



Crack Detection in Beams Using Modal Strain Energy and Frequency Based Condition Monitoring

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Abstract

This study focuses on crack detection in structural beams using Modal Strain Energy (MSE) and frequency-based condition monitoring techniques. Three beam types—cantilever, simply supported, and overhanging—are analyzed under damaged and undamaged conditions by evaluating mode shapes, natural frequencies, and modal strain energy. Finite element-based MATLAB simulations were developed and validated against ANSYS results, showing strong agreement and confirming the accuracy of the computational models. The study identifies critical regions prone to damage: near the fixed support for cantilever beams, at the midpoint for simply supported beams, and near the supports for overhanging beams. These findings demonstrate that the proposed approach enables reliable, real-time structural health monitoring, facilitating early crack detection, reducing maintenance costs, and enhancing safety in engineering applications.

Keywords: crack detection, Modal Strain Energy, Beam structures, Finite element method, Structural health monitoring.

Introduction

Beam-type structural components—such as cantilever, simply supported and overhanging beams—are ubiquitous in mechanical, civil and industrial systems. Over their service life they can develop cracks due to fatigue, overload, corrosion or manufacturing defects, which lead to local stiffness reductions and ultimately structural failure. The dynamic properties of a beam (natural frequencies, mode shapes, modal strain energy distributions) inherently depend on its physical and geometric parameters (mass, stiffness, boundary conditions). When a crack or damage occurs, it alters local stiffness and hence modifies these dynamic characteristics. Early detection of such crack-induced changes enables intervention before catastrophic failure, thereby increasing safety and reducing maintenance cost [1].

Vibration-based structural health monitoring (SHM) methods rely on extracting dynamic features of a structure and tracking changes in those features as indicators of damage. Because modal parameters (e.g., frequencies, mode shapes) are global and measurable in-situ, they offer a practical route for damage assessment [2]. Vibration-based methods capture changes in stiffness and mass distribution, and thus sensitivity to damage is inherently linked to modal shifts or energy distributions. However, limitations exist: tracking only natural frequencies is often insufficient for localization of damage because changes may be small and influenced by environmental or boundary-condition variations [3]. To overcome these, more refined damage-sensitive indices (e.g., modal curvature, modal strain energy) have been developed [4].

Computational modelling forms a cornerstone in contemporary SHM research for several reasons: first, it enables simulation of both undamaged and damaged states (e.g., various crack positions and depths) so that baseline and damaged dynamic responses—frequencies, mode shapes, MSE—can be derived and compared [5]. Second, it facilitates damage-sensitive feature extraction and the development of diagnostic indices: for instance, modal strain energy distribution can be computed element by element for

different damage scenarios, enabling localization of damage and severity estimation [6]. Third, it supports validation of experimental or real-time data and the development of monitoring algorithms before deployment. In the context of this work, MATLAB-based finite element modelling is used to generate dynamic responses (mode shapes, frequencies, MSE) for cantilever, simply supported and overhanging beams under intact and cracked conditions. Further, validation via a commercial tool (ANSYS) establishes model fidelity, enhancing confidence in real-time health monitoring applicability.

Crack identification in beams is critical for several reasons. Beams are among the most stressed elements under bending, and even small cracks at high-stress locations significantly reduce local stiffness, alter load paths, and accelerate failure. The ability to pinpoint crack locations (not just existence) is vital for targeted maintenance. Studies show that simply supported beams exhibit maximum bending moment at mid-span, cantilevers near the fixed support, and overhanging beams near the support regions—thus indicating where damage is most likely to initiate [7]. Modal strain energy-based methods are particularly attractive for beams because the MSE distribution is sensitive to local stiffness reductions: a crack introduces an abrupt change in element-wise strain energy under modal excitation, providing a high resolution index for damage localization [8]. Given the industrial relevance (machine components, train wheel axles, robot structures), using vibration-based computational methods for crack detection enhances safety and reliability.

In this investigation, dynamic responses of three beam types—cantilever, simply supported and overhanging—are analysed under undamaged and cracked conditions. Modal parameters (mode shapes, natural frequencies) and the modal strain energy (MSE) distributions are derived via a MATLAB finite-element code. The models are validated via comparison with results obtained from ANSYS. By comparing the undamaged and damaged states, damage indices based on MSE and frequency shifts are evaluated. The objective is to determine the vulnerability zones for each beam type, validate the computational model, and propose a real-time structural health monitoring tool for beam-type components.

Materials and methods

The present study investigates the dynamic behavior and crack detection in beam-type structural components using modal analysis and finite element methods. The study focuses on beams with rectangular and circular cross-sections under various boundary conditions, including cantilever, simply supported, and overhanging configurations. Both ANSYS 2022 Workbench and MATLAB environments were employed for numerical simulations, and results were cross-validated to ensure accuracy.

Materials

The beams used in this study were made of Mild Steel, a widely used engineering material for structural applications due to its uniform mechanical properties and isotropic behavior. The material properties used for numerical simulations are summarized in Table 3.1.

