IJRAR.ORG E-ISSN: 2348-1269, P-ISSN: 2349-5138

INTERNATIONAL JOURNAL OF RESEARCH AND ANALYTICAL REVIEWS (IJRAR) | IJRAR.ORG

An International Open Access, Peer-reviewed, Refereed Journal

CONFORMALLY INVARIANT ANISOTROPIC MODELS

R.N. Patra

Associate Professor

Department of Mathematics, Berhampur University, Odisha, India

Abstract : Bianchi type-I anisotropic homogeneous space-time in presence of conformally invariant scalar field is considered. The solution of the Einstein's field equations is obtained by applying the law of variation of Hubble parameter. The power land and exponential forms of the average scale factor in terms of cosmic time is generated from the exact solutions of the field equations corresponding to singular and non-singular cosmological models. The kinematical properties include a constant value of deceleration parameter and the solutions satisfies the observations of the present-day universe.

PACS: 04.20.JB;04.02-Q;98.80.Cq;04.50+h

Keywords: Cosmology, Bianchi Type-I, Conformally Invariant Scalar Fields

I. INTRODUCTION

The large-range forces are produced by Scalar fields. Gravitational physics & Cosmology focus on scalar tenser theories which provide the most natural generalizations of general relativity. The study of zero rest mass scalar fields in general relativity provides an understanding of the nature of the space-time and gravitational field associated with neutral elementary particles of zero spin. Bergmann and Leipnik [1] studied solutions of einstein's field equations in static, spherically symmetric space-time with an energy momentum tensor generated from a neutral massless scalar field. Callan et. al. [2] proposed a theory of gravitation in which the renormalized energy momentum tensor defines the same four momentum and Lorentz generators as the conventional tensor. The theory meets all the experimental tests that have been applied to general relativity. Fryland [3] studied zero-mass scalars fields coupled to the gravitational field in the static, spherically symmetric case, is completely solve for a traceless energy momentum tensor.

Accioly et.al. [4] obtained solutions of field equations to the conformally invariant scalar field with trace free energy-momentum tensor as source in Bianchi type-I space-time. Innaiah and Reddy [5] have discussed a flat Robertson-Walker-type solution treating the conformally invariant scalar field as the matter field. Venkateswarlu and Reddy [6] have obtained Bianchi type-II, VIII and IX cosmological solutions in conformally invariant scalar field with trace-free electromagnetic energy momentum tensor. Maharaj and Beeaham [7] have discussed Robertson-Walker-type universe with conformally invariant scalar field to be independent of time. Shri Ram [8,9] has obtained a exact solutions of the Einstein equations corresponding to a conformally invariant scalar field with trac-free energy momentum tensor as source in Bianchi Type-I and Bianchi VI0 space-times.

In this paper, we obtained new classes of models with constant deceleration parameters Einsteins field equations for a spatially homogeneous and anisotropic Bianchi type-I space-time in presence of conformally invariant scalar field. Sect.2 present the metric field equations. Sect.3 Present the exact solutions of the equations by using the law of variation of Hubble's parameter. These solutions correspond to singular and nonsingular models in two types of cosmologies. The physical and kinematical behaviors of the cosmological models are studied by plotting various graphs and the conclusions are summarized in section 4.

II. THE METRIC AND FIELD EQUATIONS

We consider the diagonal form of the Bianchi tuype-I metric given by

$$ds^2 = dt^2 - A^2dx^2 + B^2dy^2 + C^2dz^2 \quad (1)$$

where A(t), B(t) and C(t) are the cosmic scale factors. The field equations for the conformally invariant scalar field and the gravitational field from a conformally invariant action integral are (Accioly [4]):

$$R_{\mu\nu} f(u) = g_{\mu\nu} u_{,\alpha}u^{,\alpha} - 4u_{,\mu}u_{,\nu} + 2uu_{;\mu\nu}, \quad (2)$$

$$R = 0, \quad (3)$$

$$u^{;\upsilon}_{;\mu} = 0 \quad (4)$$

Where a comma denotes ordinary derivative and a semicolon denotes covariant derivative and

$$f(u) = 1 - u^2 \quad (5)$$

$$u = \left(\frac{k}{6}\right)^{1/2} \phi; \quad (6)$$

\ being massless scalar field. Other symbols have their usual meaning.

