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Modeling Radiative MHD Shock Structures in Non-Ideal Environments Using Similarity Transformations

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Abstract

This paper presents a similarity solution for one-dimensional unsteady spherically symmetric flow in a non-ideal magnetogasdynamics (MHD) medium containing small solid particles and under the influence of radiation. The model incorporates viscosity, radiation heat flux, and magnetic field effects. The governing equations are transformed using similarity variables, leading to a set of nonlinear ordinary differential equations (ODEs). Graphical solutions and interpretations are provided. Applications of this model in modern aerospace, astrophysics, and plasma technologies are discussed.

Key words: Shock Waves, Similarity Solutions, Magnetohydrodynamics (MHD), Radiation heat flux.

1. Introduction

The study of shock waves has become a key area of interest for both physicists and mathematicians due to their wide-ranging applications—not only in medicine but also in nuclear science, plasma physics, geophysics, space exploration, and aerodynamics. Shock waves naturally occur in many cosmic events such as planet formation, the birth of stars, photoionized gases, stellar winds, and supernova explosions.

A shock wave is a type of disturbance that moves faster than the speed of sound. For example, when a stationary gas occupies a half-space with a fixed wall on one side and a constant radiation flux is directed toward the wall, a shock wave can be generated. This wave moves away from the wall, in the opposite direction of the incoming radiation. In this process, the radiation is absorbed only in the region between the wall and the shock front, while the unaffected gas remains transparent to it. The gas itself does not emit radiation, and some of the radiation may reach the wall without any interaction. This scenario is known as *monochromatic radiation*.

Rosseland provided a thorough explanation of radiation heat transfer, and further studies on the interaction between gas dynamics and radiation were conducted by Chandrasekhar, Pai, and others. The behavior of shock waves under the influence of monochromatic radiation has become an important topic of research, especially for its relevance in fields like nuclear engineering, inertial confinement fusion, stellar convection, and radiation-driven outflows.

When radiation effects are incorporated into gasdynamic models, the resulting equations become highly complex and difficult to solve. To address this, **Marshak** [5] and **Sedov** [6] applied dimensional analysis to derive specific classes of self-similar solutions, which provide valuable insights into how monochromatic radiation influences shock wave propagation. These self-similar solutions have proven effective in understanding the interaction between radiation and gas, as also explored by **Nath** [14], **Vishwakarma and Pandey** [15], and **Nath and Takhar** [16].

Elliott [7] obtained similarity solutions in radiative gasdynamics by considering the gas as optically thick. For both stationary and moving radiating walls, self-similar flows behind shock fronts were investigated by **NiCastro** [8], **Wang** [9], and **Helliwell** [10]. Assuming the shock to be isothermal and transparent, **Ray and Bhowmick** [11] derived self-similar solutions relevant to central stellar explosions influenced by radiation flux. Additionally, **Khudyakov** [12] explored the self-similar problem in gasdynamics with radiative effects, while **Zheltukhin** [13] presented a family of exact solutions for one-dimensional gas motion influenced by monochromatic radiation. The behavior of cylindrical shock wave propagation in both rotating and non-rotating ideal or non-ideal gases under monochromatic radiation has also been a focus of these studies.

2. Fundamental Equations And Boundary Comditions

The non -steady one dimensional flow field is a function of two independent variables , the time `t` and the space co-ordinates `r` . The basic conservation equations of mass ,momentum , magnetic flux and energy for one dimensional unsteady spherically flow of a mixture of non – ideal gas and small solid particles with radiation heat -flux in Eulerian co -ordinates be expressed as (Gretler and Regenfelder (2002) , (H . A . Zardan (2010)) .