Parameter	Rectangular Beam
Young's Modulus, E (N/m ²)	2.1×10^{11}
Density, ρ (kg/m ³)	7860
Poisson's Ratio, ν	0.3
Length, L (m)	0.8
Width, b (m)	0.025
Height, h (m)	0.01

Table 1: Beam properties.

Cracks were introduced in different locations ($a/L = 0.1$ – 0.9) and depths (10%–40% of beam height or radius) to simulate realistic damage conditions. Cantilever, simply supported, and overhanging beams were analyzed with multiple cracks to assess the sensitivity of dynamic parameters to crack presence.

The overall paper involved a sequential process integrating computational modeling, finite element formulation, modal analysis, and validation. Initially, the beam geometry and material properties were defined, followed by creating uncracked and cracked models in ANSYS 2022 Workbench and MATLAB. Modal analysis was performed to obtain the first five natural frequencies, corresponding mode shapes, and modal strain energy values for each model. In MATLAB, beam models were discretized using the finite element method (FEM) to establish global stiffness and mass matrices. The MATLAB results were validated by comparing with ANSYS outcomes to ensure model consistency. Cracks were represented as local reductions in stiffness, simulating partial

structural failure. The comparison between damaged and undamaged beams provided insights into how cracks alter vibrational characteristics, which are critical for early detection and structural health monitoring applications.

Modal Analysis

Modal analysis is a vital tool for determining the dynamic response of structures subjected to vibrational excitation. It identifies the natural frequencies and corresponding mode shapes that define how structures deform when disturbed from equilibrium. In this study, both analytical and numerical modal analyses were employed to assess beam behavior under different boundary conditions—cantilever, simply supported, and overhanging. The free vibration of a beam is governed by the equation $[M]\{\ddot{U}\} + [K]\{U\} = 0$, where $[M]$ and $[K]$ are the mass and stiffness matrices, respectively. Solving this eigenvalue problem yields the natural frequencies and mode shapes. When cracks are introduced, local stiffness decreases, modifying the vibration characteristics. Modal strain energy (MSE) was computed to identify regions where the strain energy concentration increased due to cracking, indicating potential failure zones and providing a reliable method for structural health monitoring.

Finite Element Modeling

FEM divides a complex structure into discrete elements connected at nodes, approximating continuous behavior. In this study, beam elements were used, and shape functions were derived to represent transverse displacement within each element:

$$v(x) = N_1 u_1 + N_2 \phi_1 + N_3 u_2 + N_4 \phi_2$$

The elemental stiffness matrix for bending in the xy-plane with two degrees of freedom per node is:

$$[K] = \int_0^l [B]^T [D] [B] dx$$

$$[K] = \frac{EI}{L^3} \begin{bmatrix} 12 & 6l & -12 & 6l & 6l & 4l^2 & -6l & 2l^2 & -12 & -6l & 12 & -6l & 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

Where E=Young's Modulus of Elasticity

I=Moment of Inertia of the section with respect to z-axis

L=Length of the Beam

$$[M] = \int_0^l [N]^T [A] [N] dx$$

$$[M] = \frac{\rho A l}{420} \begin{bmatrix} 156 & 22l & 54 & -13l & 22l & 4l^2 & 13l & -3l^2 & 54 & 13l & 156 & -22l & -13l & -3l^2 & -22l & 4l^2 \end{bmatrix}$$

Where l=Length of the beam element in m, ρ =Density of the beam material in kg/m³ and A = Area of cross-section in m².

Beam elements were divided into 500 finite elements for high-resolution analysis. Cracked elements had reduced stiffness matrices based on crack depth and location.

ANSYS Simulation

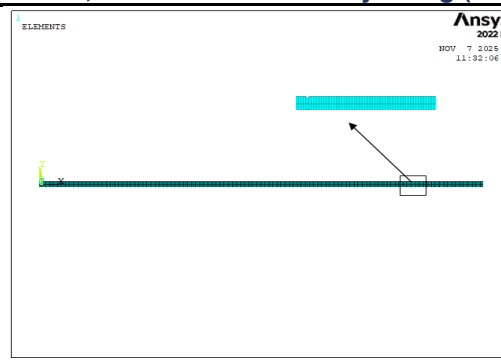
In ANSYS Workbench, beams were modeled by defining material properties, geometry, and mesh. Boundary conditions for cantilever, simply supported, and overhanging beams were applied. Modal analysis was performed to extract the first five natural frequencies and corresponding mode shapes. Post-processing allowed visualization of deformation patterns and identification of maximum stress and displacement regions.

