LRS BIANCHI TYPE-I POLYTROPIC GAS DARK ENERGY MODELS IN COSMOLOGY

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Abstract: In this paper, we have studied the Locally Rotationally Symmetric (LRS) Bianchi Type-I cosmological model in presence of Polytropic gas as dark energy in the form $P = K\rho^{1+\frac{1}{n}}$, Where $P, \rho, K$and n are the pressure, energy density, polytropic constant and polytropic index respectively and to solve the Einstein's field equations for the LRS Bianchi Type-I space time, we have applied the relation that the scalar expansion (θ) is proportional to the shear scalar(σ).Some physical and cosmological properties of the model have been obtained and discussed.

***Index Terms*- LRS Bianchi Type-I space time, Polytropic gas, Dark energy.**

I. INTRODUCTION

Cosmology is the scientific study of the large scale properties of the universe as a whole. Cosmology is study of the motion of heavenly bodies of the universe. Most of the Cosmological experiments and observations such as Type 1a Supernovae [1]-[3], Cosmic Microwave Background Radiation [4], Large Scale Structure [5], [6], Wilkinson Microwave Anisotropy Probe [7], Sloan Digital Sky Survey [8], etc. indicates that our universe expands under an accelerated expansion. In standard Friedman Lemaitre Robertson Walker (FLRW) cosmology, a new energy with negative pressure, called dark energy (DE) is responsible for this expansion [9] & [10]. But the nature of the DE is still unknown and various problems have been proposed by the researchers in this field**.** About 70% of the present energy of the universe is contained in the DE. The cosmological constant with the time independent equation of state is the earliest and simplest candidate for the dark energy [11]. Besides the cosmological constant, there are many dynamical dark energy models with the time dependent equation of state that have been proposed to explain the cosmic acceleration. Polytropic gas is one of the dynamical dark energy models to explain the cosmic acceleration of the universe [12]-[14]. The polytropic gas DE model is a phenomenological model of dark energy where the pressure is a function of energy density [15]. From the different observational data's, it is clear that our universe is homogeneous and isotropic on a large scale; however no physical evidence denies the chances of an anisotropic universe. In fact, many theoretical arguments are present promoting the existence of an anisotropic phase of the universe that approaches the isotropic phase [16]-[19]. The anisotropy plays a vital rule in the early phase of evolution of the universe and therefore studying homogeneous and anisotropic cosmological models are considered most important. The Bianchi type models are spatially homogeneous and generally anisotropy. The simplicity of the field equations made by the Bianchi type space time is very useful in constructing cosmological models which are spatially homogeneous and anisotropic. K.S.Adhav et al. have investigated Bianchi type models in different contexts [20]-[26]. Also several authors such as S.D. Katore and D.V. Kapse [27], C.P.Singh and S.Kumar [28], A.Pradhan and A.K. Vishwakarma [29], M.A. Rahman and M. Ansari [30] have investigated various cosmological models in this field.

In this paper, we have investigated Bianchi type-1 cosmological model in presence of Polytropic gas in the form $P = K\rho^{1+\frac{1}{n}}$,Where $P, \rho, K$and n are the pressure, energy density, polytropic constant and polytropic index respectively and the field equation have been solved by using the physical condition that that the scalar expansion (θ) is proportional to the shear scalar(σ). Some physical and cosmological properties of the model have been obtained and discussed.

II. METRIC AND FIELD EQUATION

We consider the LRS Bianchi type-I metric in the form

$$ds^2 = dt^2 - A(dx^2 + dy^2) - Bdz^2 \quad (1)$$

Where A and B are the metric functions of cosmic time$'t'$ only.

The Einstein's field equations are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} \quad (2)$$

Where R_{ij} is the Ricci tensor, R is the Ricci scalar and T_{ij} is the energy momentum tensor for bulk viscous cosmology
The energy conservation equation is given by

$$T^{ij}_{;j} = 0 \tag{3}$$

Where $T^{ij}_{;j} = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^j}\left(T^{ij}\sqrt{-g}\right) + T^{jk}\Gamma^i_{jk}$

The energy momentum tensor for the bulk viscous fluid is given by

$$T_{ij} = (\rho + P)u_i u_j + P g_{ij} \tag{4}$$

Where ρ is the energy density, P is the pressure and u^i is the four velocity vector satisfying $g_{ij}u^i u^j = 1$

In this model we have considered that the universe is filled with a fluid named polytropic gas.
The equation of state of the polytropic gas is given by

$$P = K\rho^{1+\frac{1}{n}} \tag{5}$$

Where K and n (>0) are constants known as polytropic constant and polytropic index respectively.
Using equation (4) for the metric (1), the Einstein's field equations becomes

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} = \rho \tag{6}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} = -P \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -P \tag{8}$$

Where overhead dot represents the differentiation with respect to cosmic time t
Using equation (1), the energy conservation equation (3) takes to the following forms

$$\dot{\rho} + \left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)(P + \rho) = 0 \tag{9}$$

The mean Hubble parameter H and scalar expansion θ are given by

$$H = \frac{1}{3}\theta = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) \tag{10}$$

Where V is the spatial volume of the universe
The average anisotropy parameter Δ and shear scalar σ^2 are given by

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{11}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^{\,2} - 3H^2\right) \tag{12}$$

Where $H_i, i = 1,2,3$ represents the directional parameters in the directions of x, y, z respectively and $\Delta = 0$ corresponds to the isotropic expansion of the universe.

