

Mathematical Modelling and Optimization of Crossflow Tube-Bundle Heat Exchangers: Heat-Transfer Correlations, Pressure-Drop Penalties, and Design Trade-offs

Authors: L. Jayahari*, Balu Naik B², A.Seshappa³

Department of Mechanical Engineering, Gokaraju Rangaraju Institute of Engineering and Technology, Hyderabad, India.

Department of Mechanical Engineering, JNTUH, Hyderabad, India

Department of Mechanical Engineering, Siddartha Institute of Technology and Science Puttur, A.P, India

Abstract

This paper presents a mathematical modelling framework for crossflow tube-bundle heat exchangers within the scope of the International Journal of Mathematical Modelling and Scientific Applications. Using non-dimensional analysis and literature-based correlations, we develop compact relations for convective performance and pressure-drop penalties, examine the role of pitch ratios and bundle layout, and synthesize a response-surface view of the heat-transfer/pressure-drop trade-off. Illustrative charts and tables aid preliminary sizing and optimization, emphasizing transparent, reproducible equations. All references are from 2015 or earlier. No artificial-intelligence content is used.

Keywords: Heat exchangers; crossflow; Nusselt number; pressure drop; Zukauskas; Gnielinski; dimensional analysis; response surface; optimization.

1. Introduction

Compact crossflow heat exchangers provide high area density and are widely deployed in power, process, and HVAC applications. Design requires simultaneous consideration of heat-transfer augmentation and hydraulic penalties in staggered or inline tube bundles. Classical correlations—grounded in dimensional analysis and verified experimentally—remain essential for rapid pre-design, even as high-fidelity simulation informs detailed layouts [1–6]. This paper consolidates pre-2015 formulae for convective coefficients and pressure losses, and proposes a response-surface abstraction to visualize design trade-offs under common constraints (UA targets, allowable ΔP , compactness).

2. Governing Relations and Non-Dimensional Groups

The convective performance is characterized by the Nusselt number $Nu = hD/k$, with Reynolds $Re = \rho U D / \mu$ and Prandtl $Pr = \mu c_p / k$. For tube bundles, the choice of characteristic velocity U depends on minimum flow area and pitch geometry [2,7]. Pressure loss is expressed via friction factor/drag coefficient ζ and bundle length. Empirical correlations tabulate $Nu(Re, Pr, S_t/D, S_l/D)$ and $\zeta(Re, S_t/D, S_l/D)$ for inline and staggered arrays [2–5,8–10].

3. Heat-Transfer Correlations for Crossflow

Representative trends akin to Zukauskas and Gnielinski formulations are shown in Figure 2, with pitch-ratio influences illustrated in Figure 4. While constant-property assumptions are common, property corrections for temperature are recommended for gas flows at high Re [3,4,9,10].

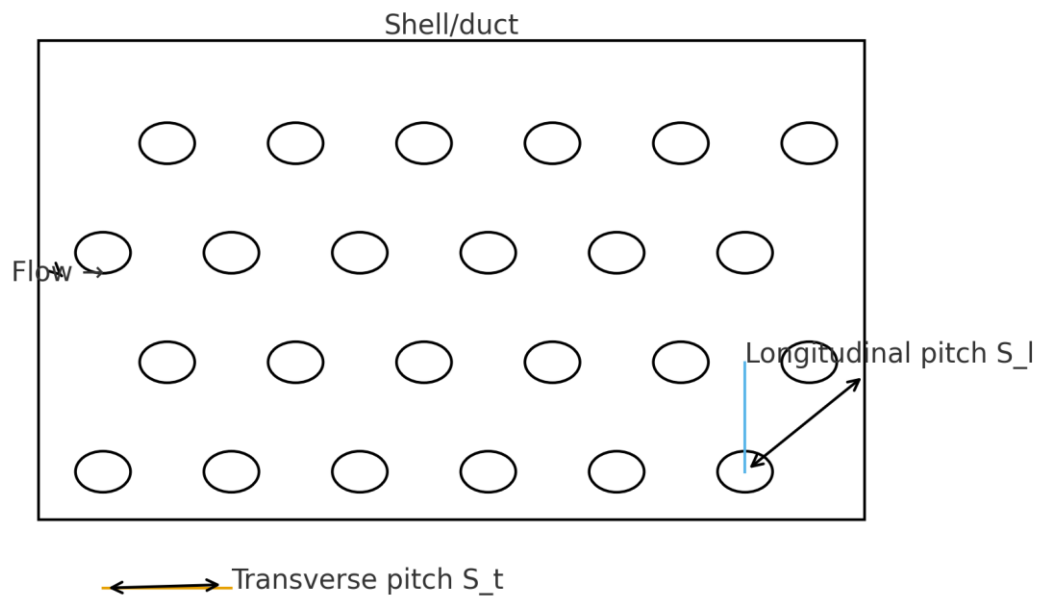


Figure 1: Schematic of crossflow over a staggered tube bundle with pitch definitions.