MATLAB Simulation

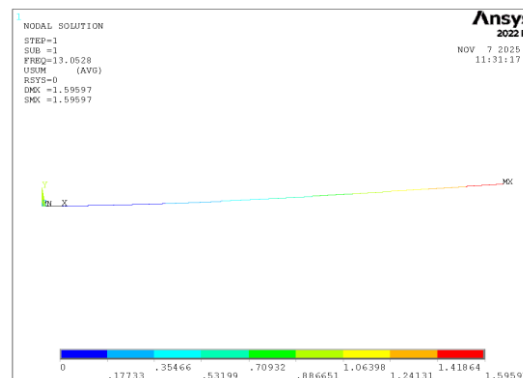
MATLAB analytical FEM was implemented by constructing global mass and stiffness matrices for cracked and un-cracked beams. Eigenvalue decomposition using MATLAB's eig function provided natural frequencies and mode shapes. Modal strain energy for each element was computed as:

$$U_{ij} = [\psi_{ij}]^T [K_i] [\psi_{ij}],$$

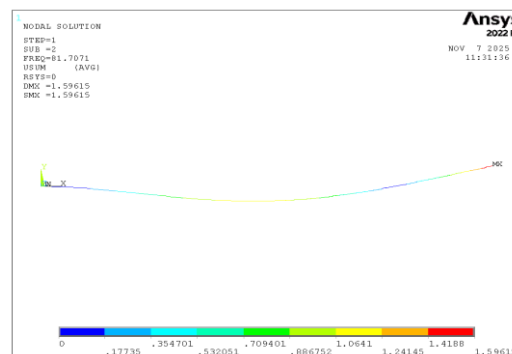
Comparison of modal parameters between undamaged and damaged beams enabled crack detection and localization. Using the above-mentioned methodology, a comprehensive set of numerical simulations was conducted to evaluate the dynamic behavior of cracked and uncracked beam structures. Both ANSYS and MATLAB environments were utilized to extract natural frequencies, mode shapes, and modal strain energy (MSE) for various beam configurations and crack conditions. The finite element approach provided accurate insight into the influence of crack depth and location on the structural response under free vibration. The results obtained from MATLAB were compared with ANSYS simulations to ensure consistency and model validation.

Fig 1: Element planning for crack at $a/l=0.9$.

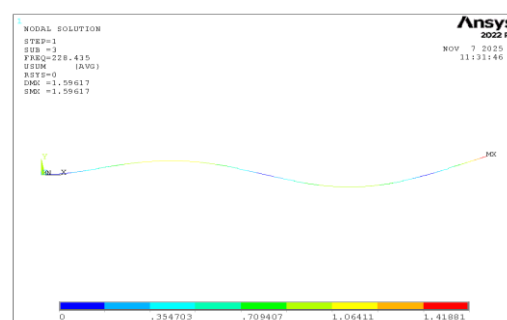
The above image illustrates the crack position on the beam, where the crack is located at 90% of the beam's length. A detailed zoomed-in view of the cracked region is also provided. In the simulation, the beam is discretized into 500 elements, and the element corresponding to the 90% position is modeled as the cracked section by reducing its depth to represent the loss of stiffness due to the crack.

Fig 2: 1st mode shape for cantilever beam and crack at $a/l=0.9$.

The above image shows the first mode shape of the cantilever beam, corresponding to a natural frequency of 13.05 Hz. In this mode, the maximum displacement occurs at the free end of the beam, as expected for a cantilever configuration. The peak displacement observed under cyclic loading is 1.595 m.

Fig 3: 2nd mode shape for cantilever beam and crack at $a/l=0.9$.

The above image illustrates the second mode shape of the cantilever beam, which corresponds to a natural frequency of 81.7 Hz. In this mode, the maximum displacement is observed near the midspan of the beam, consistent with the expected vibration behavior of a cantilever in its second mode. The peak displacement recorded under cyclic loading is 1.596 m.

Fig 4 : 3rd mode shape for cantilever beam and crack at $a/l=0.9$.

The above image illustrates the third mode shape of the cantilever beam, which corresponds to a natural frequency of 228.4 Hz. In this mode, the displacement pattern resembles a single wave, with the maximum displacement occurring near the midspan of the beam. This behavior is consistent with the expected vibration characteristics of the cantilever in its third mode. The peak displacement observed under cyclic loading is 1.597 m.

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*** NOTE *** CP = 37.703 TIME= 11:42:34
The modes requested are mass normalized (Nrmkey on MODOPT). However,
the modal masses and kinetic energies below are calculated with unit
normalized modes.

***** MODAL MASSES, KINETIC ENERGIES, AND TRANSLATIONAL EFFECTIVE MASSES SUMMARY *****

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MODE	FREQUENCY	MODAL MASS	KEKE	EFFECTIVE MASS					
				X-DIR	RATIO%	Y-DIR	RATIO%	Z-DIR	RATIO%
1	13.05	0.1326	446.1	0.000	0.00	0.9532	61.30	0.000	0.00
2	81.71	0.1108E-01	1450.	0.000	0.00	0.2960	18.84	0.000	0.00
3	228.4	0.4077E-02	4199.	0.000	0.00	0.1018	6.48	0.000	0.00
Sum				0.000	0.00	1.361	86.62	0.000	0.00

Fig 5: 3 Frequencies data for cantilever beam and crack at $a/l=0.9$.

The above image presents the output obtained from ANSYS APDL, showing the three natural frequencies of the cantilever beam. The first cyclic frequency is 13.05 Hz, the second is 81.7 Hz, and the third is 228.4 Hz. These frequencies correspond to the first, second, and third mode shapes of the beam, respectively.

