

Hesitant Fuzzy tautological sets

K.Lalitha

PG and Research Department of Mathematics,
T.K.Government Arts College, Vridhachalam - 606001, India
and

T.Muthuraji

PG and Research Department of Mathematics
Government Arts College, C. Mutlur, Chidambaram, India.

Abstract: In this paper, I study some properties of Hesitant Fuzzy Tautological Sets using Hesitant fuzzy implication operator.

Keywords: Hesitant fuzzy sets (HFS), Hesitant fuzzy implication operator (HFIO) and Hesitant fuzzy tautological sets (HFTS).

1. Introduction:

A phenomenal set theory named and introduced by Zadeh [8] several extensions of this concept have been defined. Atanassov [1] has given idea about intuitionistic fuzzy sets. Hiroshi Hashimoto [2,3] used \leftarrow , \rightarrow operators in fuzzy matrices and obtained some results. Sriram and Murugadas [6] applied this \leftarrow operator to IFM and studied about g -inverse and sub-inverse of IFMs. Murugadas and Lalitha [4] obtained some relations between the operators \leftarrow and \rightarrow . ViceneTorra [7] used the envelope of the hesitant fuzzy sets are an intuitionistic fuzzy sets. The authors [5] proved some results for implication operator on intuitionistic fuzzy Tautological Matrix. In this paper, We use \rightarrow operator in HFS and study some properties of Hesitant fuzzy Tautological sets.

2. Preliminaries

Definition 2.1. Intuitionistic Fuzzy Sets Given a reference set χ , an intuitionistic fuzzy set (or IFS) A on χ is represented in terms of two functions $\mu : \chi \rightarrow [0,1]$ and $\gamma : \chi \rightarrow [0,1]$ such that $0 \leq \mu(x) + \gamma(x) \leq 1$ for all $x \in \chi$.

Where μ represents the degree of membership and γ the degree of non-membership of x into the set A . Ifs are often represented as follows:

$\langle x, \mu_A, \gamma_A \rangle$ for $x \in \chi$

Given two IFS A and B represented by the functions μ_A, γ_A, μ_B and γ_B following operations have been defined.

- Complement

$$A^C = \{\langle x, \gamma_A(x), \mu_A(x) \rangle\}$$

- Union

$$A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\gamma_A(x), \gamma_B(x)) \rangle\}$$

- Intersection

$$A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\gamma_A(x), \gamma_B(x)) \rangle\}$$

- \oplus -union

$$A \oplus B = \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), (\gamma_A(x)\gamma_B(x)) \rangle\}$$

- \otimes -intersection

$$A \otimes B = \{\langle x, \mu_A(x) \mu_B(x), \gamma_A(x) + \gamma_B(x) - \gamma_A(x) \gamma_B(x) \rangle\}$$

Definition 2.2. Hesitant Fuzzy Sets Let χ be a reference set, then we define hesitant fuzzy set on χ in terms of a function h that when applied to χ returns a subset of $[0,1]$.

Given a hesitant fuzzy set h , we define below its lower and upper bound,

- lower bound: $h^-(x) = \min h(x)$ and
- upper bound: $h^+(x) = \max h(x)$.

It is clear that given a hesitant fuzzy set h , the pair of functions h^- , $1 - h^+$ define an intuitionistic fuzzy set.

Definition 2.3. Given a hesitant fuzzy set represented by its membership function h , we define its complement as follows:

$$h^c(x) = \bigcup_{y \in h(x)} \{1 - y\}$$

Definition 2.4. Given a hesitant fuzzy set h , we define the intuitionistic fuzzy set $A_{env}(h)$ as the envelope of h .

This set, which will be denoted by $A_{env}(h)$, is represented by $\{\langle x, \mu_A(x), \gamma_A(x) \rangle\}$ with μ and γ defined as follows:

- $\mu(x) = h^-(x)$
- $\gamma(x) = 1 - h^+(x)$

3. Results on HFTSs

Definition 3.1. An Hesitant fuzzy set is called an Hesitant fuzzy tautological set (HFTS) if and only if $h_1^- \geq 1 - h_1^+$.