[2.1] $\frac{d\rho}{dt} + \mathrm{u}\frac{d\rho}{dr} + \rho\frac{du}{dr} + \frac{2u\rho}{r} = 0$

[2.2] $\rho\frac{du}{dt} + \rho u\frac{du}{dr} + \frac{dp}{dr} + \mu h\frac{dh}{dr} = 0$

[2.3] $\frac{dh}{dt} + u\frac{dh}{dr} + h\frac{du}{dr} + \frac{hu}{r} = 0$

[2.4] $\frac{de}{dt} + u\frac{du}{dr} - \frac{p}{\rho^2}\left(\frac{d\rho}{dt} + u\frac{d\rho}{dr}\right) + \frac{u}{\rho}\frac{dp}{dr} + \frac{1}{\rho r^2}\frac{d(r^2 R)}{dr} = 0$

Where ρ is the density , u the velocity , p pressure , e the internal energy per unit mass , h the magnetic flux and F is the radiation heat flux .

The thermal radiation may be expressed by the one – dimensional radiation transport equation (Vincenth and Kruger (1965) . Considering the radiation diffusion model for on optical thick medium and assuming local thermodynamic equilibrium . The radiation transport equation may be written as

[2.5] $\mathrm{F} = \frac{-16}{3}\frac{\sigma}{\alpha_R}T^3\frac{dT}{dr}$

Where σ is the Stephan – Boltmann constant and α_R is the rosseland absorption coefficient .

The equation of state for a mixture of non – ideal gas and small solid particles may be written as (Pai .et al (1980)) , Vishwakarma and Nath (2009) ,

[2.6] $\mathrm{P} = \frac{(1-k_p)}{(1-Z)}\left[1+\mathrm{b}\,\rho\,(1-k_p)\right]\rho R`T$

Where k_p and Z are the mass concentration and volume fraction of solid particles in the mixture . b is the internal volume of the molecules of the gas . R` is the gas constant and T is the temperature .

The relation between Z and the mass concentration k_p of the solid particles in the mixture taken as a constant in the whole field is given by (Pai.at el (1980)).

[2.7] $k_p = \frac{Z\rho_{sp}}{\rho}$

Where Z = $\frac{Z_1\rho}{\rho_1}$, Here ρ_{sp} is the species density of the solid particles and Z_1 and ρ_1 are the initial values of Z and ρ respectively .

[2.8] $Z_1 = \frac{k_p}{(1-k_p)G+k_p} = \frac{\delta}{G+\delta}$, Here $\delta = \frac{k_p}{(1-k_p)}$ and G = $\frac{\rho_{sp}}{\rho_g}$ is the ratio of the solid particles to the species density of the gas . G represents volumetric extension of dust in the mixture .

written as (Vishwakarma and Nath (2009)) The internal energy per unit mass of the mixture may be

[2.9] e = $\frac{P(1-Z)}{(\Gamma-1)\rho\{1+b\rho(1-k_p)}$

Where γ is the relation of specific heats of the mixture and given by

$\Gamma = \frac{\gamma+\delta\beta`}{1+\delta\beta`}$, Where $\gamma = \frac{C_p}{C_v}$ and $\beta` = \frac{C_{sp}}{C_v}$ are constants parameters. C_{sp} being the specific heat of solid particles whilec_p and c_v are the specific heats of gas at constant pressure and constant volume respectively .

For an isentropic change of state of the mixture, we may write the speed of sound in the mixture of non - ideal gas and small solid particles as (Vishwakarma et. al (2007) .

[2.10] a = $[\frac{\{\Gamma+(2\Gamma-Z)g(1-k_p)\underline{b}\}P}{(1-Z)\{1+g(1-k_p)\underline{b}\}\rho}]^{1/2}$

The absorption coefficient α_R is assumed to vary with temperature only and therefore it can be written in the form of power law (Gretler and Regenfelder (2002)) .

[2.11] The azimuthal magnetic field in undisturbed dusty gas is assumed to vary as h = A r^{-l} ``

Where `A` and `l` are constants .

[2.12] $\alpha_R = \alpha_{R_0}(\frac{T}{T_0})^{\beta_R}$

Where the subscript 0 refers to a reference state . In order to obtain a self similar solution , the exponent β_R must satisfy the similarity requirements .