Matlab results for cantilever beam

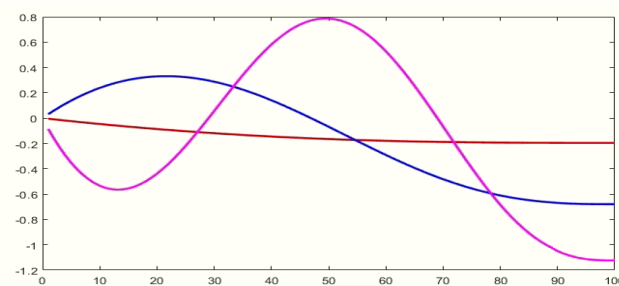


Fig 6 : Matlab results for 3 mode shapes for cantilever beam and crack at $a/l=0.9$.

The above image presents the output obtained from MATLAB, showing the mode shapes corresponding to the three natural frequencies of the cantilever beam. The red curve represents the first mode shape, the blue curve indicates the second mode shape, and the pink curve corresponds to the third mode shape.

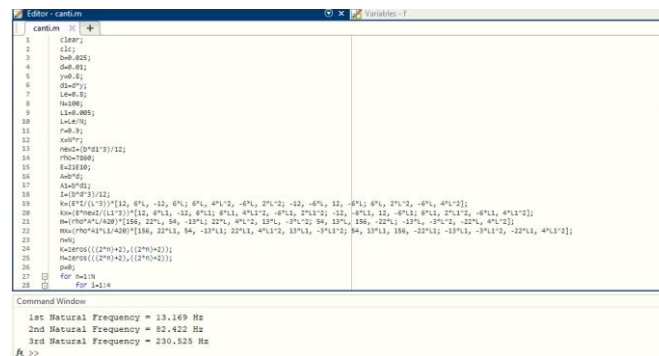


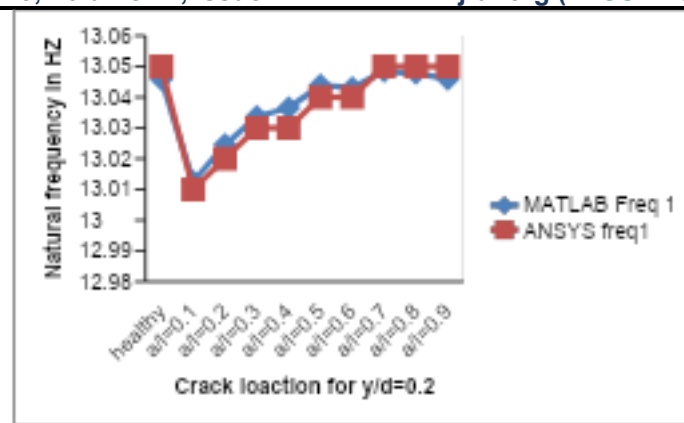
Fig 7: Matlab readings for 3 mode shapes for cantilever beam and crack at $a/l=0.9$.

The above image presents the output obtained from MATLAB, displaying the three natural frequencies of the cantilever beam. The first cyclic frequency is 13.1 Hz, the second is 82.4 Hz, and the third is 230.5 Hz. These frequencies correspond to the first, second, and third mode shapes of the beam, respectively.

Results and discussion

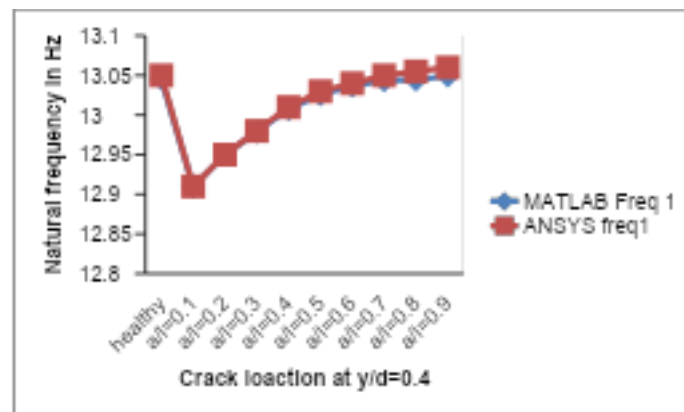
In this study, cantilever, simply supported, and overhanging beams with cracks were analyzed using ANSYS and MATLAB software to identify defect locations based on vibration behavior. The primary objective was to determine the exact position of cracks by examining changes in the beams' natural frequencies and mode shapes. Each beam was modeled with its specific boundary conditions, and cracks of varying depths and positions were introduced to study their effect on dynamic response. The results from ANSYS and MATLAB showed strong agreement, validating the reliability of both analytical and numerical approaches. It was observed that the presence of a crack reduced stiffness, leading to a decrease in natural frequency and noticeable changes in mode shapes. These variations served as clear indicators of defect locations.