Definition 3.2. For $h^-(x)$, $1 - h^+(x) \in x$, define

$$h^-(x) \rightarrow 1 - h^+(x) = 1 - h^+(x) \leftarrow h^-(x) = \begin{cases} \langle 1, 0 \rangle, & \text{if } h^-(x) \leq 1 - h^+(x) \\ 1 - h^+(x), & \text{if } h^-(x) \geq 1 - h^+(x) \end{cases}$$

Property 3.1. Let $A(h_1)$ and $A(h_2)$ be two HFSs then the following expressions are HFTSs.

1. $A_{env}(h_1) \rightarrow A_{env}(h_1)$
2. $\bar{A}_{env}(h_1) \rightarrow A_{env}(h_1)$
3. $A_{env}(h_1) \rightarrow (A_{env}(h_2) \rightarrow A_{env}(h_1))$

Proof:

1. Let $A_{env}(h_1) = \langle h_1^-, 1 - h_1^+ \rangle$

$$\text{As } \langle h_1^-, 1 - h_1^+ \rangle = \langle h_1^-, 1 - h_1^+ \rangle$$

$$\text{Then } \langle h_1^-, 1 - h_1^+ \rangle = \langle h_1^-, 1 - h_1^+ \rangle = \langle 1, 0 \rangle$$

So $A_{env}(h_1) \rightarrow A_{env}(h_2)$ is an HFTSs

2. $A_{env}(h_1) = \langle h_1^-, 1 - h_1^+ \rangle$

$$\bar{A}_{env}(h_1) = \langle 1 - h_1^+, h_1^- \rangle$$

$$\text{If } \langle 1 - h_1^+, h_1^- \rangle \leq \langle h_1^-, 1 - h_1^+ \rangle$$

$$\text{Then } \bar{A}_{env}(h_1) \rightarrow A_{env}(h_1) = \langle 1, 0 \rangle$$

3. Let $A_{env}(h_1) = \langle h_1^-, 1-h_1^+ \rangle, A_{env}(h_2) = \langle h_2^-, 1-h_2^+ \rangle$

Then $A_{env}(h_2) \rightarrow A_{env}(h_1) = \langle h_2^-, 1-h_2^+ \rangle \rightarrow \langle h_1^-, 1-h_1^+ \rangle$

Case(i)

If $\langle h_2^-, 1-h_2^+ \rangle \leq \langle h_1^-, 1-h_1^+ \rangle$, then

$$\langle h_2^-, 1-h_2^+ \rangle \rightarrow \langle h_1^-, 1-h_1^+ \rangle = \langle 1, 0 \rangle$$

$$A_{env}(h_2) \rightarrow A_{env}(h_1) = \langle 1, 0 \rangle$$

$$A_{env}(h_1) \rightarrow (A_{env}(h_2) \rightarrow A_{env}(h_1)) = A_{env}(h_1) = \langle 1, 0 \rangle$$

$$= \langle h_1^-, 1-h_1^+ \rangle \rightarrow \langle 1, 0 \rangle = \langle 1, 0 \rangle$$

Case (ii)

$\langle h_2^-, 1-h_2^+ \rangle > \langle h_1^-, 1-h_1^+ \rangle$ then

$$\langle h_2^-, 1-h_2^+ \rangle \rightarrow \langle h_1^-, 1-h_1^+ \rangle = \langle h_1^-, 1-h_1^+ \rangle$$

$$A_{env}(h_2) \rightarrow A_{env}(h_1) c$$

$$A_{env}(h_1) \rightarrow (A_{env}(h_2) \rightarrow A_{env}(h_1)) = A_{env}(h_1) \rightarrow \langle h_1^-, 1-h_1^+ \rangle$$

$$= \langle h_1^-, 1-h_1^+ \rangle \rightarrow \langle h_1^-, 1-h_1^+ \rangle = \langle 1, 0 \rangle$$

Thus $A_{env}(h_1) \rightarrow (A_{env}(h_2) \rightarrow A_{env}(h_1))$ is an HFTSs.