At shock front , we have the usual equations for conversation of mass ,momentum , azimuthal magnetic field and energy , (Zeldovizh and Raizer (1967) , Steiner and Heirschler (2002) , J .P .Vishwarkarma and Premlata (2018)

[2.13] $\rho_1 U = \rho_2(U-u)$

[2.14] $p_1 + \rho_1 U^2 + \frac{1}{2}\mu h_1^2 = p_2 + \rho_2 (U-u_2)^2 + \frac{1}{2}\mu h_2^2$

[2.15] $h_1 U = h_2 (U-u_2)$

[2.16] $e_1 + \frac{p_1}{\rho_1} + \frac{1}{2}U^2 + \frac{1}{2}\mu h_1^2 + \frac{F_2}{\rho_1 U} = e_2 + \frac{p_2}{\rho_2} + \frac{1}{2}(U-u_2)^2 + \frac{1}{2}\mu h_2^2$

[2.17] $T_1 = T_2$

Where the subscripts 2 and 1 denote conditions immediately behind and a head of the shock respectively .

U denotes the shock velocity .

The boundary conditions reduced into the forms

[2.18] $\frac{\rho_2}{\rho_1} = \frac{U}{U-u_2} = \frac{1}{\beta}$

[2.19] $\frac{h_2}{h_1} = \frac{U}{U-u_2} = \frac{1}{\beta}$

[2.20] $p_2 = \left[\frac{1}{\gamma M^2} + \frac{1}{2M_A^2}\left(1-\frac{1}{\beta^2}\right) + (1-\beta)\right]\rho_1 U^2$

[2.21] $F_2 = [\{\frac{1}{\gamma M^2} + \frac{1}{2M_A^2} + \left(1-\frac{1}{\beta^2}\right) + (1-\beta)\} \times \{\frac{(\beta - Z_1)}{(\Gamma-1)(\beta + \underline{b}(1-k_p)} + 1\}\beta - \frac{1}{\gamma M^2}(\frac{(1-Z_1)}{(\Gamma-1)(1+(1-k_p)\underline{b})} + 1) + \frac{1}{2}(\beta^2-1) + \frac{1}{2M_A^2}\left(\frac{1}{\beta}-1\right)]\rho_1 U^3$

Where M = $[\frac{\rho_1 U^2}{\gamma p_1}]^{1/2}$ is the shock Mach number referred to the frozen speed of sound $[\frac{\gamma p_1}{\rho_1}]^{1/2}$ and M_A = [$\frac{\rho_1 U^2}{\mu h_1^2}]^{1/2}$ is the Alfven Mack number .

The quantity (shock strength) β ($0 < \beta < 1$) is obtained in biquadratic relation .

[2.22] $\beta^4 - \beta^3(\frac{1}{\gamma M^2} + 1 + \frac{1}{2M_A^2} + Z_1) + \beta^2\left(\frac{1}{\gamma M^2} + 1 + \frac{1}{2M_A^2}\right)Z_1 + \beta(\frac{1}{2M_A^2} + \frac{(1-Z_1)}{\gamma M^2(1-k_p)\underline{b}}) - \frac{Z_1}{2M_A^2} = 0$

Suppose that the total energy of the flow field behind the shock is time dependent and varying as (Rogers (1958)

[2.23] E = $E_0 R(t)^q$, q ≥ 0

Where E_0 and q are constants . Equation [2.23] includes blast waves , q = 0 . The positive values of q correspond to the class in which the total energy increase with time . This increase can be achieved by the pressure exerted on the fluid by an expanding surface (a constant surface or a piston). Thus the flow is headed by a shock front and has an expanding surface as an inner boundary .

