



Slip-Driven Buongiorno Heat and Mass Transport in Unsteady Williamson Nanofluid

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Abstract - Many engineering and industrial applications, including, biomedical, food processing, and heat exchangers, demand thorough analysis of heat and mass transfer, particularly when Williamson nanofluids are used due to their non-Newtonian characteristics and improved thermal efficiency. The thermal and boundary slip of Unsteady Williamson nanofluid flow over a stretching sheet with effects of magnetic field, viscous dissipation and linear chemical reaction effects are studied numerically. Buongiorno model is utilized to explore heat transfer phenomenon caused by Brownian motion and thermophoresis. By using the suitable similarity transformation of unsteady flow, the fundamental PDE's of the following model are reduced to non-dimensional ODE's. The created model is solved utilizing the inbuilt operation bvp4c method of MATLAB software. The outcomes reveal that rising Ec , W and Pr helps in rising the heat transfer characteristics, while rising Sc and C_r modifies mass transfer.

Keywords: Chemical reaction, Slip Conditions, Viscous Dissipation, Williamson Nanofluid

Nomenclature

A	Stretching parameter
B_0	Applied magnetic field strength (T)
B^*	Dimensional Thermal slip coefficient
C	Nanoparticle concentration (kg m^{-3})
C_∞	Ambient nanoparticle concentration (kg m^{-3})
C_w	Wall nanoparticle concentration (kg m^{-3})
C_r	Chemical reaction parameter
C_p	Specific heat at constant pressure ($\text{J kg}^{-1} \text{K}^{-1}$)
C_f	Skin friction coefficient
D_B	Brownian diffusion coefficient ($\text{m}^2 \text{s}^{-1}$)
D_T	Thermophoretic diffusion coefficient ($\text{m}^2 \text{s}^{-1}$)
Ec	Eckert number
g	Dimensionless stream function
K	Permeability parameter

K_p	Permeability parameter (Non-dimensional)
K_r	Chemical reaction rate constant
M	Magnetic parameter
N_b	Brownian motion parameter
N_t	Thermophoresis parameter
Nu_x	Local Nusselt number
Pr	Prandtl number
q_m	Mass flux ($\text{kg m}^{-2} \text{s}^{-1}$)
q_w	Heat flux (W m^{-2})
Re	Reynolds number
Sc	Schmidt number
Sh_x	Local Sherwood number
T	Fluid temperature (K)
T_∞	Ambient temperature (K)
T_w	Wall temperature (K)
u, v	Velocity components in x and y directions (m s^{-1})
u_w	Stretching sheet velocity (m s^{-1})
W	Williamson fluid parameter (Non-dimensional)
x, y	Cartesian coordinates

Greek

α	Unsteadiness parameter
β	Thermal slip parameter
δ	Velocity slip parameter
δ^*	Dimensional velocity slip coefficient
η	Similarity variable
θ	Dimensionless temperature
ϕ	Dimensionless concentration
κ	Thermal conductivity ($\text{W m}^{-1} \text{K}^{-1}$)
μ	Dynamic viscosity ($\text{kg m}^{-1} \text{s}^{-1}$)
ϑ	Kinematic viscosity ($\text{m}^2 \text{s}^{-1}$)
ρ	Fluid density (kg m^{-3})

ρC_p	Volumetric heat capacity ($\text{J m}^{-3} \text{K}^{-1}$)
σ_e	Electrical conductivity (S m^{-1})
τ_w	Wall shear stress (N m^{-2})
I	Williamson fluid parameter (s)
ψ	Stream function

Introduction

In modern trends of fluid flow and their property analysis, nanofluids have emerged as a revolutionary class of engineered fluids that blend nanotechnology with classical thermofluidic systems. Among the myriad models that describe non-Newtonian behavior in nanofluids, the Williamson nanofluid model stands out for its refined capacity to capture shear-thinning characteristics, named after the physicist R.L. Williamson. This model characterizes fluids whose apparent viscosity diminishes with rising shear rate, making it particularly suitable for polymers, biofluids, and controlled flow behavior making it an ideal candidate for high-precision cooling systems, medical diagnostics, and smart material processing. In the findings of Krishnamurthy *et al.*¹, Williamson nanofluid belongs to the class of Visco-inelastic fluids. The foundational study on velocity boundary layers over flat plates was conducted by Blasius², which was later extended by Sakiadis³, who explored both approximate and exact theoretical methods for laminar and turbulent flows in flat and cylindrical boundary layers. Building on these classical models, Ramesh *et al.*⁴ concluded in their study that the thermal boundary layer in Blasius flow is thicker as compared to the Sakiadis flow. Prior to that, Khan and Khan⁵ examined a wide area of models say, Blasius, Sakiadis, stretching and stagnation-point flows. Srinivasulu and Goud⁶ worked on Williamson nanofluid flow and concluded that enhanced magnetic parameter thickens the boundary layer. Krishnamurthy *et al.*⁷ studied the impacts of heat source and chemical reaction on magneto Williamson nanofluid in the porous medium whereas Hashim and Hamid⁸ reviewed convective flows of Williamson fluid.

The phenomena of velocity slip and thermal slip have garnered widespread scholarly attention, owing to their profound practical significance in the realm of microscale and nanoscale applications. Unlike traditional no-slip boundary conditions where fluid velocity and temperature at a surface are assumed to match those of the boundary velocity slip allows the fluid to move tangentially relative to the surface, while thermal slip permits a finite temperature difference between the fluid and the surface at the interface. These modified boundary conditions offer a more realistic depiction of fluid behavior, especially in environments where classical assumptions fail, such as in microfluidics, nanotechnology, and rarefied gas flows. Sajadifar *et al.*⁹ taken slip velocity and temperature jump boundary conditions for fluid flow and heat transfer of non-Newtonian nanofluid in a microtube. The thermal slip condition for nanofluid flow past stretching cylinder being explored by Mishra and Kumar¹⁰. Khan *et al.*¹¹ conducted the investigation of heat transfer under effect of velocity slip boundary condition in carbon nanotubes suspended nanofluid flow. Jat *et al.*¹² taken heat and mass transfer analysis for Casson nanofluid. Raju *et al.*¹³ worked on Williamson Hydromagnetic Fluid Flow and found out that the thermal property uplifts on rising Brownian motion, thermophoresis, thermal radiation, and heat source parameters. Shahsavari *et al.*¹⁴ conducted a comprehensive investigation regarding the effects of nanoparticle volume concentration on hybrid nanofluid flow. Zueco and Ahmed¹⁵ conducted coupled heat and mass transfer study of fluid flow. In general, considering slip conditions plays a vital role in advancing modern technologies that demand high precision, efficiency, and compact design.

Unsteady flow describes the fluid motion where the flow properties change with time at any given point of the flow, these flows capture the dynamic, time-dependent behavior of fluids, making it essential for accurately defining many real-world engineering and natural systems. Ludford *et al.*¹⁶ studied unsteady flow of compressible viscous fluids and setting a mile stone in this way. Venkateswaran and Merkle¹⁷ numerically investigated the dissipation control for viscous and unsteady flows. Further, Chen¹⁸ worked for unsteady flow of non-Newtonian fluids. Unglehart and Manhart¹⁹ prepared a model of unsteady flow for linear dissipation rate across porous media. Mahmood *et al.*²⁰ explored the combined influence of thermal properties on unsteady hybrid nanofluid flow utilizing double stratification. Swami *et al.*²¹ findings demonstrate that the interaction of non-Newtonian fluid behavior, heat generation, and reactive effects. Challa *et al.*²² presented that a rise in Brownian motion increases the fluid's temperature while decreasing

nanoparticle concentration. Shahzad and Sagheer²³ illustrated that the diffusion and Forchheimer flow exhibit a strong influence on the flow regimes. This study contributes to the optimization of processes in industries such as polymer extrusion, chemical engineering, and biomedical applications. It is crucial in applications involving fluid-structure interactions, oscillating boundary conditions, and sudden changes in flow geometry or pressure gradients.