The field equations (2) – (6) for metric (1), in comoving coordinates, lead to

$$\left(\frac{\ddot{A}}{A}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}\right)f(u)=\dot{u}^2+2u\dot{u}\frac{\dot{A}}{A}, \quad (7)$$

$$\left(\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}\right)f(u)=\dot{u}^2+2u\dot{u}\frac{\dot{B}}{B}, \quad (8)$$

$$\left(\frac{\ddot{C}}{C}+\frac{A\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}\right)f(u)=\dot{u}^2+2u\dot{u}\frac{\dot{C}}{C}, \quad (9)$$

$$\left(\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}\right)f(u)=-3\dot{u}^2+2u\ddot{u}, \quad (10)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}=0, \quad (11)$$

$$\ddot{u}+\dot{u}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \quad (12)$$

Here a dot represent differentiation with respect to time t. The average scale factor R of the metric (1) is defined as

$$R=(ABC)^{1/3} \quad (13)$$

Equation (12), on integration, gives

$$\dot{u}=\frac{k_1}{R^3} \quad (14)$$

Where k_1 is a constant of integration.

The volume scale factor V is given by

$$V=R^3=ABC. \quad (15)$$

We define the generalized mean Hubble's para1meter H as

$$H=\frac{1}{3}(H_1+H_2+H_3), \quad (16)$$

where $H_1=\frac{\dot{A}}{A}, H_2=\frac{\dot{B}}{B}$ and $H_3=\frac{\dot{C}}{C}$ are directional Hubble's parameters in the direction of x, y and z respectively.

From(13), (15) and (16), we obtain

$$H=\frac{\dot{R}}{R}=\frac{1}{3}(H_1+H_2+H_3). \quad (17)$$

For the metric (1) in comoving coordinates the expansion scalar \, which determines the volume behaviour of the fluid, is given by

$$\theta=\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=3H \quad (18)$$

An important quantity q, the deceleration parameter in cosmology, is given by

$$q=-\frac{R\ddot{R}}{\dot{R}^2}. \quad (19)$$

This parameter measures the rate of expansion at which the expansion of the universe is changing with time in terms of scale factors, and its sign characterizes accelerating or decelerating nature of the universe. In case $q<0$, the universe decelerates whereas $q<0$, describes an accelerating universe. It may be noted that though the current observations of Sne Ia and CMB favour accelerating models($q<0$), they do not altogether rule out the decelerating ones which are also consistent with these observations (Vishwakarms, [10]).

Now, following the approach of Saha and Rikhvitsky [11], Singh et. al.[12] and shri Ram et. al., [13,14], we solve the field equations (7)-(12). Substracting (7) from (8), (8) from (9) and 97) from (9), we get the following three relations respectively:

$$\frac{B}{A}=\alpha_1 exp\left[\gamma_1\int \frac{dt}{(1-u^2)R^3}\right], \quad (20)$$

$$\frac{C}{B}=\alpha_2 exp\left[\gamma_2\int \frac{dt}{(1-u^2)R^3}\right], \quad (21)$$

$$\frac{C}{B}=\alpha_3 exp\left[\gamma_3\int \frac{dt}{(1-u^2)R^3}\right] \quad (22)$$

where $\alpha_1,\alpha_2,\alpha_3$ and $\gamma_1,\gamma_2,\gamma_3$ are constants of integration. From (20)-(22), the metric functions can be written respectively as

$$A=m_1R\ exp\ exp\left[\frac{\square_1}{3}\int \frac{dt}{(1-u^2)R^3}\right], \quad (23)$$

$$B=m_2R\ exp\ exp\left[\frac{\square_2}{3}\int \frac{dt}{(1-u^2)R^3}\right], \quad (24)$$

$$C=m_3R\ exp\ exp\left[\frac{\square_3}{3}\int \frac{dt}{(1-u^2)R^3}\right] \quad (25)$$

where $\lambda_1=\frac{-(2\gamma_1+\gamma_2)}{3}, \lambda_2=\frac{\gamma_1-\gamma_2}{3}, \gamma_3=\frac{\gamma_1+2\gamma_2}{3}$ and