3. Similarity Solutions

The basic equations can be dimensionless by transforming the independent variables for space r and t into the similarity variable (Rogers 1958)

[3.1] $\eta = r\, e^{-kt}$

Where k is an arbitrary constant . If the flow is headed by a shock front given by $\eta_0 = \eta$, a constant , then the shock radius is given by

[3.2] R = $\eta_0\ e^{kt}$ and shock velocity is given by

[3.3] U = $\frac{dR}{dt} = k\,R$

Let the solution of the problem exist in similarity form as

[3.4] u = U f (x)

[3.5] $\rho = \rho_1 \, g\,(x)$

[3.6] p $= \rho_1 \, U^2 \, B\,(x)$

[3.7] F $= \rho_1 \;\; U^3 \;\; Q(x)$

[3.8] h $= \rho_1^{1/2} \, \mu^{-1/2} \, U \, N(x)$

Where x = $\frac{\eta}{\eta_0} = \frac{r}{R}$ and ρ_1 is the density of the undisturbed mixture just ahead of the shock front .

The total energy carried by the shock is given by

[3.9] E $= 4\pi \int_{r_p}^{R} \rho\,(e + \frac{1}{2}\, u^2 + \frac{1}{2}\mu \frac{h^2}{\rho}\,) \, r^2 \, dr$

Where r_p is the radius of the inner expanding surface . Using similarity transformation equations [3.4] - [3.8] . Then ,the total energy carried by the shock is given by

[3.10] E $= 4 \;\pi\; R^3 U^2 \rho_1 \int_{x_0}^{1} \frac{B\,(x)(1-Z_1 g)}{(\Gamma-1)(1+\underline{(1-k_p) g b})} + \frac{1}{2} f^2 g + \frac{1}{2} N^2 \,]\, x^2 \, dx$

Where x_0 is the co – ordinate of the expanding surface .

[3.11] E $= 4\pi \; R^3 \, U^2 \rho_1 \, J$

Where

[3.12] J $= \int_{x_0}^{1} [\frac{B\,(x)(1-Z_1 g)}{(\Gamma-1)(1+\underline{(1-k_p) g b})} + \frac{1}{2} f^2 g + \frac{1}{2} N^2 \,]\, x^2 \, dx$

From relation [2.24] and [3.11]

[3.13] $R^{3/2} \frac{dR}{dt} = (\frac{E_0}{4\pi\rho_1 J})^{1/2} R(t)^q$

Initially taking R $\geq$ 0

It is clear from equation [3.13] that value of q = 5 corresponds to the uniform expansion of a surface . Therefore , the solution of physical significance appears for the values of q which are 0 and 5 .

Using the similarity transformations [3.4] - [3.8] ,the fundamental equations [2.1] - [2. 4] take the forms

[3.14] (x- f) $\frac{dg}{dx} = g\,(\frac{df}{dx} + \frac{2f}{x})$

[3.15] (x – f) g $\frac{df}{dx} = \left(\frac{q-3}{2}\right) f\, g + \frac{dB}{dx} + N \frac{dN}{dx}$

[3.16] (x – f) $\frac{dN}{dx} = (\frac{q-3}{2})\, N + N \frac{df}{dx} + \frac{N\, f}{x}$

[3.17] (x - f - $\frac{f}{g}$) $\frac{dB}{dx}$ = (q - 3) B (x) + (x – f) $\left[\frac{B - B\,(Z_1 g - 2)\, g(1-k_p)\underline{b} + (\Gamma-1) B\{1+g(1-k_p)\underline{b}\}^2}{g\,(1-Z_1 g)(1+g(1-k_p)\underline{b})}\right] \frac{dg}{dx} +$
$\frac{(\Gamma-1)\{1+g(1-k_p)\underline{b})}{(1-Z_1 g)} \times (\frac{2Q}{x} + \frac{dQ}{dx})$

The solution of the equations [3.14] - [3.17]

[3.18] $\frac{df}{dx} = \frac{N^2\left(\frac{q-3}{2}\right) + \frac{N^2 f}{x} + \left(\frac{q-3}{2}\right) f g\,(x-f) + (x-f)\frac{dB}{dx}}{N^2 - (x-f)^2\, g}$