In the realm of advanced fluid dynamics, the integration of magnetohydrodynamic (MHD) effects alongside viscous dissipation is paramount for the precise modeling and effective optimization of a wide spectrum of engineering and industrial processes involving electrically conducting fluids. Li *et al.*²⁴ worked on the effects of buoyancy on magneto MgO–TiO₂ considering slip conditions. With the rise of Williamson parameter velocity declines whereas temperature and nanoparticle volume fraction profiles uplifts as simulated according to Reddy *et al.*²⁵. Zhu *et al.*²⁶ utilized Buongiorno model for examining the properties for Williamson nanofluid. Sharma and Chaudhary²⁷ studied magneto flows, considering shape factor of nanofluids and presented new findings. Idris *et al.*²⁸ inspects magnetized hybrid ferrofluid flow considering the magnetic parameter and Eckert number that reduced the heat transfer. Chaudhary and Chouhan²⁹ discovered flow of Prandtl-Eyring nanofluid considering coupled chemical reaction effects. Chaudhary *et al.*³⁰ analysed magneto flows and Aghalya and Tamizharasi³¹ analysed non-linear thermal radiation in Casson fluids, these effects are especially important in applications like, nuclear fusion reactors, Cooling of nuclear and electronic systems, Magneto-thermo-mechanical systems, astrophysical and geophysical flows.

Notwithstanding the wide applicability of non-Newtonian fluid models, the Williamson model remains relatively unexplored in the context of unsteady stretching surfaces subjected to velocity and thermal slip effects. Based on the available literature and the gaps identified therein, the present investigation addresses the unsteady Williamson nanofluid flow incorporating velocity and thermal slip effects within the Buongiorno framework.

Mathematical Formulation

Consider the two-dimensional time dependent incompressible flow of Williamson nanofluid in presence of vertically impressed magnetic field over stretching sheet as presented in Fig. 1. The velocity components are (u, v) taken in (x, y) axes. The velocity at the boundary layer is taken to be $u_w(x, t) = \frac{ax}{(1-\alpha t)}$, where a is a constant. The velocity slip and thermal slip are incorporated with effects of viscous dissipation. A linear chemical reaction effect is also incorporated to study the effects of mass transfer and effects of heat transfer are utilized using the Buongiorno model. The governing two-dimensional boundary layer equations for the unsteady flow of a non-Newtonian Williamson nanofluid are expressed as follows-

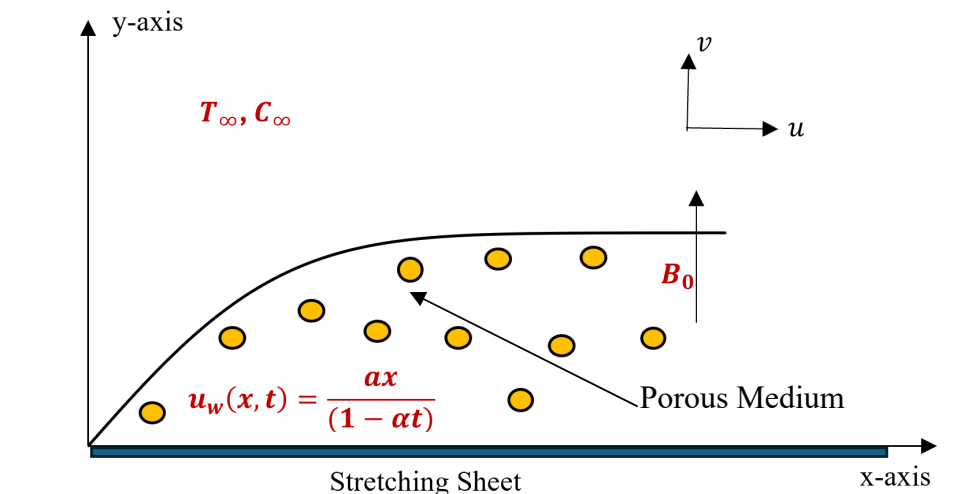


Fig.1- Schematic of flow geometry

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_e B_0^2}{\rho} u - \frac{\mu u}{\rho K} + \sqrt{2} \Gamma \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{(\rho C)_p}{(\rho C)_f} \left(D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right) + \frac{\mu}{(\rho C_p)} \left(\frac{\partial u}{\partial y} \right)^2 \quad (3)$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} - K_r (C - C_\infty) \quad (4)$$

The boundary conditions incorporated with flow are given by

$$u = u_w + \delta^* \mu \left(\frac{\partial u}{\partial y} \right), v = 0, T = T_w + B^* \left(\frac{\partial T}{\partial y} \right), C = C_w \quad \text{at } y = 0$$

$$u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{as } y \rightarrow \infty \quad (5)$$

Here, δ^* and B^* are the velocity and thermal slip parameters. The PDE's in the above equations are highly non-linear, in order to solve them numerically we can change them in non-dimensional ODE's. So, for this purpose we will use the following similarity transformations.

$$\psi = \sqrt{\frac{a\vartheta}{(1-at)}} g(\eta), \quad \eta = \sqrt{\frac{a}{\vartheta(1-at)}} y, \quad \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \quad \phi(\eta) = \frac{C-C_\infty}{C_w-C_\infty} \quad (6)$$

Where, g , θ and ϕ are functions of η and prime (') denotes derivative with respect to η .

Equation of Continuity given in Eq. (1) satisfy the equations $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$. The transformation in Eq. (6), reduces the Eqs. (2)-(4) and boundary conditions (5) in the following form

$$g'''(1 + Wg'') = A \left(\frac{\eta}{2} g'' + g' \right) + g'^2 - gg'' - (M + K_p)g' \quad (7)$$

$$\frac{1}{Pr} \theta'' = A \frac{\eta}{2} \theta' - g\theta' - N_b \phi' \theta' - N_t \theta'^2 + Ec g''^2 \quad (8)$$

$$\frac{1}{Sc} \phi'' = A \frac{\eta}{2} \phi' - g\phi' - \frac{N_t}{N_b Sc} \theta'' + C_r \phi \quad (9)$$

With the following boundary conditions

$$\eta = 0: g(0) = 0; \quad g'(0) = 1 + \delta g''(0); \quad \theta(0) = 1 + \beta \theta'(0); \quad \phi(0) = 1$$

$$\eta \rightarrow \infty: g'(\infty) \rightarrow 0; \quad \theta(\infty) \rightarrow 0; \quad \phi(\infty) \rightarrow 0 \quad (10)$$

Where, $A = \frac{\alpha}{a}$ is the stretching parameter, $M = \frac{\sigma_e B_0^2 (1-at)}{\rho a}$ is the magnetic parameter, $K_p = \frac{\vartheta}{aK}$ is the permeability parameter, $W = \Gamma \sqrt{\frac{2a^3}{\vartheta^3(1-at)^3}}$ is the non-dimensional Williamson nanofluid parameter, $Pr = \frac{\vartheta}{a}$ is the Prandtl number, $N_t = \frac{(\rho C)_p D_T (T_w - T_\infty)}{(\rho C)_f T_\infty \vartheta}$ is the thermophoresis parameter, $N_b = \frac{(\rho C)_p D_B}{(\rho C)_f \vartheta} (C_w - C_\infty)$ is the Brownian motion parameter, $Ec = \frac{a^2 x^2}{C_p (T_w - T_\infty) (1-at)^2}$ is the Eckart number, $Sc = \frac{\vartheta}{D_B}$ is the Schmidt number and $C_r = \frac{K_r (1-at)}{a}$ is the chemical reaction parameter and δ and β are the non-dimensional velocity and thermal slip parameter.