Property 3.2. If $A_{env}(h_1)$ and $A_{env}(h_2)$ be to HFTSs then

$$1. A_{env}(h_1) \rightarrow (A_{env}(h_1) \vee A_{env}(h_2))$$

$$2. A_{env}(h_2) \rightarrow (A_{env}(h_1) \vee A_{env}(h_2))$$

$$3. A_{env}(h_1) \rightarrow A_{env}(h_2) \rightarrow (A_{env}(h_1) \wedge A_{env}(h_2)) \text{ are HFTSs.}$$

Proof

$$(A_{env}(h_1) \vee A_{env}(h_2)) = \langle h_1^-, 1-h_1^+ \rangle \vee \langle h_2^-, 1-h_2^+ \rangle$$

Case (i)

If $\langle h_1^-, 1-h_1^+ \rangle \leq \langle h_2^-, 1-h_2^+ \rangle$

$$A_{env}(h_1) \rightarrow (A_{env}(h_1) \vee A_{env}(h_2)) = A_{env}(h_1) \rightarrow \langle h_2^-, 1-h_2^+ \rangle$$

$$= \langle h_1^-, 1-h_1^+ \rangle \rightarrow \langle h_2^-, 1-h_2^+ \rangle = \langle 1, 0 \rangle$$

Case (ii)

If $\langle h_1^-, 1-h_1^+ \rangle > \langle h_2^-, 1-h_2^+ \rangle$

$$A_{env}(h_1) \rightarrow (A_{env}(h_1) \vee A_{env}(h_2)) = A_{env}(h_1) \rightarrow \langle h_1^-, 1-h_1^+ \rangle$$

$$= \langle h_1^-, 1-h_1^+ \rangle \rightarrow \langle h_1^-, 1-h_1^+ \rangle = \langle 1, 0 \rangle$$

So, $A_{env}(h_1) \rightarrow (A_{env}(h_1) \vee A_{env}(h_2))$ is an HFTSs.

$$2. A_{env}(h_2) \rightarrow (A_{env}(h_1) \vee A_{env}(h_2))$$

Case (i)

If $\langle h_1^-, 1-h_1^+ \rangle \leq \langle h_2^-, 1-h_2^+ \rangle$

$$A_{env}(h_2) \rightarrow \langle h_2^-, 1-h_2^+ \rangle = \langle h_1^-, 1-h_1^+ \rangle \rightarrow \langle h_2^-, 1-h_2^+ \rangle = \langle 1, 0 \rangle$$

Case (ii)

$$\langle h_1^-, 1-h_1^+ \rangle > \langle h_2^-, 1-h_2^+ \rangle$$

$$A_{env}(\mathbf{h}_2) \rightarrow \langle \mathbf{h}_1^-, 1-\mathbf{h}_1^+ \rangle = \langle \mathbf{h}_1^-, 1-\mathbf{h}_1^+ \rangle \rightarrow \langle \mathbf{h}_1^-, 1-\mathbf{h}_1^+ \rangle = \langle 1, 0 \rangle$$

So, $A_{env}(\mathbf{h}_2) \rightarrow (A_{env}(\mathbf{h}_1) \vee A_{env}(\mathbf{h}_2))$ is an HFTSs.

$$3. A_{env}(\mathbf{h}_1) \rightarrow A_{env}(\mathbf{h}_2) \rightarrow (A_{env}(\mathbf{h}_1) \wedge A_{env}(\mathbf{h}_2))$$

Case (i)

$$\text{If } \langle \mathbf{h}_1^-, 1-\mathbf{h}_1^+ \rangle \leq \langle \mathbf{h}_2^-, 1-\mathbf{h}_2^+ \rangle$$

$$\begin{aligned} A_{env}(\mathbf{h}_2) \rightarrow (A_{env}(\mathbf{h}_1) \wedge A_{env}(\mathbf{h}_2)) &= A_{env}(\mathbf{h}_2) \rightarrow \langle \mathbf{h}_1^-, 1-\mathbf{h}_1^+ \rangle \\ &= \langle \mathbf{h}_2^-, 1-\mathbf{h}_2^+ \rangle \rightarrow \langle \mathbf{h}_1^-, 1-\mathbf{h}_1^+ \rangle = \langle \mathbf{h}_1^-, 1-\mathbf{h}_1^+ \rangle \end{aligned}$$