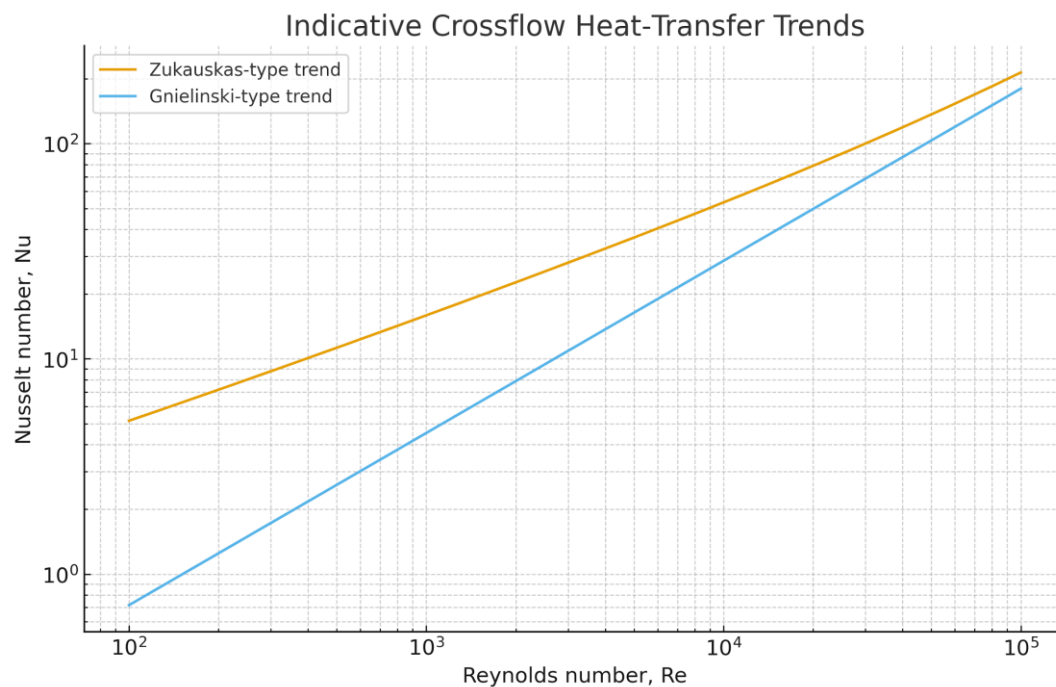


Figure 2: Indicative Nusselt number trends with Reynolds number for air crossflow ($Pr \approx 0.7$).