Physical Quantities of Interest

Some important fluid flow parameters are skin friction coefficient, Nusselt number and Sherwood number, that are useful in determining the flow behavior and are given by

$$C_f = \frac{\tau_w}{\rho u_w^2}, \quad Nu_x = \frac{x q_w}{\kappa (T - T_\infty)} \quad \text{and} \quad Sh_x = \frac{x q_m}{D_B (C - C_\infty)} \quad (11)$$

Where, $\tau_w = \mu \left[\left(\frac{\partial u}{\partial y} \right)_{y=0} + \frac{\Gamma}{2} \left(\frac{\partial y}{\partial x} \right)_{y=0}^2 \right]$ is the skin friction or shear stress at the surface of stretching surface, $q_w = -\kappa \left(\frac{\partial T}{\partial y} \right)_{y=0}$ is the heat flux from the surface and $q_m = -D_B \left(\frac{\partial C}{\partial y} \right)_{y=0}$ is the surface mass flux. The reduced values of these useful parameters are as follows

$$(Re)^{0.5}C_f = \frac{W}{2}g''(0)^2 + g''(0) \quad (12)$$

$$(Re)^{-0.5}Nu_x = -\theta'(0) \quad (13)$$

$$(Re)^{-0.5}Sh_x = -\phi'(0) \quad (14)$$

Numerical Scheme and Validation

The dimensionless Eqs. (7), (8) and (9) so obtained are coupled, along with corresponding boundary situations in Eq. (10), are encoded for solving the system of differential equations using `bvp4c` of MATLAB. A relative tolerance of 10^{-5} is specified to control solution accuracy. The initial guess is updated to get solution of expected tolerance.

To solve system of equations via `bvp4c`, we firstly convert this system of equations along with boundary conditions as follows:

Let $g = f_1$, $\theta = f_4$ and $\phi = f_6$ then

$$\begin{aligned} f'_1 &= f_2; f'_2 = f_3; f'_3 = \left(\frac{1}{1+Wf_3}\right)(A(0.5\eta)f_3 + f_2) + f_2^2 - f_1f_3 - (M + K_p)f_2; f'_4 = f_5; f'_5 = \\ &Pr Pr (A(0.5\eta)f_5 - f_1f_5 - N_b f_5 f_7 - N_t f_5^2 + Ec f_3^2); f'_6 = f_7; f'_7 = Sc (A(0.5\eta)f_7 - f_1f_7 - C_r f_6) - \\ &\left(\frac{N_t}{N_b}f_6\right) \end{aligned} \quad (15)$$

Boundary conditions:

$$f_1(0) = 0; f_2(0) = 1 + \delta f_3(0); f_4(0) = 1 + \beta f_5(0); f_6(0) = 1; f_2(\infty) = 0; f_4(\infty) = 0; f_6(\infty) = 0 \quad (16)$$

By addressing the problem, the procedure begins with carefully formulating the system in a manner compatible with MATLAB's boundary value problem solver, `bvp4c`. The higher-order differential equations are first rewritten as a system of first-order equations, which are then implemented in a function named `bvpfun`, as described in Eq. (15). The boundary conditions associated with the problem are implemented in `bcfun`, as specified in Eq. (16). An initial approximation of the solution is then provided using the `bvpinit` function. The problem is subsequently solved using the command `sol = bvp4c(@bvpfun, @bcfun, solinit)`, and the results are plotted using the command `plot(sol.x, sol.y)`. From a computational standpoint, $\eta_\infty = 6$ is used for velocity, temperature and concentration profiles.

Under the specific circumstances employed in Kho *et al.*³², comparison made with the current work and is found to be in excellent agreement with the existing literature as shown in Table 1.

Table 1 - Comparison of values of $g''(0)$ with Kho *et al.*³² by taking $\beta = 1$, $W = 0.5$, $Pr = 7$, $N_t = 2$, $N_b = 2.5$, $Sc = 5$ and A, Ec, C_r, M, K_p all zero for different values of δ

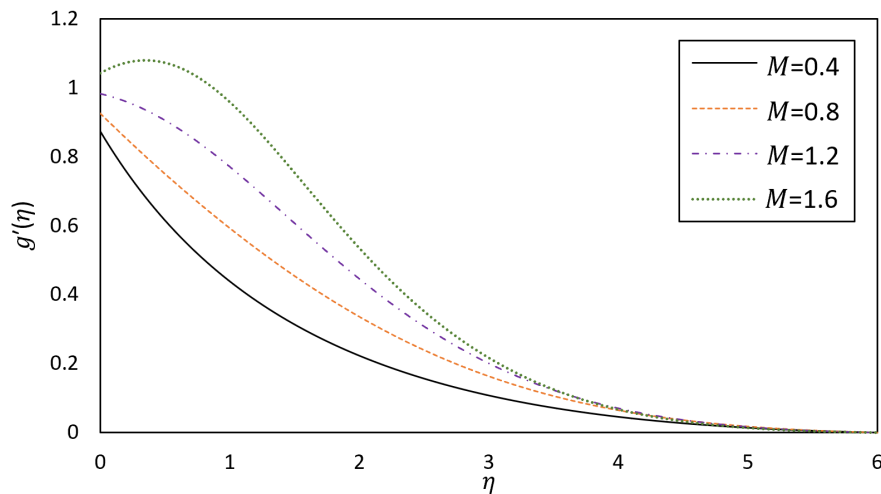
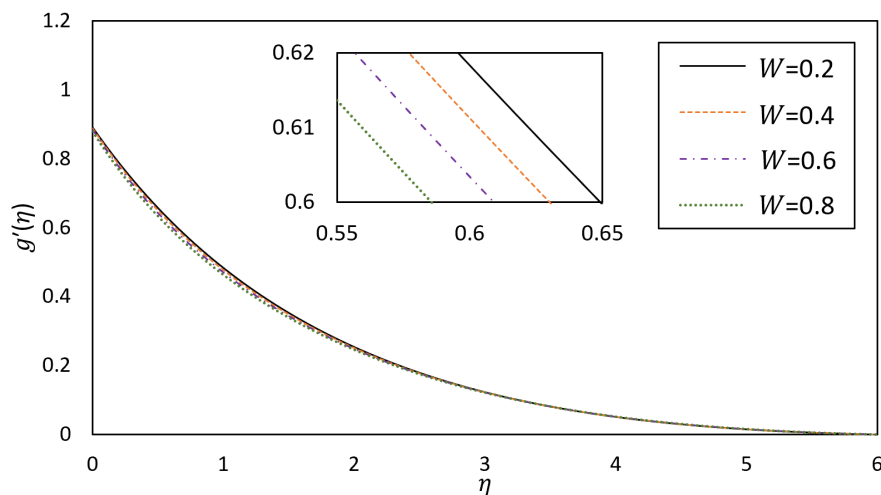
δ	$g''(0)$	
	Kho <i>et al.</i> ³²	Present results
0.25	-0.824627	-0.823986
0.75	-0.521313	-0.521792
1.25	-0.391317	-0.391846
1.75	-0.316124	-0.316187