III. SOLUTIONS OF THE FIELD EQUATIONS

To solve the Einstein's Field equations (6), (7) & (8) we assume that the scalar expansion ( ) is proportional to the shear scalar (σ^2) . Thus by assuming it we can take a relation as follows

$$B = A^m \tag{13}$$

Where $m(> 0)$ is a constant
With the help of the equation (13), the field equations (6), (7) and (8) can be reduced to the following forms

$$(2m + 1)\frac{\dot{A}^2}{A^2} = \rho \tag{14}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} = -P \tag{15}$$

$$(m + 1)\frac{\ddot{A}}{A} + m^2\frac{\dot{A}^2}{A} = -P \tag{16}$$

Solving the equations (15) & (16) we get

$$\left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2}\right) + (m + 2)\frac{\dot{A}^2}{A^2} = 0 \tag{17}$$

Integrating it we get

$$A(t) = a_0[t_0 + (m + 2)t]^{\frac{1}{m+2}} \tag{18}$$

Where a_0 and t_0 are constants of integration.
From the equations (13) and (18) we get

$$B(t) = \left[a_0\{t_0 + (m + 2)t\}^{\frac{1}{m+2}}\right]^m$$

$$= a_0^m[t_0 + (m + 2)t]^{\frac{m}{m+2}} \tag{19}$$

By a suitable choice of constants ($a_0 = 1, t_0 = 0$), the metric equation (1) can be written as

$$ds^2 = dt^2 - [(m+2)t]^{\frac{1}{m+2}}(dx^2 + dy^2) - [(m+2)t]^{\frac{m}{m+2}} dz^2 \quad (20)$$

IV. PHYSICAL AND COSMOLOGICAL PROPERTIES OF THE MODEL

From the equation (14) we get the energy density as

$$\rho = \frac{(2m+1)}{(m+2)^2 t^2} \quad (21)$$

Using the equation (21) in the equation (5) we get the pressure as

$$P = K\left[\frac{(2m+1)}{(m+2)^2 t^2}\right]^{1+\frac{1}{n}} \quad (22)$$

The mean Hubble parameter (H) and scalar expansion $(\theta\ )$are given by

$$H = \frac{1}{3t} \quad (23)$$

$$\theta = \frac{1}{t} \quad (24)$$

The spatial volume $(\ V)$ of the universe is given by

$$V = (m+2)t \quad (25)$$

The shear scalar σ^2 is given by

$$\sigma^2 = \frac{(m-1)^2}{(m+2)^2 t^2} = \frac{(m-1)^2\theta^2}{(m+2)^2} \quad (26)$$

The average anisotropy parameter (Δ) is given by

$$\Delta = \frac{2(m-1)^2}{(m+2)^2} = \frac{2\sigma^2}{\theta^2} \quad (27)$$

The deceleration parameter(q) is given by

$$q = -\frac{\dot{H}}{H^2} - 1 = 2 \quad (28)$$

The equation of state parameter(ω) is given by

$$\omega = \frac{P}{\rho} = K\left[\frac{(2m+1)}{(m+2)^2 t^2}\right]^{\frac{1}{n}} \quad (29)$$

Also the energy density parameter(Ω) is given by

$$\Omega = \frac{\rho}{3H^2} = \frac{3(2m+1)}{(m+2)^2} \quad (30)$$

For $m = 1,$ we have $A = B, \Delta = 0, \sigma^2 = 0, \Omega = 1$, this corresponds to an isotropic flat universe and the universe would expand forever but the rate of expansion would slow after infinite amount of time, Also for $m \neq 1$, we have Δ=constant, $\frac{\sigma}{\theta}$ =constant,$\Omega < 1$,this corresponds to an anisotropic open universe and the universe would expand forever.

V. CONCLUSION

A study of the Bianchi type I cosmological model with the polytropic gas has been done. The physical and cosmological parameters which play a key role in the discussion of the model are obtained. It is noted that the spatial volume of the universe is zero at $t = 0$ and increases infinitely as $t \to \infty$. The energy density, the pressure, the average Hubble parameter, expansion scalar and shear scalar are infinite at $t = 0$ and approaches 0 as $t \to \infty$. The universe is isotropic and flat when $m = 1$ and it is anisotropic and open when $m \neq 1$. Therefore the model is anisotropic throughout the evolution of the universe except at $m = 1$.In this model the average anisotropy parameter (Δ) and energy density parameter(Ω) are independent of the cosmic time.

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