$$A_{env}(\mathbf{h}_1) \rightarrow A_{env}(\mathbf{h}_2) \rightarrow (A_{env}(\mathbf{h}_1) \wedge A_{env}(\mathbf{h}_2))$$

$$A_{env}(\mathbf{h}_1) \rightarrow \langle \mathbf{h}_1^-, 1-\mathbf{h}_1^+ \rangle = \langle \mathbf{h}_1^-, 1-\mathbf{h}_1^+ \rangle \rightarrow \langle \mathbf{h}_1^-, 1-\mathbf{h}_1^+ \rangle = \langle 1, 0 \rangle$$

Case (ii)

$$\text{If } \langle \mathbf{h}_1^-, 1-\mathbf{h}_1^+ \rangle > \langle \mathbf{h}_2^-, 1-\mathbf{h}_2^+ \rangle$$

$$\begin{aligned} A_{env}(\mathbf{h}_2) \rightarrow (A_{env}(\mathbf{h}_1) \wedge A_{env}(\mathbf{h}_2)) &= A_{env}(\mathbf{h}_2) \rightarrow \langle \mathbf{h}_2^-, 1-\mathbf{h}_2^+ \rangle \\ &= \langle \mathbf{h}_2^-, 1-\mathbf{h}_2^+ \rangle \rightarrow \langle \mathbf{h}_2^-, 1-\mathbf{h}_2^+ \rangle = \langle 1, 0 \rangle \end{aligned}$$

$$A_{env}(\mathbf{h}_1) \rightarrow A_{env}(\mathbf{h}_2) \rightarrow (A_{env}(\mathbf{h}_1) \wedge A_{env}(\mathbf{h}_2)) = A_{env}(\mathbf{h}_1) \rightarrow \langle 1, 0 \rangle$$

$$= \langle \mathbf{h}_1^-, 1-\mathbf{h}_1^+ \rangle \rightarrow \langle 1, 0 \rangle \rightarrow \langle 1, 0 \rangle$$

So, $A_{env}(\mathbf{h}_1) \rightarrow A_{env}(\mathbf{h}_2) \rightarrow (A_{env}(\mathbf{h}_1) \wedge A_{env}(\mathbf{h}_2))$ is an HFTSs.

Property 3.3. If $A_{env}(\mathbf{h}_1)$ and $A_{env}(\mathbf{h}_2)$ are HFTSs then

$$(i) A_{env}(\mathbf{h}_1) \wedge A_{env}(\mathbf{h}_2) \rightarrow A_{env}(\mathbf{h}_1)$$

$$(ii) A_{env}(\mathbf{h}_1) \wedge A_{env}(\mathbf{h}_2) \rightarrow A_{env}(\mathbf{h}_2) \text{ are HFTSs.}$$

Proof:

$$(i) A_{env}(\mathbf{h}_1) \wedge A_{env}(\mathbf{h}_2) = \langle \mathbf{h}_1^-, 1-\mathbf{h}_1^+ \rangle \wedge \langle \mathbf{h}_2^-, 1-\mathbf{h}_2^+ \rangle$$

Case (i)

$$\text{If } \langle \mathbf{h}_1^-, 1-\mathbf{h}_1^+ \rangle \leq \langle \mathbf{h}_2^-, 1-\mathbf{h}_2^+ \rangle$$

$$A_{env}(\mathbf{h}_1) \wedge A_{env}(\mathbf{h}_2) \rightarrow A_{env}(\mathbf{h}_1) = \langle \mathbf{h}_1^-, 1-\mathbf{h}_1^+ \rangle \rightarrow \langle \mathbf{h}_1^-, 1-\mathbf{h}_1^+ \rangle = \langle 1, 0 \rangle$$