$m_1=\sqrt[3]{d_1^{-2}d_2^{-1}}, m_2=\sqrt[3]{d_1d_2^{-1}}, m_3=\sqrt[3]{d_1d_2^2}.$

The constants $\lambda_1,\lambda_2,\lambda_3$ and m_1,m_2,m_3satisfies the relations

$$\lambda_1+\lambda_2+\lambda_3=0, m_1m_2m_3=1 \quad (26)$$

It is clear, from(23)-(25), that once we get the value of the average scale factor R, we can obtain solutions of the metric functions A, B, C.

III. VARIATION LAW FOR HUBBLE PARAMETER

In order to solve the field equations, let us assume that the Hubble's parameter H is related to the average scale factor R by relation

$$H = lR^{-n}, \tag{27}$$

where $l > 0$ and $n \geq 0$ are constants.

From (17) and (27), we get

$$\dot{R} = lR^{-n+1}, \tag{28}$$

$$\ddot{R} = -l^2(n-1)R^{-2n+1}. \tag{29}$$

From (19) , (28) and (29), we obtain

$$q = n - 1. \tag{30}$$

Equation (30) indicates that the deceleration parameter q is constant. Thus the special law of variation of Hubble's parameter (27) yields a constant value of q. The sine of q indicates whether the model inflates or not. The positive sign of q corresponds to standard decelerating model whereas the negative sine indicates inflation. Intergrating (28), we obtain the laws of variation for the average scale factor R as

$$R = (nlt + c_1)^{1/n}, n \neq 0, \tag{31}$$

$$R = c_2 exp(lt), \tag{32}$$

where c1 and c2 are integration constants. Equation (30) implies that the condition for expanding universe is n = 1 + q > 0.

IV. EXACT SOLUTIONS

We now find the solution of integrals in (23)-(25) in two types of cosmologies for $n \neq 0$ and n = 0 explicitly.

A. POWER LAW SOLUTION ($n \neq 0$)

From (14) and (31), we obtain

$$u = \frac{\gamma_1}{(n-3)l}\left[(nlt + c_1)^{n-3/n} + \gamma_2\right] \tag{33}$$

where γ_2 is an integration constant. Using the values of u and R into (23)-(25) we obtain solutions of A, B and C as follows:

$$A(t) = l_1(nlt + c_1)^{1/n}\left[\frac{(n-3)l+\gamma_1(nlt+c_1)^{n-3/n}+\gamma_2}{(n-3)l-\gamma_1(nlt+c_1)^{n-\frac{3}{n}}-\gamma_2}\right]^{\frac{x_1}{6\gamma_1}}, \tag{34}$$

$$B(t) = m_1(nlt + c_1)^{1/n}\left[\frac{(n-3)l+\gamma_1(nlt+c_1)^{n-3/n}+\gamma_2}{(n-3)l-\gamma_1(nlt+c_1)^{n-\frac{3}{n}}-\gamma_2}\right]^{\frac{x_2}{6\gamma_1}} \tag{35}$$

$$C(t) = m_1(nlt + c_1)^{1/n}\left[\frac{(n-3)l+\gamma_1(nlt+c_1)^{n-3/n}+\gamma_2}{(n-3)l-\gamma_1(nlt+c_1)^{n-\frac{3}{n}}-\gamma_2}\right]^{\frac{x_3}{6\gamma_1}} \tag{36}$$

