.

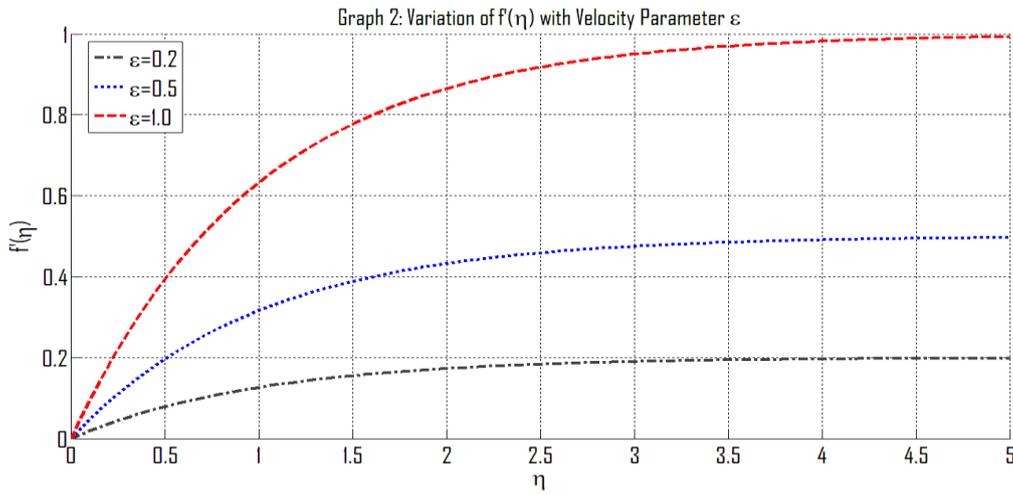
V Result and Discussion

The transformed system of nonlinear ordinary differential equations, represented by equations (10), is solved using the MATLAB ODE Solver while adhering to the specified boundary conditions (11) and (12). The graphical outputs effectively demonstrate the variations in the flow parameters.

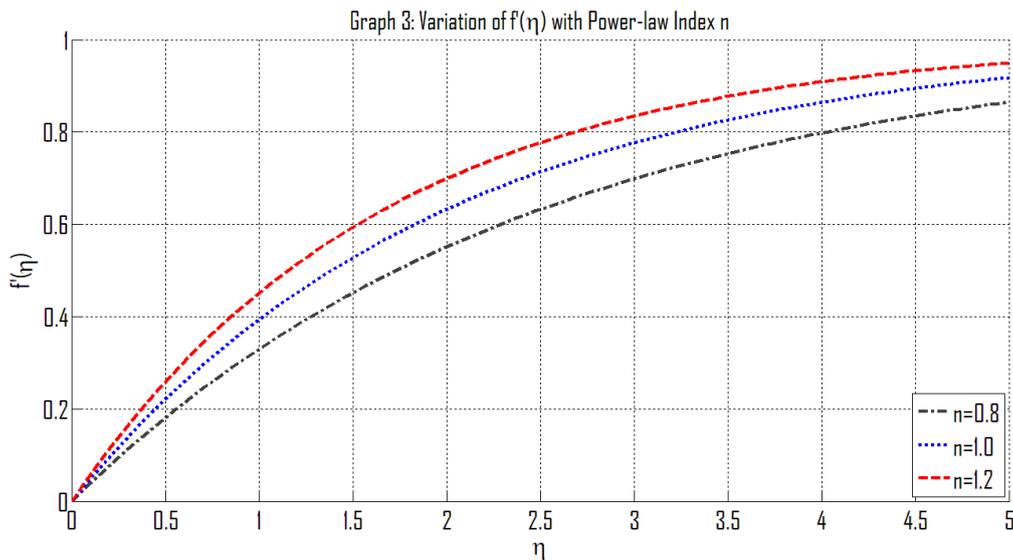
The graph (1) illustrates the influence of the power-law index n on the velocity profile $f'(\eta)$ in the boundary layer of a non-Newtonian fluid flowing over a moving surface under the influence of a magnetic field. As the index n increases from 0.8 (pseudo-plastic fluid) to 1.2 (dilatant fluid), the velocity profile rises more rapidly and reaches the asymptotic value closer to the surface. This behavior indicates that higher values of n , corresponding to shear-thickening fluids, enhance momentum diffusion and accelerate the flow near the surface. In contrast, lower values of n show a more gradual increase in velocity, typical of shear-thinning behavior. The result demonstrates how non-Newtonian characteristics significantly affect the structure of the boundary layer, with dilatant fluids promoting faster velocity recovery compared to pseudo-plastic ones.



The graph (2) shows how the fluid velocity profile responds to different values of the velocity ratio parameter $\epsilon = \frac{U_w}{U_\infty}$, where U_w is the plate velocity and U_∞ is the free-stream velocity. The graph demonstrates that as ϵ increases from 0.2 to 1.0, the velocity profile $f'(\eta)$ rises more quickly and approaches its asymptotic value faster. For lower ϵ , the profile increases gradually, indicating slower fluid motion near the surface. When $\epsilon = 1.0$, the wall and fluid move at the same speed, resulting in a rapid acceleration of the fluid. This behavior signifies that a higher wall-to-fluid velocity ratio enhances momentum transfer from the surface to the fluid, thereby thickening the boundary layer and accelerating the fluid more effectively.



The graph (3) illustrates the impact of the power-law index n on the velocity distribution of a non-Newtonian fluid within the boundary layer. The power-law index characterizes the fluid's rheological behavior—where $n < 1$ corresponds to shear-thinning (pseudo-plastic) fluids, $n = 1$ to Newtonian fluids, and $n > 1$ to shear-thickening (dilatant) fluids. As shown in the graph, increasing n from 0.8 to 1.2 results in a steeper velocity gradient and faster rise in $f'(\eta)$, indicating more rapid fluid acceleration near the wall. This suggests that dilatant fluids develop a thicker boundary layer and attain higher velocities faster compared to shear-thinning fluids. Therefore, the graph highlights that the fluid's response to applied shear significantly influences the momentum transport within the boundary layer, with higher n values enhancing the velocity profile throughout the domain.



Conclusion

The velocity profile of non-Newtonian fluid flow in a magneto-hydrodynamic context is significantly affected by the magnetic parameter M , velocity ratio ϵ , and power-law index n . Higher values of M and n correspond to increased shear thickening and magnetic resistance, which promote faster velocity development within the boundary layer. Similarly, increasing ϵ intensifies momentum transfer from the surface, accelerating the fluid motion. These findings demonstrate that manipulating these parameters can effectively control fluid flow dynamics in engineering and industrial applications involving non-Newtonian magneto-fluid systems.

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