Case (ii)

$$\text{If } \langle \mathbf{h}_1^-, 1-\mathbf{h}_1^+ \rangle > \langle \mathbf{h}_2^-, 1-\mathbf{h}_2^+ \rangle$$

$$A_{env}(\mathbf{h}_1) \wedge A_{env}(\mathbf{h}_2) \rightarrow A_{env}(\mathbf{h}_1) = \langle \mathbf{h}_2^-, 1-\mathbf{h}_2^+ \rangle \rightarrow \langle \mathbf{h}_1^-, 1-\mathbf{h}_1^+ \rangle = \langle 1, 0 \rangle$$

Hence $A_{env}(\mathbf{h}_1) \wedge A_{env}(\mathbf{h}_2) \rightarrow A_{env}(\mathbf{h}_1)$ is an HFTSs.

$$(ii) A_{env}(\mathbf{h}_1) \wedge A_{env}(\mathbf{h}_2) \rightarrow A_{env}(\mathbf{h}_2)$$

Case (i)

$$\text{If } \langle \mathbf{h}_1^-, 1-\mathbf{h}_1^+ \rangle \leq \langle \mathbf{h}_2^-, 1-\mathbf{h}_2^+ \rangle$$

$$A_{env}(\mathbf{h}_1) \wedge A_{env}(\mathbf{h}_2) \rightarrow A_{env}(\mathbf{h}_2) = \langle \mathbf{h}_1^-, 1-\mathbf{h}_1^+ \rangle \rightarrow \langle \mathbf{h}_2^-, 1-\mathbf{h}_2^+ \rangle = \langle 1, 0 \rangle$$

Case (ii)

$$\text{If } \langle \mathbf{h}_1^-, 1-\mathbf{h}_1^+ \rangle > \langle \mathbf{h}_2^-, 1-\mathbf{h}_2^+ \rangle$$

$$A_{env}(\mathbf{h}_1) \wedge A_{env}(\mathbf{h}_2) \rightarrow A_{env}(\mathbf{h}_2) = \langle \mathbf{h}_2^-, 1-\mathbf{h}_2^+ \rangle \rightarrow \langle \mathbf{h}_2^-, 1-\mathbf{h}_2^+ \rangle = \langle 1, 0 \rangle$$

Hence $A_{env}(\mathbf{h}_1) \wedge A_{env}(\mathbf{h}_2) \rightarrow A_{env}(\mathbf{h}_2)$ is an HFTSs.

Property 3.4. If $A_{env}(h_1)$ and $A_{env}(h_2)$ are HFTSs and $A_{env}(h_1) < A_{env}(h_2)$ then $A_{env}(h_1) \rightarrow A_{env}(h_2)$ is an HFTSs.

Proof

Given $A_{env}(h_1) < A_{env}(h_2) \rightarrow \langle h_1^-, 1-h_1^+ \rangle < \langle h_2^-, 1-h_2^+ \rangle$

Then $A_{env}(h_1) < A_{env}(h_2) = \langle 1, 0 \rangle$

So $A_{env}(h_1) < A_{env}(h_2)$ is an HFTSs.

Property 3.5. If $A_{env}(h_1), A_{env}(h_2)$ are HFTSs then the following expressions are HFTSs.

1. $(A_{env}(h_1) \wedge A_{env}(h_1) \rightarrow A_{env}(h_2)) \rightarrow A_{env}(h_2)$
2. $A_{env}(h_1) \rightarrow ((A_{env}(h_1) \leftarrow A_{env}(h_2)) \vee A_{env}(h_2))$
3. $A_{env}(h_2) \rightarrow ((A_{env}(h_1) \rightarrow A_{env}(h_2)) \vee A_{env}(h_1))$
4. $A_{env}(h_2) \rightarrow ((A_{env}(h_1) \vee A_{env}(h_2)) \rightarrow A_{env}(h_2))$

Proof:

1. **Case(i)**

If $\langle h_1^-, 1-h_1^+ \rangle \leq \langle h_2^-, 1-h_2^+ \rangle$

$A_{env}(h_1) \rightarrow A_{env}(h_2) = \langle h_1^-, 1-h_1^+ \rangle \rightarrow \langle h_2^-, 1-h_2^+ \rangle = \langle 1, 0 \rangle$