The directional Hubble's Parameters H1, H2, H3 and Hubble parameter are obtained as

$$H_1 = \frac{l}{nlt+c_1} + \frac{2\lambda_1(n-3)^2l^2(nlt+c_1)^{-3/n}}{3\gamma_1[(n-3)^2l^2-\{\gamma_1(nlt+c_1)+\gamma_2\}^2]}, \tag{37}$$

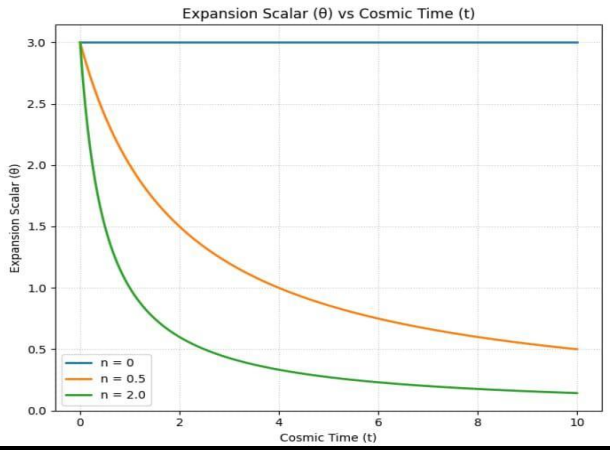
$$H_2 = \frac{l}{nlt+c_1} + \frac{2\lambda_1(n-3)^2l^2(nlt+c_1)^{-3/n}}{3\gamma_1[(n-3)^2l^2-\{\gamma_1(nlt+c_1)+\gamma_2\}^2]}, \tag{38}$$

$$H_3 = \frac{l}{nlt+c_1} + \frac{2\lambda_3(n-3)^2l^2(nlt+c_1)^{-3/n}}{3\gamma_1[(n-3)^2l^2-\{\gamma_1(nlt+c_1)+\gamma_2\}^2]}, \tag{39}$$

$$H = \frac{3l}{nlt+c_1} \tag{40}$$

The expansion scalar $\theta = 3H$ is

$$\theta = \frac{3l}{(nlt+c_1)}. \tag{41}$$



It is observed that the spatial volume is zero and the expansion scalar is infinite at t = t_1 where $t_1 = \frac{-c_1}{nl}$, which shows that the universe starts evolving with zero volume at t = t_1 with an infinite rate of expansion. The directional scale factors vanish at t = t_1 and therefore the model has a point singularity at the initial epoch. the universe exhibits the power-law expansion after the Big-Bang impulse. as t increases the scale factors and the spatial volume increase but the expansion scalar decreases. Thus, the rate of expansion slows down as the cosmic evolution progress.

The positive value of q indicates that the universe is decelerating. The scalar function u, being infinite at t = t_1, decreases as time increases and ultimately becomes a constant for large time.

B. EXPONENTIAL SOLUTION (n = 0)

In this case, we obtain an exponentially expanding non-singular cosmological model describing the accelerating phase of the present day universe.

From (14) and (32), we obtain

$$u = \gamma_3 - \frac{\gamma_1}{3lc_2} exp(-3lt) \tag{42}$$

where γ_3 is an integration constant. Using the value of u and R into (23)-(25), we obtain

$$A = m_1 exp(lt) \left[\frac{2c_2 l+\gamma_1(\gamma_3-exp(-3lt))}{2c_2 l-\gamma_1(\gamma_3-exp(-3lt))}\right]^{\frac{\gamma_1}{6c_2\gamma_1}} \tag{43}$$

$$B = m_2 exp(lt) \left[\frac{2c_2 l+\gamma_1(\gamma_3-exp(-3lt))}{2c_2 l-\gamma_1(\gamma_3-exp(-3lt))}\right]^{\frac{\gamma_2}{6c_2\gamma_1}} \tag{44}$$

$$C = m_3 exp(lt) \left[\frac{2c_2 l+\gamma_1(\gamma_3-exp(-3lt))}{2c_2 l-\gamma_1(\gamma_3-exp(-3lt))}\right]^{\frac{\gamma_3}{6c_2\gamma_1}} \tag{45}$$