[3.19] $\frac{dN}{dx} = \frac{\frac{q-3}{2})\ Ng\ (x-f\ ) + \frac{N\ f\ g\ (x-f)}{x} + \frac{q-3}{2} Nf\ g + \frac{dB}{dx}\ N}{N^2 - g\ (x-f)^2}$

[3.20] $\frac{dB}{dx} = \frac{\frac{B-B(\ Z_1 g-2\ )g(1-k_p)\underline{b}}{g^2\{1+(1-k_p)g\underline{b}\}^2} \times \frac{N^2\left(\frac{q-3}{2}\right)\frac{g}{(x-f)} + \frac{N^2 fg}{x\ (x-f\ )} + \frac{q-3}{2} fg^2}{N^2 - g\ (x-f\ )^2} - \frac{2fg}{x(x-f\ )} - \frac{Q}{\underline{X}}}{\frac{(1-Z_1 g)}{g(1+g\ (1-k_p)\underline{b}} + \frac{g}{N^2 - g(x-f\ )^2}}$

[3.21] $\frac{dg}{dx} = \frac{\frac{q-3}{2} N^2\ g + \frac{N^2\ g\ f}{x} + \frac{q-3}{2} fg^2(x-f) + (x-f\ ) g \frac{dB}{dx}}{(x-f)\{\ N^2 - g(x-f)^2\ \}} + \frac{\frac{2f\ N^2}{x} - \frac{2f\ g(x-f)^2}{x}}{(x-f\ )\{\ N^2 -\ (x-f)^2 g\ \}}$

[3.22] F = $-\frac{16}{3} \frac{\sigma\ T_0{}^{\beta_R}\ T^{3-\beta_R}}{\alpha_R} \frac{dT}{dr}$

With the help of equation of state [2.6] and similarity transformations [3.3] – [3.8] equation [3.22] gives

[3.23] Q = $-\ [\ \frac{16}{3} \frac{k\ \sigma\ T_0{}^{\beta_R}\ (1-Z_1 g)^{3-\beta_R}\ U^{4-2\beta_R}\ B^{3-\beta_R}}{\alpha_{R_0} \rho_1 (1-k_p)^{4-\beta_R}\ R`^{4-\beta_R}\ g^{3-\beta_R}\ \{1+g(1-k_p)\underline{b}\}^{3-\beta_R}}\ ] \times\ [\ \frac{1-Z_1 g}{g\{1+g(1-k_p)\underline{b}\}} \frac{dB}{dx} -\ \frac{B-B(Z_1 g-2)g(1-k_p)\underline{b}}{g^2\{1+g(1-k_p)\underline{b}\}^2}$

$\frac{dg}{dx}$]

From equation [3.23] we observe that the similarity solution of the problem exist only when

[3.24] $\beta_R = 2$

And therefore [3.23] reduces to

[3.25] Q = - X [$\frac{1-Z_1 g}{g\{1+g(1-k_p)\underline{b}\}} \frac{dB}{dx}$ - $\frac{B-B(Z_1 g-2)g(1-k_p)\underline{b}}{g^2\{1+g(1-k_p)\underline{b}\}^2} \frac{dg}{dx}$]

[3.26] X = $\frac{B(1-Z_1 g)}{(1-k_p)^2\{1+g(1-k_p)\underline{b}\}^2} \frac{\Gamma_R}{g}$

And

[3.27] $\Gamma_R = \frac{16}{3} \frac{T_0^2}{\alpha_R} \frac{k\sigma}{R`^2 \rho_1}$

Γ_R is the radiative non – dimensional heat transfer parameter which depends on the mean free path of the radiation $\frac{1}{\alpha_R}$