$(A_{env}(h_1) \wedge (A_{env}(h_1) \rightarrow A_{env}(h_2))) = \langle h_1^-, 1-h_1^+ \rangle \wedge \langle 1, 0 \rangle = \langle 1, 0 \rangle$

$(A_{env}(h_1) \wedge (A_{env}(h_1) \rightarrow A_{env}(h_2))) \rightarrow A_{env}(h_2) = \langle h_1^-, 1-h_1^+ \rangle \rightarrow \langle h_2^-, 1-h_2^+ \rangle = \langle 1, 0 \rangle$

Case(ii)

If $\langle h_1^-, 1-h_1^+ \rangle \leq \langle h_2^-, 1-h_2^+ \rangle$

$A_{env}(h_1) \rightarrow A_{env}(h_2) = \langle h_1^-, 1-h_1^+ \rangle$

$A_{env}(h_1) \wedge (A_{env}(h_1) \rightarrow A_{env}(h_2)) = A_{env}(h_1) \wedge \langle h_2^-, 1-h_2^+ \rangle$

$= \langle h_1^-, 1-h_1^+ \rangle \wedge \langle h_2^-, 1-h_2^+ \rangle$

$= \langle h_2^-, 1-h_2^+ \rangle$

$(A_{env}(h_1) \wedge (A_{env}(h_1) \rightarrow A_{env}(h_2))) A_{env}(h_2) = \langle h_2^-, 1-h_2^+ \rangle \rightarrow A_{env}(h_2) = \langle h_2^-, 1-h_2^+ \rangle \rightarrow \langle h_2^-, 1-h_2^+ \rangle = \langle 1, 0 \rangle$

So $(A_{env}(h_1) \wedge (A_{env}(h_1) \rightarrow A_{env}(h_2))) A_{env}(h_2)$ is an HFTSs.

2. **Case(i)**

If $\langle h_1^-, 1-h_1^+ \rangle < \langle h_2^-, 1-h_2^+ \rangle$

$A_{env}(h_1) \rightarrow ((A_{env}(h_1) \leftarrow A_{env}(h_2)) \vee A_{env}(h_2))$

$= A_{env}(h_1) \rightarrow (A_{env}(h_1) \vee A_{env}(h_2))$

$= A_{env}(h_1) \rightarrow A_{env}(h_2) = \langle 1, 0 \rangle$

Case(ii)

If $\langle h_1^-, 1-h_1^+ \rangle \leq \langle h_2^-, 1-h_2^+ \rangle$

$A_{env}(h_1) \rightarrow (\langle 1, 0 \rangle \vee A_{env}(h_2))$

$A_{env}(h_1) \rightarrow \langle 1, 0 \rangle = \langle h_1^-, 1-h_1^+ \rangle \rightarrow \langle 1, 0 \rangle = \langle 1, 0 \rangle$

So $A_{env}(h_1) \rightarrow ((A_{env}(h_1) \leftarrow A_{env}(h_2)) \vee A_{env}(h_2))$ is an HFTSs.

3. Case (i)

If $\langle h_1^-, 1-h_1^+ \rangle \leq \langle h_2^-, 1-h_2^+ \rangle$

$A_{env}(h_1) \rightarrow A_{env}(h_2) = \langle 1, 0 \rangle$

$(A_{env}(h_1) \rightarrow A_{env}(h_2)) \vee A_{env}(h_1) = \langle 1, 0 \rangle \vee \langle h_1^-, 1-h_1^+ \rangle = \langle 1, 0 \rangle$

$A_{env}(h_2) \rightarrow ((A_{env}(h_1) \rightarrow A_{env}(h_2)) \vee A_{env}(h_1)) = \langle h_2^-, 1-h_2^+ \rangle \rightarrow \langle 1, 0 \rangle = \langle 1, 0 \rangle$

Case (ii)

If $\langle h_1^-, 1-h_1^+ \rangle > \langle h_2^-, 1-h_2^+ \rangle$

$A_{env}(h_1) \rightarrow A_{env}(h_2) = \langle h_2^-, 1-h_2^+ \rangle$

$((A_{env}(h_1) \rightarrow A_{env}(h_2)) \vee A_{env}(h_1)) = \langle \langle h_2^-, 1-h_2^+ \rangle \vee \langle h_1^-, 1-h_1^+ \rangle \rangle$

$= \langle h_1^-, 1-h_1^+ \rangle$

$A_{env}(h_2) \rightarrow \langle h_1^-, 1-h_1^+ \rangle = \langle h_2^-, 1-h_2^+ \rangle \rightarrow \langle h_1^-, 1-h_1^+ \rangle = \langle 1, 0 \rangle$

So $A_{env}(h_2) \rightarrow ((A_{env}(h_1) \rightarrow A_{env}(h_2)) \vee A_{env}(h_1))$ is an HFTSs.