Cantilever beam results



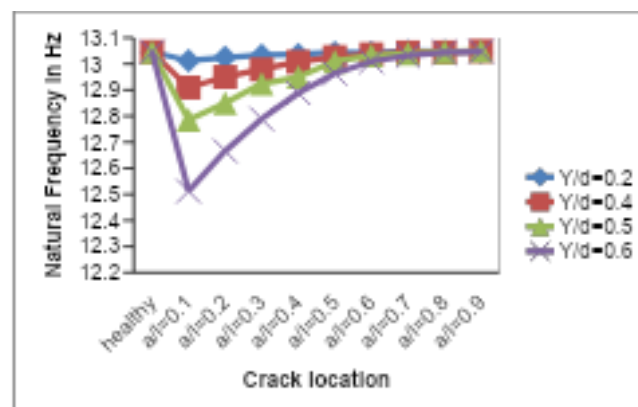
Graph 1 : Natural frequency readings for Y/d=0.2.

The graph illustrates the variation of natural frequency for a crack depth ratio of ($y/d = 0.2$), comparing MATLAB and ANSYS results for the first vibration mode. As the crack moves along the beam, a gradual frequency change is observed, with a maximum value of 13.04587 Hz at ($a/L = 0.1$), indicating a nearly healthy beam condition. The close agreement between MATLAB and ANSYS results confirms the accuracy and reliability of both methods in predicting the beam's dynamic behavior under minor crack conditions.



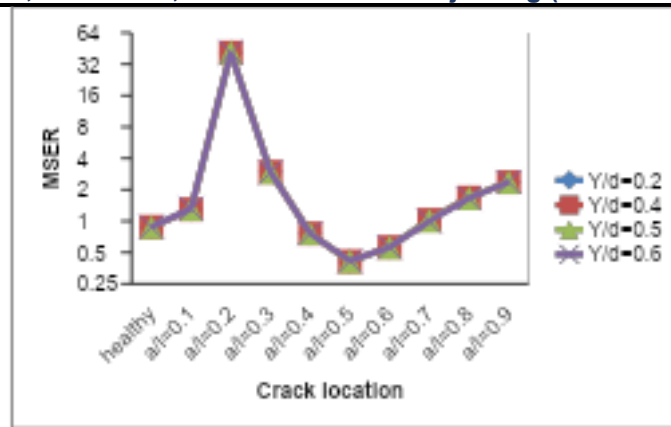
Graph 2 : Natural frequency readings for Y/d=0.4.

The graph illustrates the variation of natural frequency for a crack depth ratio of ($y/d = 0.2$), comparing first-mode frequencies obtained from MATLAB and ANSYS. As the crack location shifts toward ($a/L = 0.9$), corresponding to the healthier region of the beam, the frequency attains a maximum value of 13.05 Hz. Conversely, when the crack is nearer to the fixed end ($a/L = 0.1$), the frequency slightly decreases due to stiffness reduction.



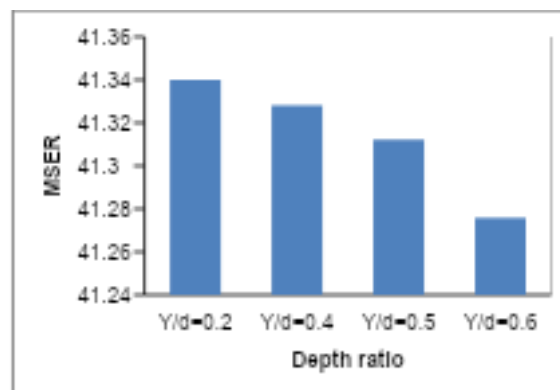
Graph 3: First natural frequency for different Y/d & a/L.

The graph illustrates the variation of the first natural frequency with different crack depth ratios (y/d) and crack locations (a/L). Results are plotted for four depth ratios: ($y/d = 0.2$), ($y/d = 0.4$), ($y/d = 0.5$), and ($y/d = 0.6$). The maximum natural frequency of about 13.024 Hz remains nearly constant for shallower cracks, showing minimal stiffness reduction. As the crack depth increases, the frequency gradually decreases due to reduced structural rigidity. The lowest frequency of 12.51192 Hz occurs at ($y/d = 0.6$), confirming that deeper cracks significantly affect the beam's dynamic behavior by lowering stiffness and natural frequency.



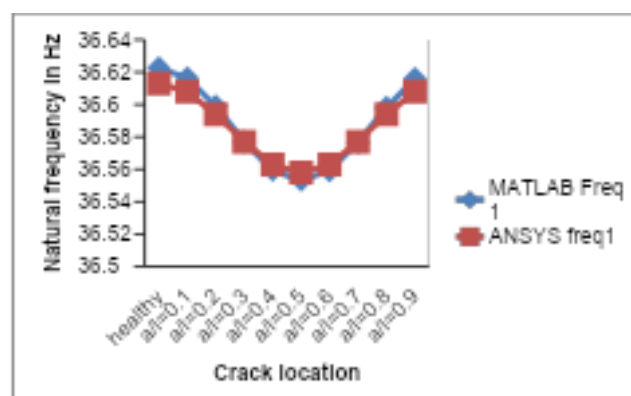
Graph 4 : MSER reading for different Y/d & a/L.