The directional Hubble's parameters H_1, H_2 and H_3 are obtained as

$$H_1 = l + \frac{3\gamma_1 l^2}{9c_2 l^2-\gamma_1^2(\gamma_3-exp-3lt)^2}, \tag{46}$$

$$H_2 = l + \frac{3\gamma_2 l^2}{9c_2 l^2-\gamma_1^2(\gamma_3-exp-3lt)^2}, \tag{47}$$

$$H_3 = l + \frac{3\gamma_3 l^2}{9c_2 l^2-\gamma_1^2(\gamma_3-exp-3lt)^2}, \tag{48}$$

whereas the average generalized Hubble's parameter H = l

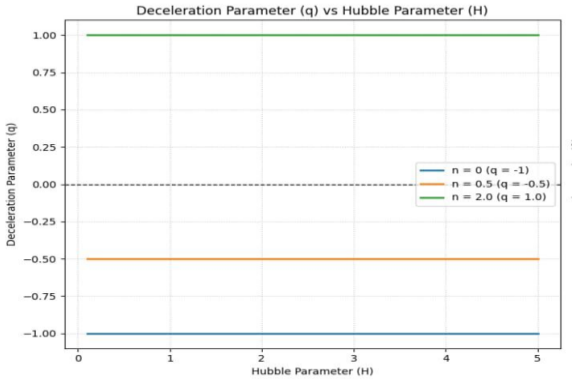
The spatial volume, expansion scalar and deceleration parameter are obtained as

$$V = c_2^3\, exp\, exp\,(3lt)\,, \tag{49}$$

$$\theta = 3l, \tag{50}$$

$$q = -1. \tag{51}$$

From (32) it is observed that as $t \to -\infty, R \to 0$ which shows the exponential inflationary phase of the universe is the universe is infinitely old. The directional scale factors and spatial volume of the universe increase exponentially with cosmic time, whereas the



mean Hubble parameter and the expansion scalar are constant throughout.

As $t \to \infty$, the scale factors and volume of the universe become infinitely large, which shows that the universe is dominated by vacuum energy which drives the accelerated expansion of the universe. The scalar function u is a decreasing function of time, which tends to zero for large time. $q = -1, \frac{dH}{dt} = 0$, implies the greatest value of the Hubble parameter and the fastest rate of expansion of the universe.

V. CONCLUSION

In this paper we have investigated spatially homogeneous and an anisotropic Bianchi type-I cosmological models of the universe with constant deceleration parameter within the framework of conformally invariant scalar field. We have obtained solutions by applying the law of variation of Hubble parameter in two types of cosmologies, one with power-law expansion and other one with exponential expansion. The cosmological model with power-law expansion has a point singularity at t = t1. The rate pf expansion is a decreasing function of time and finally tends to zero as $t \to -\infty$. The scalar function is infinite at the initial singularity and becomes negligible for large time. The scalar factors and the spatial volume become infinitely large as $t \to -\infty$ which would give essentially an infinite universe. The positive value of deceleration parameter shows that the universe is decelerating one. The cosmological model with exponential expansion has no finite singularity. The directional scale factors are time dependent while the average Hubble parameter is constant. The expansion of the universe right from the beginning. The scalar function is always

decreasing function of time and tends to zero for large time. Since q = -1, this model describes the accelerating phase of the present-day universe.

VI. ACKNOWLEDGMENT

I convey thanks to Dr. R.R. Swain, Department of Physics, U.P. Sc. College, Sheragada, GM for her support to carry out the research work.

REFERENCES

[1] O. Bergmann and R. Leipnik, Space-Time Structure of a Static Spherically Symmetric Scalar Field, Phys. Rev., 107 (1957), 1157-1161.