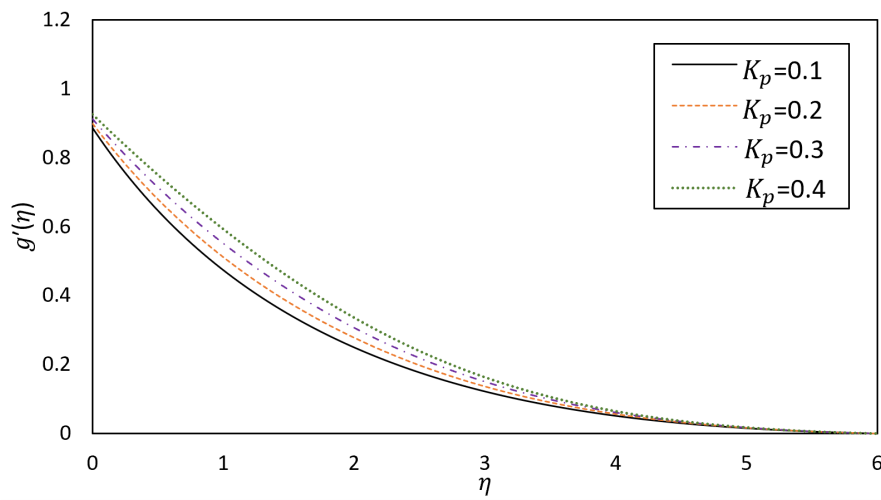
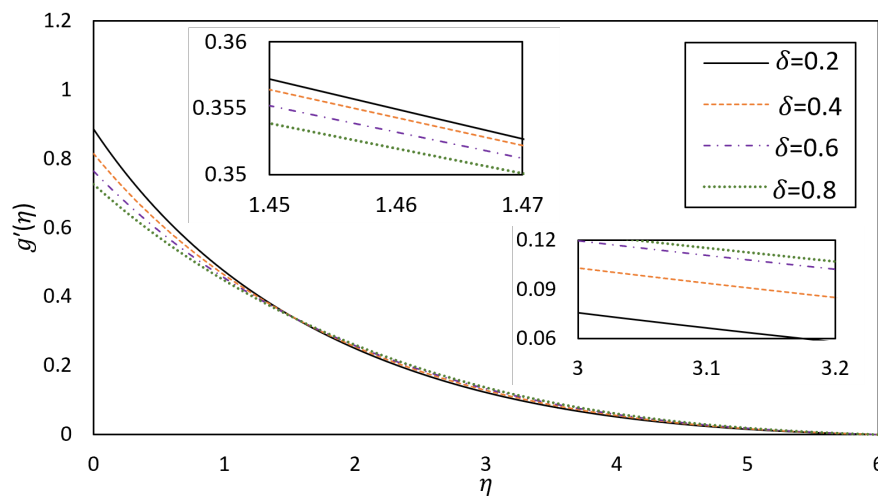
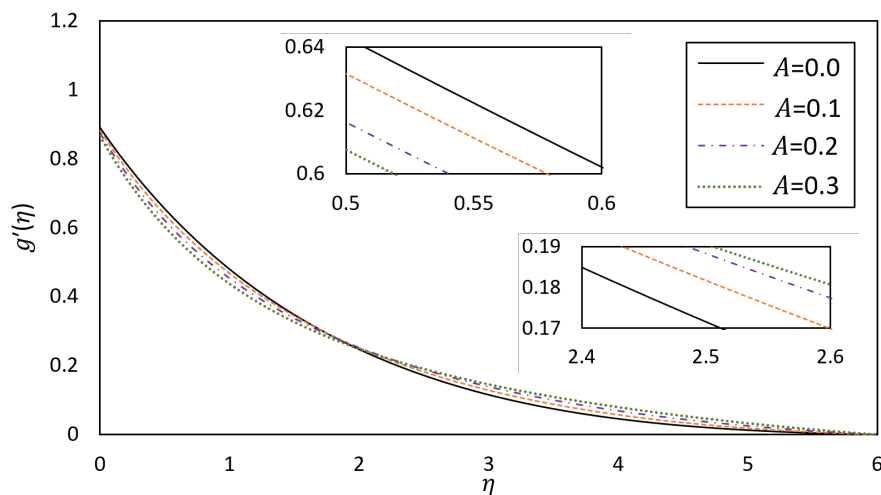
Results and Discussion

The proposed work addresses the consequences of flow parameters on unsteady Williamson nanofluid, considering $\delta = 0.2$, $\beta = 0.3$, $W = 0.5$, $Pr = 4.5$, $N_t = 0.1$, $N_b = 0.3$, $Sc = 1.5$ and $A = 0.05$, $Ec = 0.3$, $C_r = 0.2$, $M = 0.5$, $K_p = 0.1$ for obtaining numerical solutions and visualizing the effects of each parameter on key flow characteristics.

Velocity distribution profiles- Fig. 2 illustrates that with increase of M , significantly uplifts the velocity of the Williamson nanofluid throughout the boundary layer region. Fig. 3 demonstrates that the velocity boundary level declines as the Williamson parameter increases. This takes place as the values of W enhance

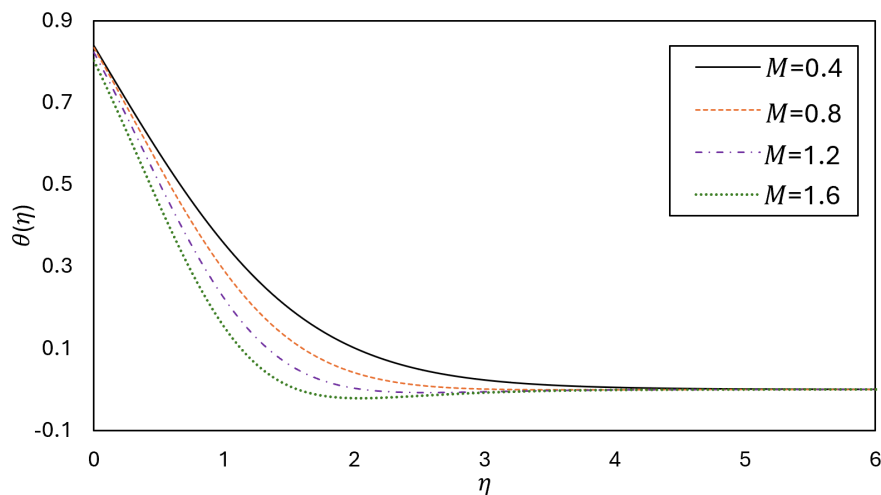
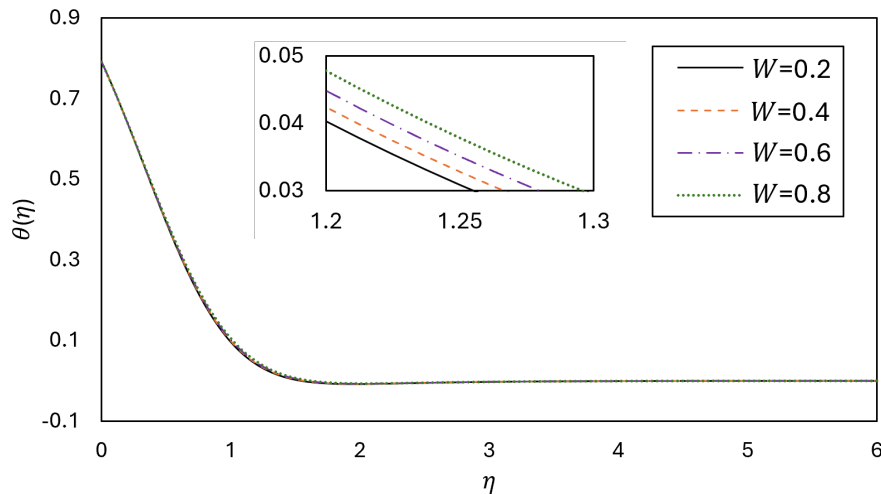
the fluid's non-Newtonian characteristic, increasing the effective viscosity of the Williamson nanofluid resulting in resistance to flow and becomes stronger, which reduces the fluid velocity within the boundary layer. Fig. 4 focuses that the nanofluid's velocity increases with rise in values of K_p . The porosity parameter is inverse to permeability parameter and physically, the porous medium introduces resistance to the fluid motion due to drag forces within the pores. This resistance slows down the flow and leads to a thin velocity boundary layer. Fig. 5 shows in interior range of η increase in δ leads to a reduction in the velocity distribution across the boundary layer but it kept on increasing in outer range of η . Fig. 6 presents that the velocity profile firstly decreases in interior range of η while accelerates in outer range of η with rise of A . Practically, due to higher acceleration enhances the surface's stretching, which drives the fluid more strongly along the sheet. This consequently thickens the momentum boundary layer and enhances nanofluid's velocity.