The graph illustrates the variation of the Modal Strain Energy Ratio (MSER) with different crack depth ratios (y/d) and crack locations (a/L). The MSER distribution shows a sinusoidal trend, indicating periodic variations in strain energy concentration caused by cracks. Results are plotted for four depth ratios: $y/d = 0.2$ (blue), $y/d = 0.4$ (red), $y/d = 0.5$ (green), and $y/d = 0.6$ (purple). The maximum MSER value of about 41.33 occurs at $a/L = 0.2$, showing strong energy localization near this crack position, while the minimum value of around 12.51 appears at $a/L = 0.6$. The overall trend suggests that MSER patterns remain consistent for all crack depths, with crack location having a dominant influence on strain energy distribution along the beam.

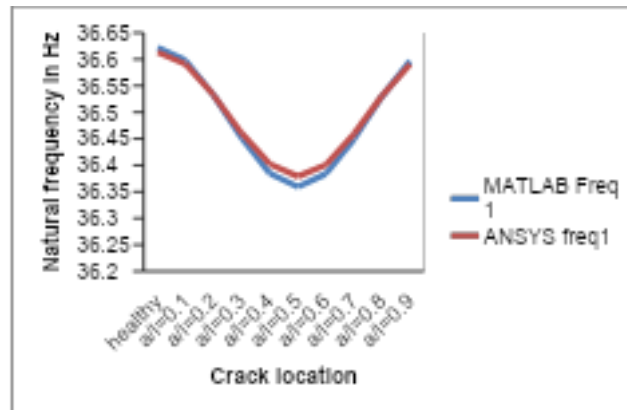
Graph 5: MSER reading for different Y/d at $a/L(\text{MAX})=0.2$.

The graph shown above represents the Modal Strain Energy Ratio readings for different crack depth ratios ((Y/d)) at a fixed crack location of ($a/L = 0.2$). The analyzed depth ratios include ($Y/d = 0.2$), (0.4), (0.5), and (0.6). From the results, it is observed that the maximum MSER occurs at ($Y/d = 0.2$), while the minimum MSER is recorded at ($Y/d = 0.6$). This trend indicates an inverse relationship between the crack depth ratio and the MSER value—meaning that as the crack depth increases, the modal strain energy ratio decreases. Physically, this implies that shallower cracks result in higher strain energy concentration near the defect, whereas deeper cracks tend to dissipate the strain energy more evenly along the beam. Hence, the MSER serves as a sensitive parameter for identifying and quantifying the severity of cracks in beam structures.

4.2 Simply supported beam results

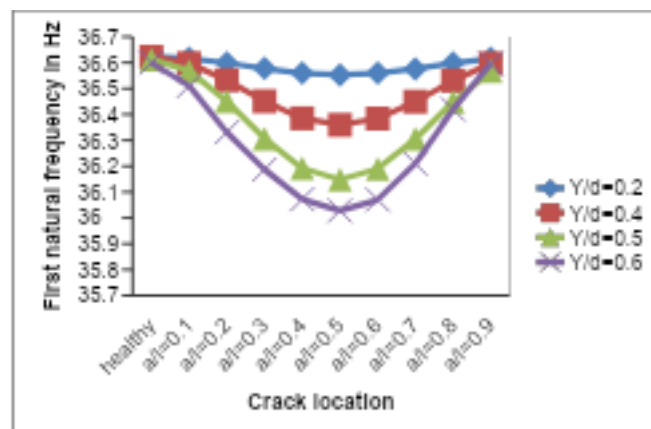
Graph 6: Natural frequency readings for $Y/d=0.2$.

The graph shown above illustrates the natural frequency variation for ($Y/d = 0.2$), comparing results obtained from MATLAB and ANSYS for the first mode of vibration. The MATLAB frequencies are depicted in blue, and the ANSYS frequencies are shown in red. The plotted data follows a sinusoidal trend, indicating the sensitivity of the natural frequency to changes in crack position along the beam. As the crack location moves toward the free end (healthy section), the natural frequency reaches its maximum value of 13.04587 Hz, whereas the minimum frequency of 36.55 Hz occurs at ($a/L = 0.5$).



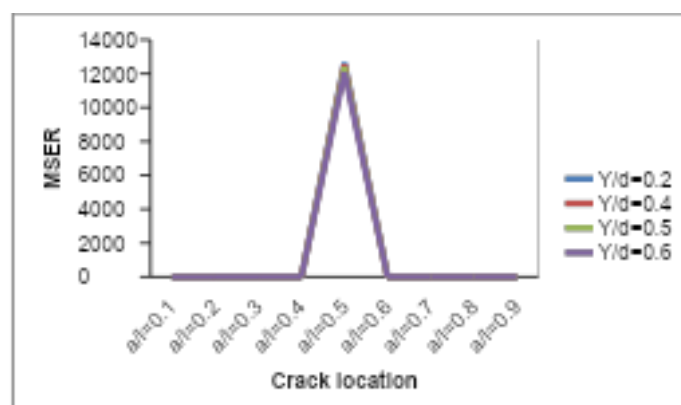
Graph 7 : Natural frequency readings for $Y/d=0.4$.

The graph shown above represents the natural frequency variation for ($Y/d = 0.4$), comparing the results obtained from MATLAB and ANSYS for the first mode of vibration. The MATLAB frequencies are shown in blue, and the ANSYS frequencies are represented in red. As the crack location moves toward the healthy region of the beam, the natural frequency attains its maximum value of 36.22 Hz, while the minimum frequency, observed at ($a/L = 0.5$), is 36.35 Hz.