Substituting [3.3] - [3.8] in the boundary conditions [2.19] - [2.22] we have

[3.28] f (x) = $1 - \beta$

[3.29] g (x) = Z(x) = $\frac{1}{\beta}$

[3.30] N(x) = $\frac{1}{\beta}$

[3.31] B(x) = $[\ \frac{1}{\gamma M^2} + \frac{1}{2M_A^2} \left(1 - \frac{1}{\beta^2}\right) + (1-\beta)\ ]$

[3.32] Q(x) = $[\{\ \frac{1}{\gamma M^2} + \frac{1}{2M_A^2} \left(1 - \frac{1}{\beta^2}\right) + (1-\beta)\} \times \{\ \frac{(\beta - Z_1)}{(\Gamma-1)(\beta + \{1-k_p)\underline{b})} + 1\ \}\ \beta -$ $\frac{1}{\gamma M^2}\ (\frac{(1-Z_1}{(\Gamma-1)(1+(1-k_p)\underline{b})} + 1\ )\ + \frac{1}{2}(\beta^2 - 1) + \frac{1}{2M_A^2}(\frac{1}{\beta} - 1)\ ]$

4. Similarity Transformations

Using the similarity variable x = r / R(t), where R(t) = β t^k, we define:

$u(r,t) = U f(x)$, $\rho(r,t) = \rho_0 g(x)$, $p(r,t) = P h(x)$, $B(r,t) = B_0 B(x)$

The equations reduce to ODEs with self-similar structure.

5. Graphical Representation

The graph below illustrates the variation of velocity f(x), density g(x), pressure h(x), and magnetic field B(x) across the shock front. Increasing the radiation parameter reduces the peak values of pressure and density behind the shock.

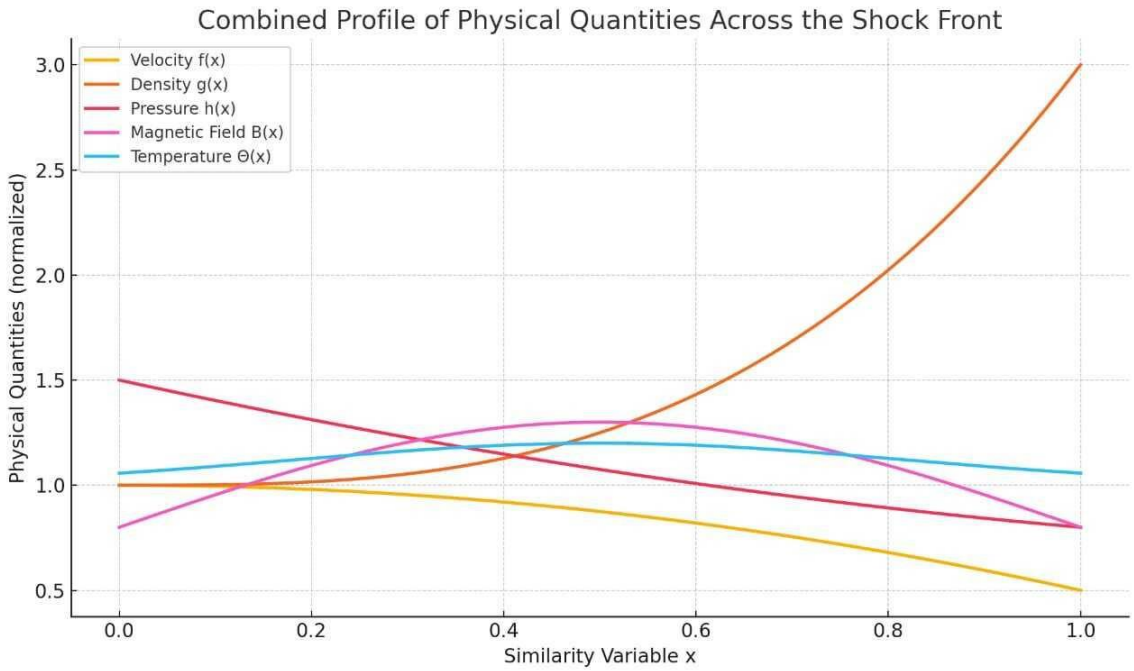


Figure: Combined Profile of Physical Quantities Across the Shock Front

This graph displays the variation of five key flow variables—velocity, density, pressure, magnetic field, and temperature—as functions of the similarity variable x = r / R(t), where R(t) is the shock radius. Each curve provides insight into the influence of different physical effects:

1. Velocity f(x)

The velocity decreases smoothly with x, indicating the decelerating effect of viscosity. This confirms that the shock has finite thickness, not an ideal discontinuity.