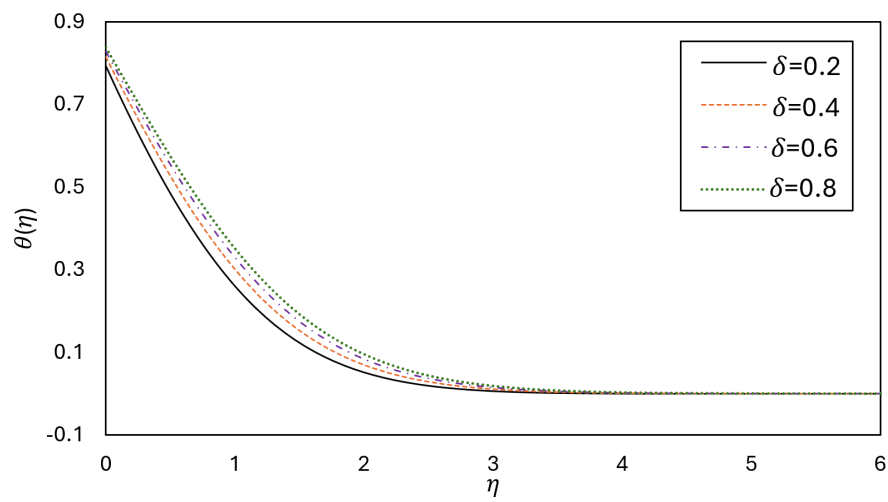
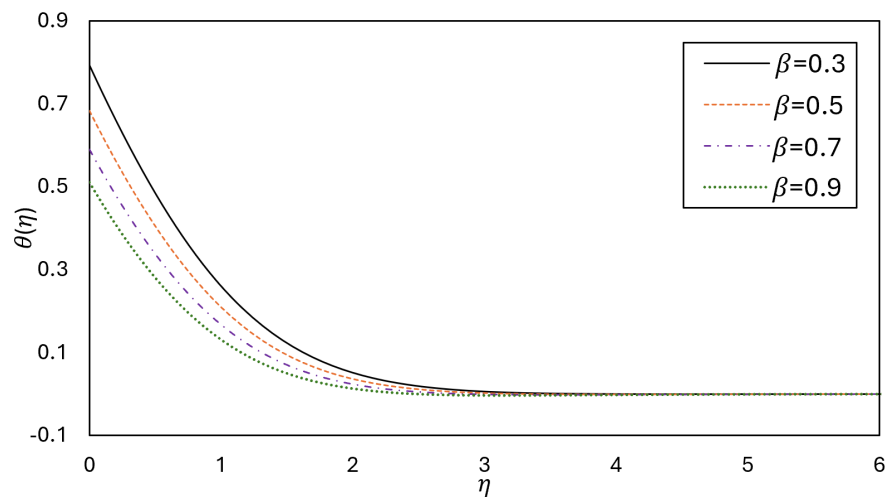
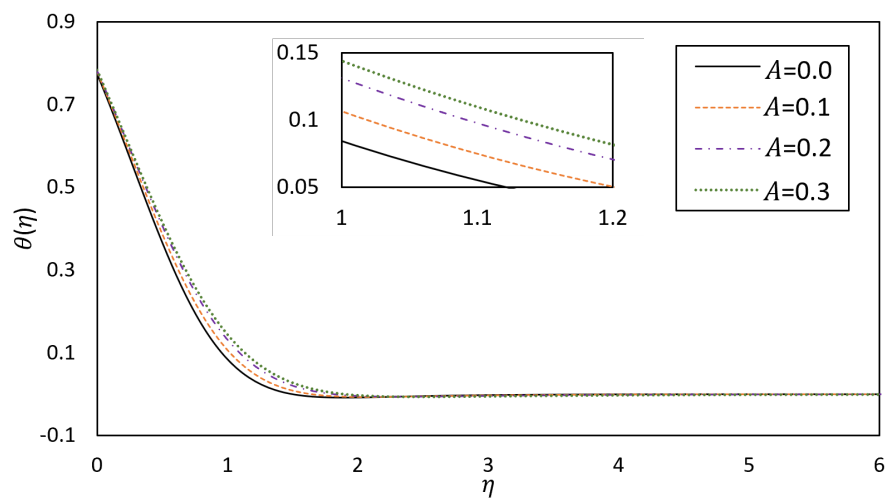
Fig.2- Plot of $g'(\eta)$ vs M Fig.3- Plot of $g'(\eta)$ vs W

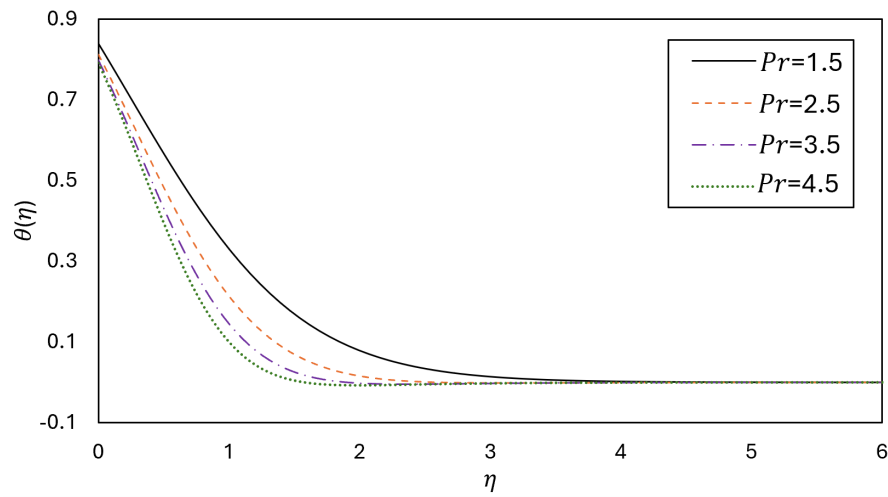
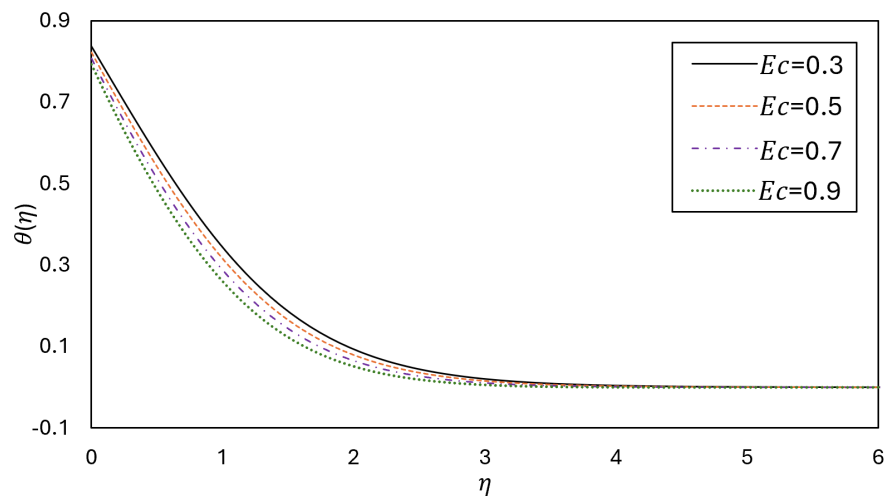
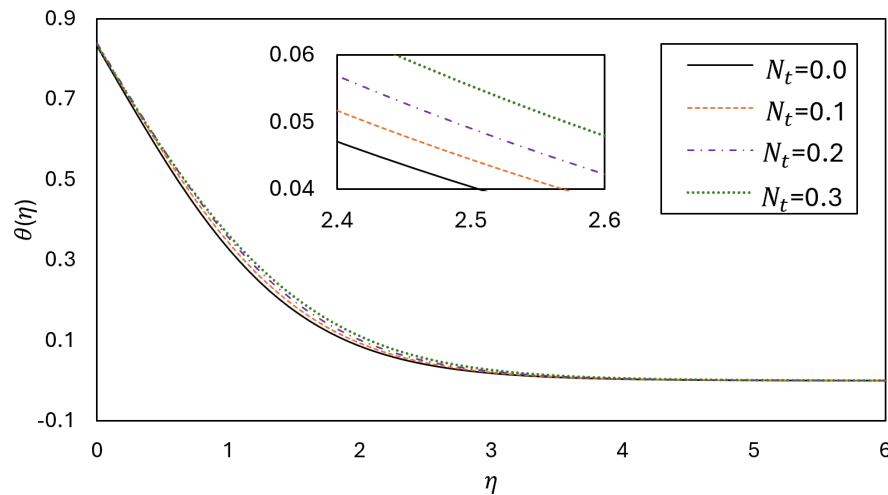
Fig.4- Plot of $g'(\eta)$ vs K_p Fig.5- Plot of $g'(\eta)$ vs δ Fig.6- Plot of $g'(\eta)$ vs A