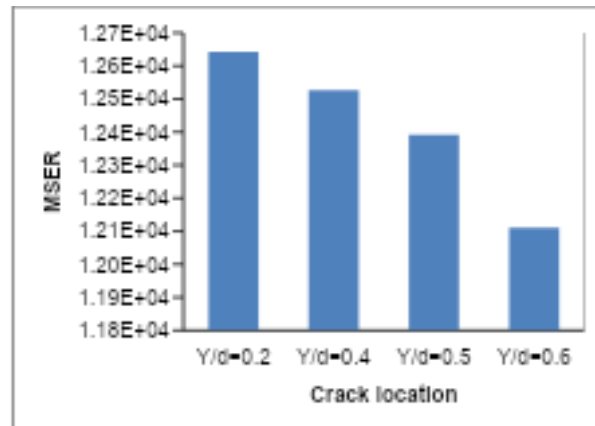
Graph 8 : First natural frequency for different Y/d & a/L .

The graph shown above represents the first natural frequency variation for different (Y/d) ratios and crack locations ((a/L)). Four depth ratios were considered — ($Y/d = 0.2, 0.4, 0.5,$) and (0.6) — where the curves for each are color-coded: blue for ($Y/d = 0.2$), red for ($Y/d = 0.4$), green for ($Y/d = 0.5$), and purple for ($Y/d = 0.6$). From the graph, it can be observed that the maximum natural frequency is obtained for ($Y/d = 0.6$), with a value of 13.024 Hz, whereas the minimum natural frequency occurs at ($a/L = 0.5$), which is 12.51192 Hz for ($Y/d = 0.6$). This behavior indicates that as the crack depth increases, the stiffness of the beam decreases, resulting in a corresponding reduction in natural frequency.



Graph 9: MSER reading for different Y/d & a/L .

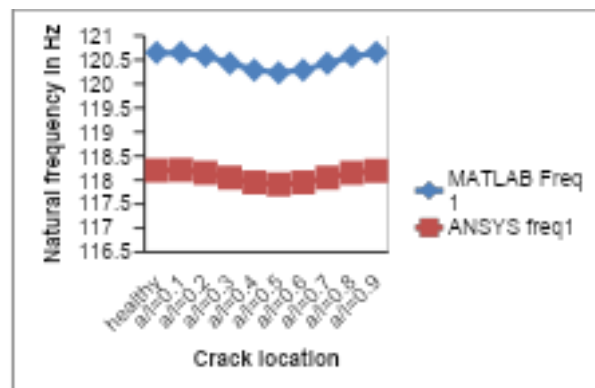
The graph shown above represents the Modal Strain Energy Ratio variation for different (Y/d) ratios and crack locations (a/L). The curve exhibits a triangular waveform pattern, illustrating the fluctuation of strain energy with changing crack positions. The depth ratios considered are (Y/d = 0.2, 0.4, 0.5, and 0.6), where each is color-coded as follows: blue for (Y/d = 0.2), red for (Y/d = 0.4), green for (Y/d = 0.5), and purple for (Y/d = 0.6). From the graph, it can be observed that the maximum MSER for all (Y/d) values is approximately 1.26×10^4 , occurring at (a/L = 0.5).



Graph 10: MSER reading for different Y/d at a/L(MAX)=0.5.

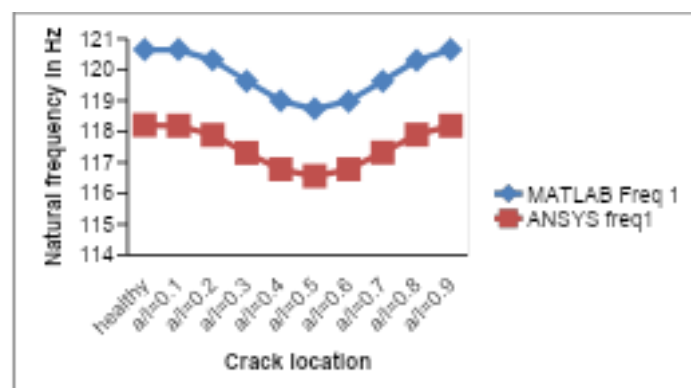
The graph shown above illustrates the Modal Strain Energy Ratio variation for different (Y/d) ratios at a fixed crack location of (a/L = 0.5). The considered depth ratios are 0.2, 0.4, 0.5, and 0.6. From the graph, it is observed that the maximum MSER occurs at (Y/d = 0.2), while the minimum MSER is recorded at (Y/d = 0.6). This trend clearly indicates that as the crack depth increases, the modal strain energy ratio decreases.