4. Case (i)

If $\langle h_1^-, 1-h_1^+ \rangle \geq \langle h_2^-, 1-h_2^+ \rangle$

$(A_{env}(h_1) \vee A_{env}(h_2)) = \langle h_1^-, 1-h_1^+ \rangle$

$(A_{env}(h_1) \vee A_{env}(h_2)) \rightarrow A_{env}(h_2) = \langle h_1^-, 1-h_1^+ \rangle \rightarrow \langle h_2^-, 1-h_2^+ \rangle$

$= \langle h_2^-, 1-h_2^+ \rangle$

$A_{env}(h_2) \rightarrow ((A_{env}(h_1) \vee A_{env}(h_2)) \rightarrow A_{env}(h_2)) = A_{env}(h_2) \rightarrow \langle h_2^-, 1-h_2^+ \rangle$

$\langle h_2^-, 1-h_2^+ \rangle \rightarrow \langle h_2^-, 1-h_2^+ \rangle = \langle 1, 0 \rangle$

Case (ii)

If $\langle h_1^-, 1-h_1^+ \rangle < \langle h_2^-, 1-h_2^+ \rangle$

$A_{env}(h_1) \vee A_{env}(h_2) = \langle h_2^-, 1-h_2^+ \rangle$

$((A_{env}(h_1) \vee A_{env}(h_2)) \rightarrow A_{env}(h_2)) = \langle \langle h_2^-, 1-h_2^+ \rangle \rightarrow \langle h_2^-, 1-h_2^+ \rangle \rangle = \langle 1, 0 \rangle$

$A_{env}(h_2) \rightarrow ((A_{env}(h_1) \vee A_{env}(h_2)) \rightarrow A_{env}(h_2)) = A_{env}(h_2) \rightarrow \langle 1, 0 \rangle$

$= \langle h_2^-, 1-h_2^+ \rangle \rightarrow \langle 1, 0 \rangle = \langle 1, 0 \rangle$

So $A_{env}(h_2) \rightarrow ((A_{env}(h_1) \vee A_{env}(h_2)) \rightarrow A_{env}(h_2))$ is an HFTSs.

REFERENCES

[1].K.T.Ananassov, Intuitionistic Fuzzy sets,VII ITKR'S session,Sofia,June 1983.

[2].S.Sriram,P.Murugadas, Sub-inverse of Intuitionistic Fuzzy Matrices,AclaaCieneiaIndica(Mathematics),XXXVIIM(1)(2011),41-56.

[3].P. Murugadas and K. Lalitha,Dual implication operator in intuitionistic Fuzzy Matrices, Intuitionistic Conference on Mathematical Modelling and its applications, December 22,24(2012),organized by Department of Mathematics, Annamalai University.

[4].P. Murugadas and K. Lalitha, Implication operator on intuitionistic Fuzzy Tautological Matrix,International Journal of Fuzzy Mathematical Archive and Informatics ISSN 2320-3242,vol.5(2)(2014),79-84.

[5].ViceneTorra,Hesitant Fuzzy Sets,International Journal of Intelligent systems, Vol.25,(2010),529-539.

[6].L.A.Zadeh,Fuzzy Sets,Information and Control,8(1965)338-353.

[7].H.Hashimoto, Sub-inverse of fuzzy matrices, Fuzzy Sets and Systems,12(1984)155-168.

[8].H.Hashimoto,Traces of fuzzy relations under dual operations,Journal of Advanced Computational Intelligence and Intelligent Informatics,9(5)(2005),563-569.