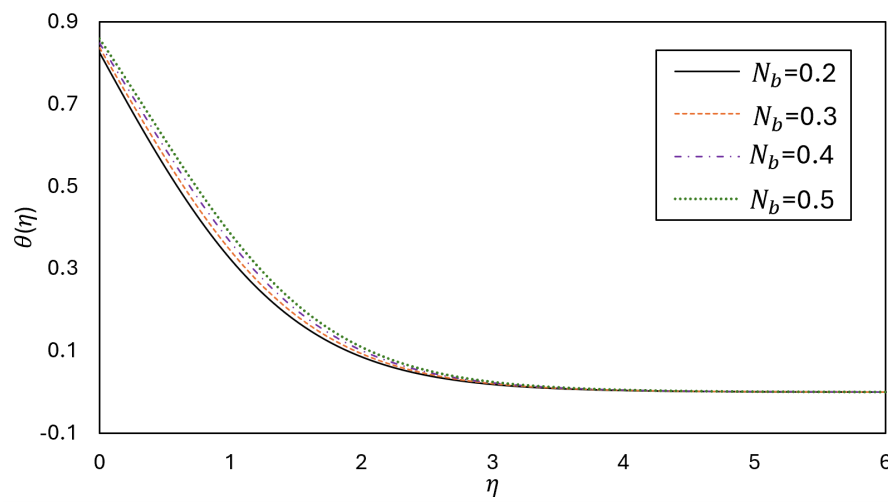
Temperature distribution profiles- Fig. 7 demonstrates that with rise of M , thermal boundary layer declines. It takes place due to a stronger magnetic field resists the fluid's motion, consequently, lowers the thermal energy within the boundary layer. Fig. 8 depicts that due to increment of W , the temperature of the nanofluid increases. Larger values of the Williamson parameter strengthen fluid's non-Newtonian characteristics and increase the effective viscosity, which leads to enhanced viscous effects and generate additional heat within the fluid, which results in an enhanced thermal distribution within the fluid. Fig. 9 shows that the thermal profile rises with increasing values of δ , it is because higher velocity slip reduces the interaction between the fluid and the surface, which weakens heat transfer at the wall and then causes the

fluid temperature within the boundary layer to rise. Fig. 10 illustrates that $\theta(\eta)$ decreases significantly with increasing values of β . It happens because higher thermal slip diminishes the transfer of heat from the wall to the fluid, leading to a decline in thermal boundary layer. Fig. 11 gives that the temperature increases as A increases. Fig. 12 demonstrates that thermal profiles are negatively influenced with rise of Pr . Physically, higher Prandtl numbers correspond to fluids with lower thermal diffusivity, which reduces heat diffusion in the boundary layer and thus results in a thinner thermal boundary layer. Fig. 13 shows evidently that the fluid's temperature decline with higher values of Ec . Fig. 14 clearly depicts that $\theta(\eta)$ upgrades as the values of N_t rises. The thermophoretic force drives nanoparticles from hotter regions toward colder regions, which enhances heat transport within the fluid and increases the temperature distribution. Fig. 15 illustrates that $\theta(\eta)$ increases with rise in values of N_b . This behavior occurs because stronger Brownian motion enhances the random dispersion of nanoparticles and it results enhanced energy exchange within the fluid and thus enhance fluid's temperature.

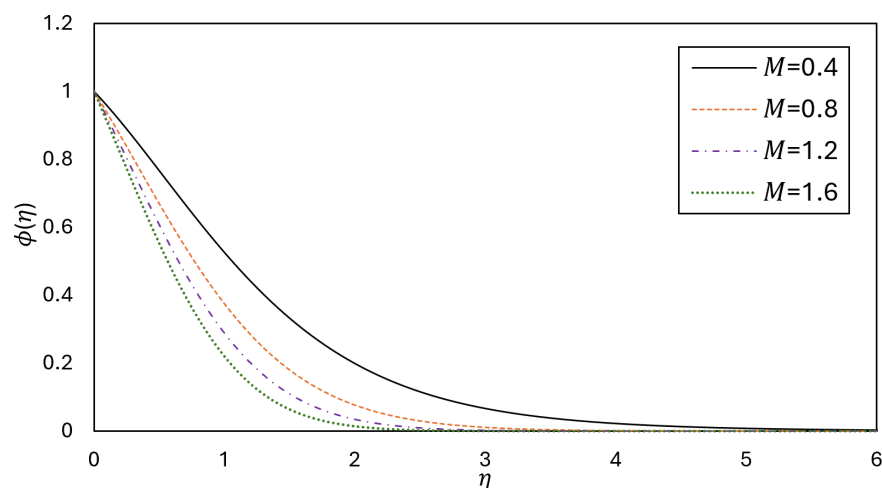
Fig.7- Plot of $\theta(\eta)$ vs M Fig.8- Plot of $\theta(\eta)$ vs W