[2] C.G.Callan, J.S. Coleman and R. Jackiw, A New Improved Energy Momentum Tensor, Ann. Phys., 59 (1970), 42-73.

[3] J. Frφyland, Phys. Rev. D, 25 (1982), 1470-1473.

[4] A.J. accioly, A.N. Vaidya and M. M. Som, Bianchi type-I Models with Conformally Invariant Scalar Field, Phys. Rev. D., 27 (1983), 2282-2284.

[5] O. Innaiah and D.R.K. Reddy, Robertson-Walker Type Model with Conformally Invariant Scalar Field with Trace-free Energy Momentum Tensor, Astrophys. Space. Sci., 117 (1985), 65-67.

[6] R. Venkateswarlu and D. R. K. Reddy, Bianchi type-II, VII and IX Cosmologies with Conformally Invariant Scalar Field and Electromagnetic Field, Astrophys. Space. Sci., 184 (1991), 153-156.

[7] S. D. Maharaj and A. Beesham, Robertson-walker-Type Universes with Conformally Invariant scalar Field, Astrophys. Space. Sci., 136 (1987), 315-320.

[8] S. Ram, Bianchi Type-I Models with Conformally Invariant Scalar Field Trace-free Energy Momentum Tensor, IL Nuovo Cime. B, 81 (1984), 1-8.

[9] S. Ram, Bianchi Type-Vio Conformally Invariant Scalar Fields, Astrophys. Space. Sci.,, 104 (1984), 151-162.

[10] R.G. Vishwakarma, a Study of angular Size Redshift Relation for Models in Which Lambda Decays as the Energy Density, Class. Quant. Grav., 17 (2000), 3833-3842.

[11] B. Saha and V. Rikhvitsky, Bianchi Type-I Universe with Viscous Fluid and a Λ, Phusica D, 219 (2006), 168-176.

[12] C. P. Singh, S. Ram and M. Zeyauddin, Bianchi Type-V Perfect Fluid space-time Model in General Relativity, Astrophys. Space. Sci., 315 (2008), 181-187.

[13] S. Ram, M. K. Verma and M. Zeyauddin, Homogeneous anisotripic Cosmological Models with Viscous and Heat Flow in Lyra's Geometry, Mod. Phys. Lett. A, 24 (2009), 1847-1856.

[14] S. Ram, M. K. verma and M. Zeyauddin, spatially Homogeneous Bianchi type V cosmological Models in Scale Covariant Theory of Gravitation, Chin. Phys. Lett., 26 (2009), 089802-1-4.

[15] M. S. Berman, a Special Law of variation for Hubble's Parameter, Nuovo cime. B., 74 (1983), 182-186.

[16] M. S. Berman and F. Fomide, Cosmological Models with Constant Deceleration parameter, Gen. Rel. Grav., 20 (1988), 191-198.

[17] B. Saha, Anisotropic Cosmological Models with Perfect Fluid and a Λ Term, Astrophys. Space. Sci., 302 (2006), 83-91.

[18] J. P. Singh and P. S. Baghel, Bianchi Type-V Cosmological Models with Constant Deceleration Parameter in General Relativity, Int. J. Theor. Phys. 48 (2009), 449-462.

[19] A. Pradhan and K. Jotania, Some Exact Bianchi Type-V Perfect Fluid Cosmological Models with Heat Flow and Vacuum Eergy Density Λ Expressions for Some Observable Quantities, Int. J. Theor. Phys., 49 (2010), 1719-1738.

[20] A. Pradhan, H. Amirhaschi and B. Saha, Bianchi Type-I Anisotropic Dark energy Models with Constant Deceleration Parameter Int. J. Theor. Phys., 50, (2011), 2923-2930.

[21] A. Pradhan, Anisotropic Bianchi Type-I Magnetized String Cosmological Models with Decaying Vacuum Energy Density Λ, Commun. Theor. Phys., 55 (2011), 931-941.