4.3 Overhanging beam



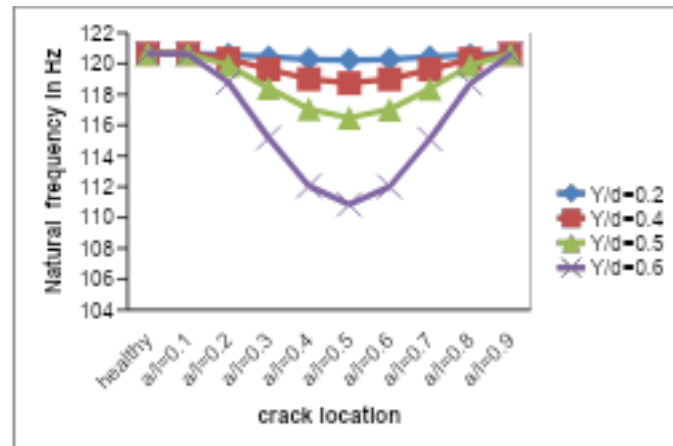
Graph11 : Natural frequency readings for Y/d=0.2.

The graph shown above illustrates the natural frequency variation for (Y/d = 0.2), comparing the results obtained from MATLAB and ANSYS for the first mode of vibration. The MATLAB frequencies are shown in blue, and the ANSYS frequencies are represented in red. From the graph, it is observed that as the crack location moves toward the healthy region of the beam, the natural frequency increases, reaching a maximum value of 120.6492 Hz. The minimum natural frequency is recorded at (a/L = 0.5), with a value of 120.2306 Hz.



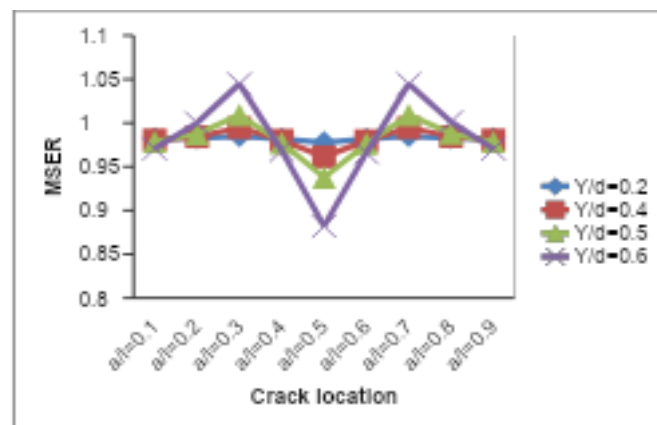
Graph 12: Natural frequency readings for Y/d=0.4.

The graph shown above represents the natural frequency variation for ($Y/d = 0.4$), comparing the results obtained from MATLAB and ANSYS for the first mode of vibration. The MATLAB frequencies are shown in blue, and the ANSYS frequencies are represented in red. The graph exhibits a sinusoidal variation, demonstrating the fluctuation of natural frequency with respect to crack position. As the crack location moves toward the healthy region, the natural frequency reaches its maximum value of 120.6492 Hz, whereas the minimum natural frequency, observed at ($a/L = 0.5$), is 118.7339 Hz.



Graph 13 : First natural frequency for different Y/d & a/L .

The graph shown above illustrates the first natural frequency variation for different (Y/d) ratios and crack locations ((a/L)). Four different depth ratios were considered — ($Y/d = 0.2, 0.4, 0.5,$) and (0.6) — with color representations as follows: blue for ($Y/d = 0.2$), red for ($Y/d = 0.4$), green for ($Y/d = 0.5$), and purple for ($Y/d = 0.6$). From the graph, it can be observed that the maximum natural frequency for all (Y/d) values occurs at the healthy region, reaching 120.6492 Hz, while the minimum natural frequency is recorded at ($a/L = 0.5$) for ($Y/d = 0.6$), with a value of 120.2306 Hz.



Graph 14 : MSER reading for different Y/d & a/L .

The graph shown above illustrates the Modal Strain Energy Ratio variation for different (Y/d) ratios and crack locations ((a/L)). The curve exhibits a triangular wave pattern, showing the variation in strain energy distribution with respect to the crack position. The considered depth ratios are ($Y/d = 0.2, 0.4, 0.5,$) and (0.6), represented by blue, red, green, and purple colors, respectively. From the graph, it is observed that the maximum MSER occurs for ($Y/d = 0.6$) at ($a/L = 0.7$), with a value of 1.044743, while the minimum MSER is found for ($Y/d = 0.2$) at ($a/L = 0.6$), with a value of 0.981787.

Conclusion

In this study, damage detection in beams was performed using natural frequency analysis and the Modal Strain Energy Ratio (MSER) method for cantilever, simply supported, and overhanging configurations. Cracks with varying depth ratios ($(y/d = 0.2-0.6)$) and locations ($(a/L = 0.1-0.9)$) were analyzed using ANSYS and MATLAB, showing strong agreement between both platforms. The MSER method effectively identified crack locations through stiffness variation, with maximum strain energy near critical regions such as the fixed support or midspan. Results showed that natural frequency decreased with increasing crack depth, confirming stiffness reduction. For cantilever, simply supported, and overhanging beams, the first natural frequencies obtained from ANSYS and MATLAB closely matched, validating the method's reliability. The combined use of natural frequency and MSER analysis proved to be a robust, accurate, and computationally efficient approach for structural health monitoring, offering early crack detection capabilities of engineering structures.

References

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