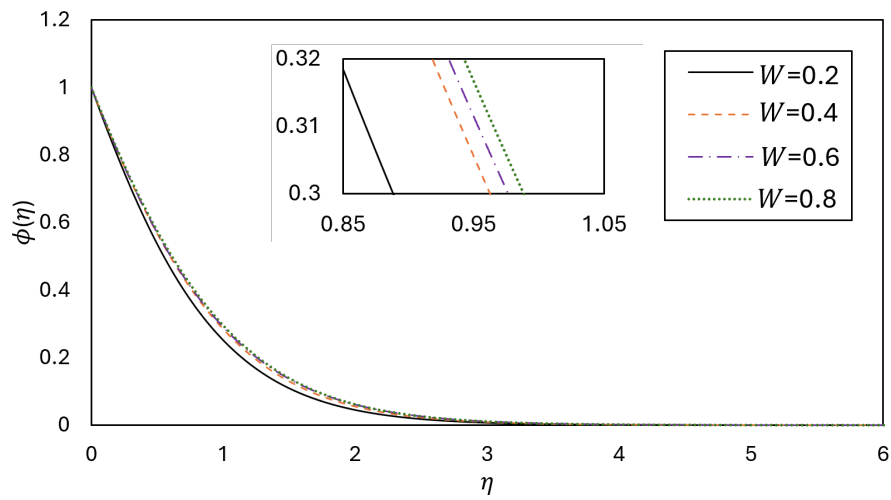
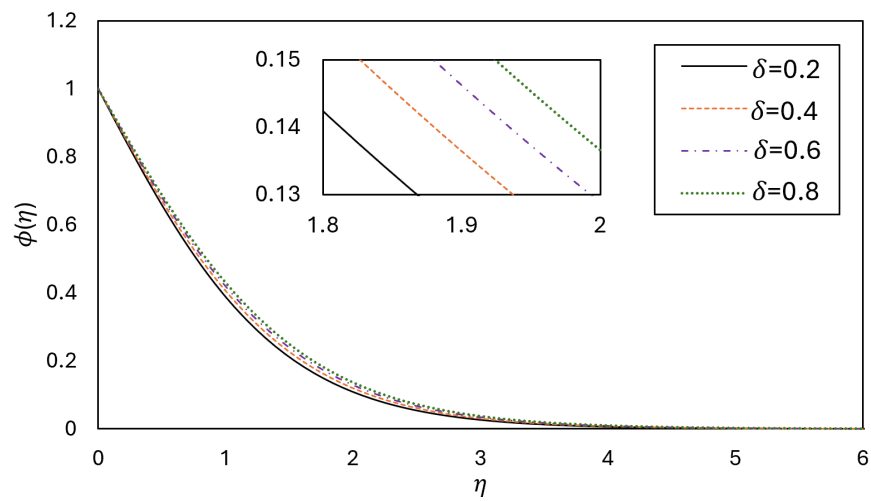
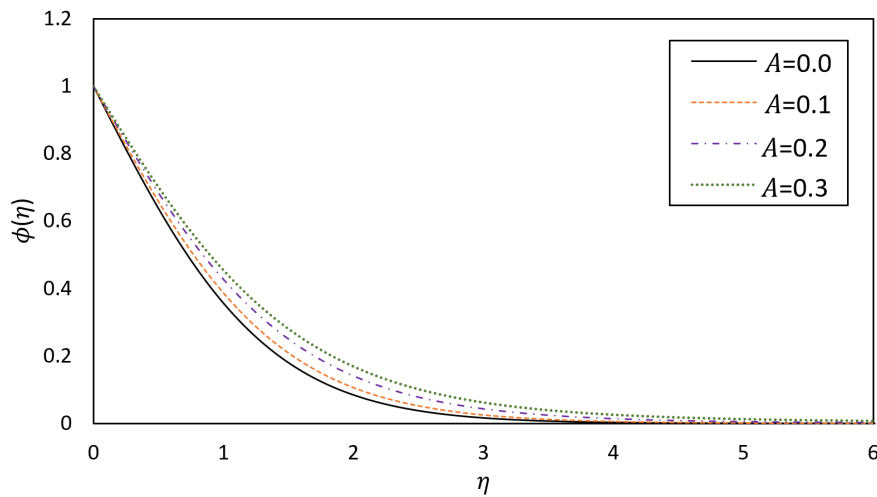
Fig.9- Plot of $\theta(\eta)$ vs δ Fig.10- Plot of $\theta(\eta)$ vs β Fig.11- Plot of $\theta(\eta)$ vs A

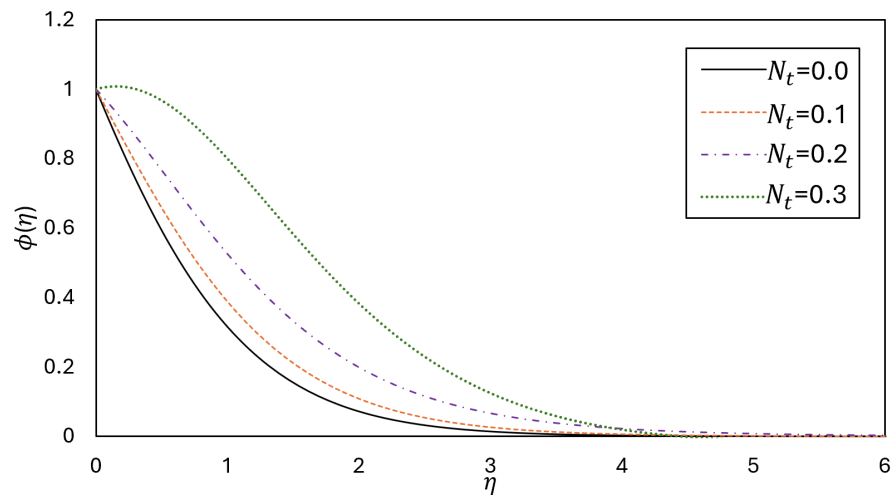
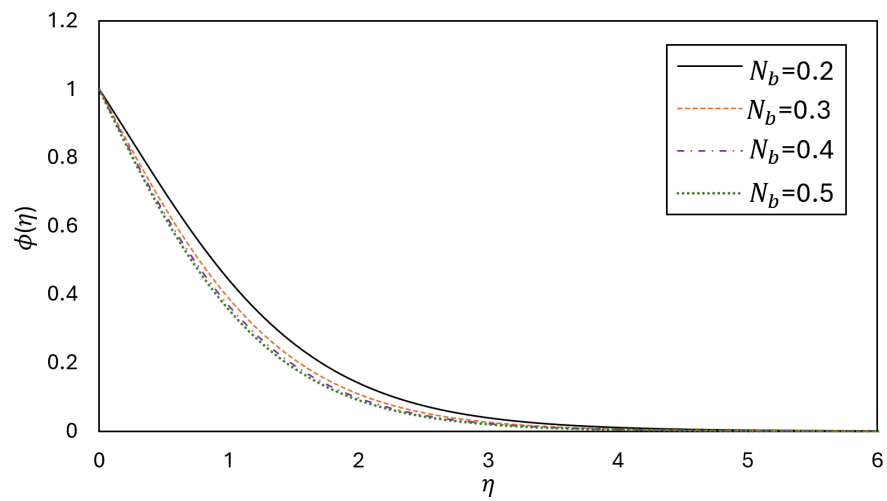
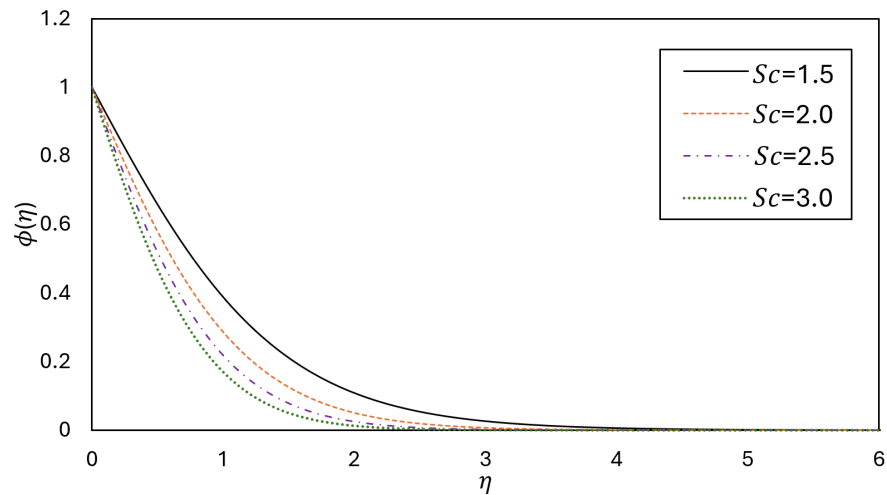
Fig.12- Plot of $\theta(\eta)$ vs Pr Fig.13- Plot of $\theta(\eta)$ vs Ec Fig.14- Plot of $\theta(\eta)$ vs N_t