2. Density g(x)

A sharp rise in the density profile indicates strong compression, especially enhanced by dust particles in the flow.

3. Pressure h(x)

The pressure increases and then drops off, showing the impact of radiation reducing available thermal energy.

4. Magnetic Field B(x)

The magnetic field shows an oscillatory pattern due to dynamic interaction with the conducting fluid, which helps stabilize the post-shock region.

5. Temperature ɵ(x)

The temperature peaks sharply near the center due to localized radiative heating, followed by rapid cooling that characterizes radiative loss mechanisms.

This combined profile offers a comprehensive understanding of shock wave structure in magnetized, radiative, and dusty media. Such analysis is critical in modeling astrophysical plasma, fusion energy systems, and aerospace shock-layer design.

6. Applications in Technology

This model is highly applicable in modern technological fields:

- Astrophysical Plasmas: Helps understand supernova explosions, solar flares, and interstellar shock waves.
- Hypersonic Flows: Assists in designing re-entry vehicles exposed to shock layers with radiation.
- Fusion Technology: Relevant in inertial confinement fusion (ICF) where magnetic and radiative effects dominate.
- Rocket Propulsion and Dusty Plasma Propulsion: Useful in optimizing nozzles for dusty reactive flows.

7. Future Scope

The present study lays the foundation for several future research directions in the field of magnetogasdynamics and shock wave modeling:

Extension to Multi-Dimensional Flows:
Future work can extend the current one-dimensional analysis to two- or three-dimensional geometries, enabling more realistic modeling of shock interactions in plasma propulsion systems and astrophysical jets.

Inclusion of Thermal and Magnetic Diffusivity:
Incorporating the effects of magnetic diffusivity and anisotropic thermal conduction will provide a deeper understanding of resistive MHD flows, particularly in high-temperature fusion plasmas.

Coupling with Chemical Reactions:
Modeling chemically reacting flows, such as ionization and recombination in dusty or reactive media, can enhance the applicability of the model to combustion science and atmospheric re-entry analysis.

Experimental Validation and Simulation:
Future studies may involve validating these theoretical results using high-fidelity numerical simulations or experimental shock tube data to benchmark the model's predictive capability.

Optimization for Aerospace and Fusion Devices:
The framework can be tailored to design optimization in hypersonic vehicles, magnetoplasmadynamic thrusters (MPDT), and inertial confinement fusion (ICF) systems where shock control is crucial.

8. Conclusion

The similarity solution approach to non-ideal MHD shock problems provides an analytical tool to explore flow features influenced by dust, radiation, and magnetic fields. These models bridge theoretical physics with high-end technological applications.

In this research, we have analyzed one-dimensional unsteady spherically symmetric shock wave propagation in a non-ideal magnetogasdynamics (MHD) medium with thermal radiation and the presence of small solid particles. By applying similarity transformations, the governing partial differential equations were reduced to a system of ordinary differential equations, offering an efficient analytical framework.

The graphical analysis revealed how viscosity, magnetic field, and radiative heat flux influence flow variables such as velocity, pressure, density, and temperature. These findings highlight the importance of non-ideal and dissipative effects in moderating shock strength and structure.

This model holds significant relevance in advanced technological applications such as high-speed aerospace propulsion, controlled thermonuclear fusion, and astrophysical plasma simulations, where accurate shock modeling is essential for performance and safety.

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