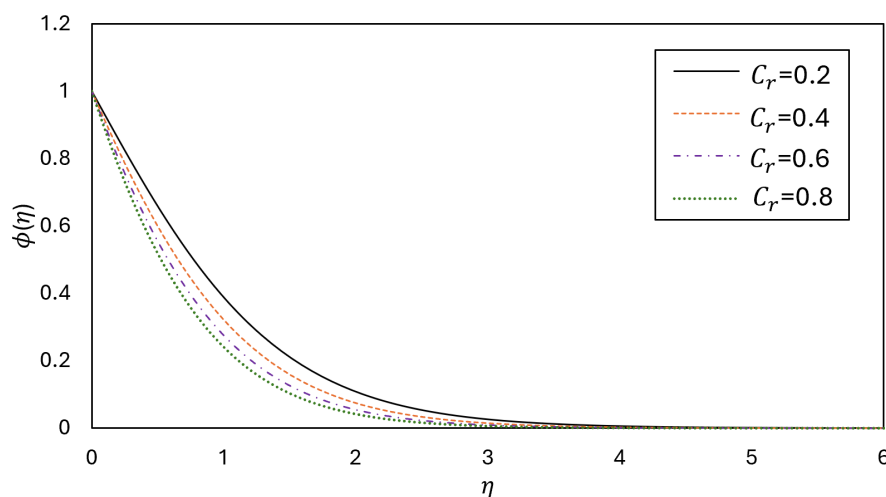
Fig.15- Plot of $\theta(\eta)$ vs N_b

Concentration profiles- Fig. 16 demonstrates that the concentration profile decreases with rise of M . Fig. 17 shows the positive influence of increasing W on concentration profile, it occurs because higher values of W enhance fluid's non-Newtonian characteristics, which increases fluid's resistance and slows down nanoparticle diffusion and thereby increasing concentration boundary layer's thickness. Fig. 18 clearly depicts that increasing values of δ enhances the concentration distribution, this physically occurs because of higher slip at the surface and hence reduces the resistance between the fluid and the wall, which promotes mass diffusion. Fig. 19 evidently shows that $\phi(\eta)$ increases with rise in stretching parameter. This phenomenon is observed due to enhances fluid motion near the boundary by stronger stretching, which promotes mass diffusion and leads to a rise in the thickness of concentration boundary layer. Fig. 20 presents the influence of N_t on $\phi(\eta)$. It is observed that the concentration profile increases with increasing values of N_t . Fig. 21 illustrates the effect of the Brownian motion parameter N_b on the concentration profile. It is noticed that the concentration decreases with increasing values of N_b . Fig. 22 shows the variation of $\phi(\eta)$ for different values of Sc . It is observed that the fluid's concentration declines as Sc increases. Physically, larger values of Sc correspond to lower mass diffusivity, which reduces nanoparticle diffusion within the boundary layer and results in a thinner concentration boundary layer. Fig. 23 presents that $\phi(\eta)$ declines with upliftment of C_r . Practically, due to consumption of nanoparticles takes place within the fluid due to stronger chemical reactions, thereby it works to reduce the concentration and thinning the concentration boundary layer.

Fig.16- Plot of $\phi(\eta)$ vs M

Fig.17- Plot of $\phi(\eta)$ vs W Fig.18- Plot of $\phi(\eta)$ vs δ Fig.19- Plot of $\phi(\eta)$ vs A

Fig.20- Plot of $\phi(\eta)$ vs N_t Fig.21- Plot of $\phi(\eta)$ vs N_b Fig.22- Plot of $\phi(\eta)$ vs Sc

Fig.23- Plot of $\phi(\eta)$ vs C_r

Physical quantities of interest

The useful parameters as mentioned in Eqs. (12), (13) and (14) are useful in determining behaviour of fluid mentioned in Table 2. By upgrading the stretching parameter A , the transport quantities say C_f , Nu_x and Sh_x decreases marginally. The Williamson parameter indicates enhancement in Nu_x while C_f and Sh_x decline gradually. Ec and Pr has a significant effect on heat transfer, Nu_x increases considerably on increasing Ec and Pr . When N_t and N_b increase, indicating a reduction in heat transfer rate while enhanced nanoparticle mass transfer is observed. All three, C_f , Nu_x and Sh_x decreases on increasing δ . On increasing β , the Nusselt decrease slightly. The Schmidt number, on increasing results in a decrease in Nu_x but a substantial increase in Sh_x . The chemical reaction parameter C_r enhances mass transfer.

Table 2 - Values of $(Re)^{0.5}C_f$, $(Re)^{-0.5}Nu_x$ and $(Re)^{-0.5}Sh_x$ with variation of various parameters

A	W	M	Pr	Ec	N_t	N_b	δ	β	Sc	C_r	$(Re)^{0.5}C_f$	$(Re)^{-0.5}Nu_x$	$(Re)^{-0.5}Sh_x$
0.05	0.5	0.5	4.5	0.3	0.1	0.3	0.2	0.3	1.5	0.2	-0.496500	0.745551	0.733996
0.01											-0.514758	0.750391	0.758262
0.03											-0.509453	0.747043	0.749874
	0.0										-0.541406	0.742660	0.745309
	0.2										-0.524384	0.743832	0.741155
	0.4										-0.506161	0.744994	0.736527
		1.0									-0.224368	0.675063	1.071126
		1.5									0.059812	0.709475	1.125213
		2.0									0.580536	0.797092	1.215703
			1.5									0.547798	
			3.5									0.709232	
			5.5									0.767846	
				0.2								0.701973	
				0.6								0.874836	
				1.0								1.043933	
					0.2							0.711620	0.461149
					0.3							0.555689	0.613238
					0.4							0.313817	1.441216
						0.4						0.644354	0.788422
						0.5						0.551799	0.819416
						0.6						0.468498	0.839466
							0.3				-0.452169	0.715516	0.720069
							0.4				-0.416070	0.691973	0.708257
							0.5				-0.385933	0.672876	0.698040
								0.4				0.708481	
								0.5				0.674265	
								0.6				0.642682	
									2.5			0.659905	1.097593
									3.0			0.632831	1.241297
									3.5			0.611211	1.370667
										0.3			0.829499

Conclusion

The conclusion section presents the major findings obtained from the present investigation, it summarizes the substantial influence of the key physical parameters on the fluid's behavior. The main conclusions of the present study are listed as follows-

- The velocity boundary layer declines with rise of M , K_p and A while it diminishes with rise in W and δ .
- The temperature distribution rises with improvement of N_b , N_t and δ parameters while it declines with higher values of M , W , β , Ec , Pr and A .
- The concentration profiles uplift with rise in N_t , A and δ while N_b and W , indicating reduced nanoparticle accumulation within the boundary layer.
- Skin friction coefficient intensifies with upliftment of M , while it diminishes with increase in A , W and δ .
- Ec , W and Pr intensifies the heat transfer characteristics, and thus enhancing Nu_x .
- Sc and C_r enhance mass transfer.

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Conflict of Interest

The authors declare no conflict of interest.

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