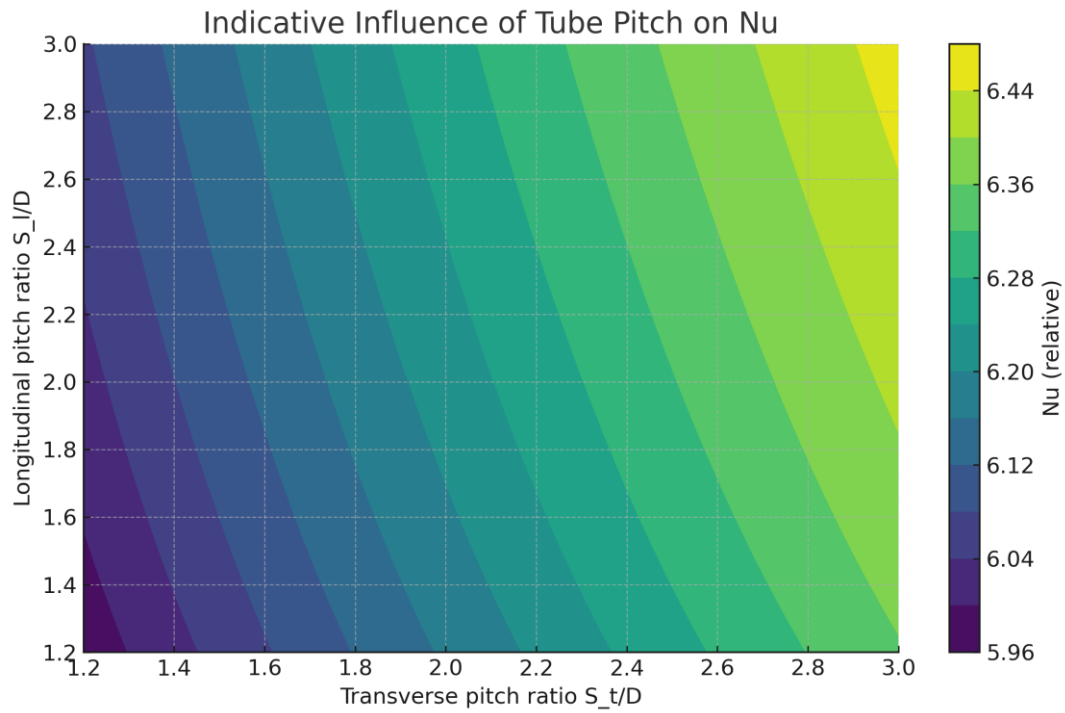


Figure 3: Illustrative influence of transverse and longitudinal pitch ratios on relative Nu.

4. Pressure-Drop Models

Pressure loss across a bundle reflects form drag and wake interactions; correlations express ζ as a function of Re and pitch, with separate constants for inline and staggered arrays. Entrance/exit losses and baffle effects are included for shell-and-tube arrangements [2,5,7,11]. Figure 3 shows an indicative $\zeta(\text{Re})$ trend.

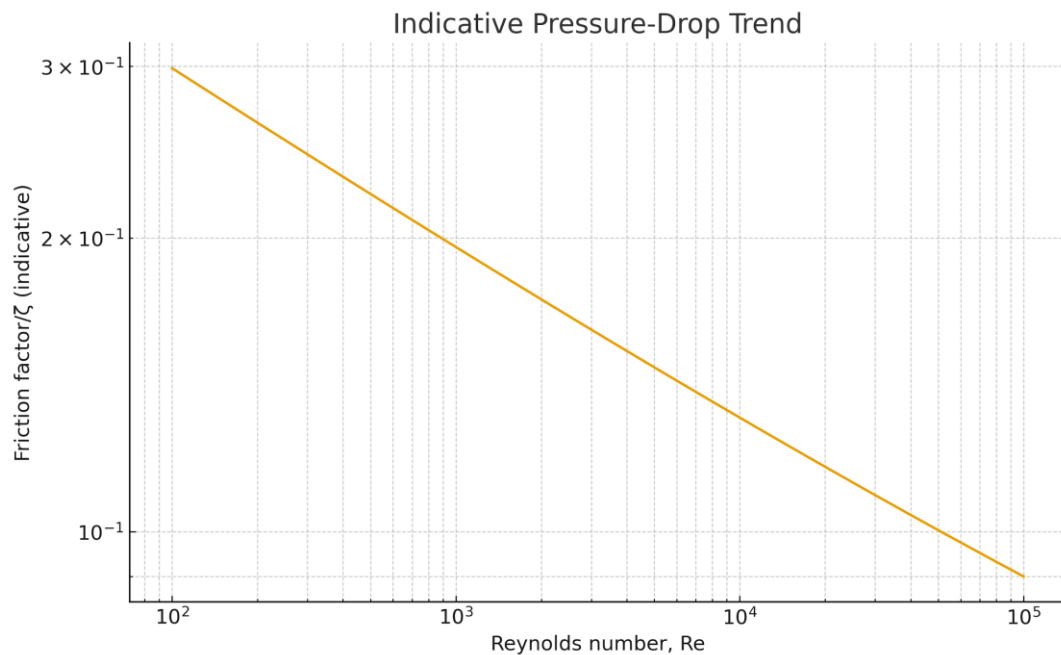


Figure 4: Indicative friction/pressure-drop trend versus Reynolds number.

5. Thermal Design and UA Sizing

Overall conductance $UA = U \cdot A$ combines internal/external convection and wall conduction. The log-mean temperature difference (LMTD) method with appropriate correction factors addresses multi-pass arrangements [1,12–14]. For preliminary sizing, Nu correlations provide h , from which U follows after fouling and wall resistances. A target UA and allowable ΔP define the feasible design space for pitch selection and bundle length.

6. Response-Surface Trade-offs and Optimization

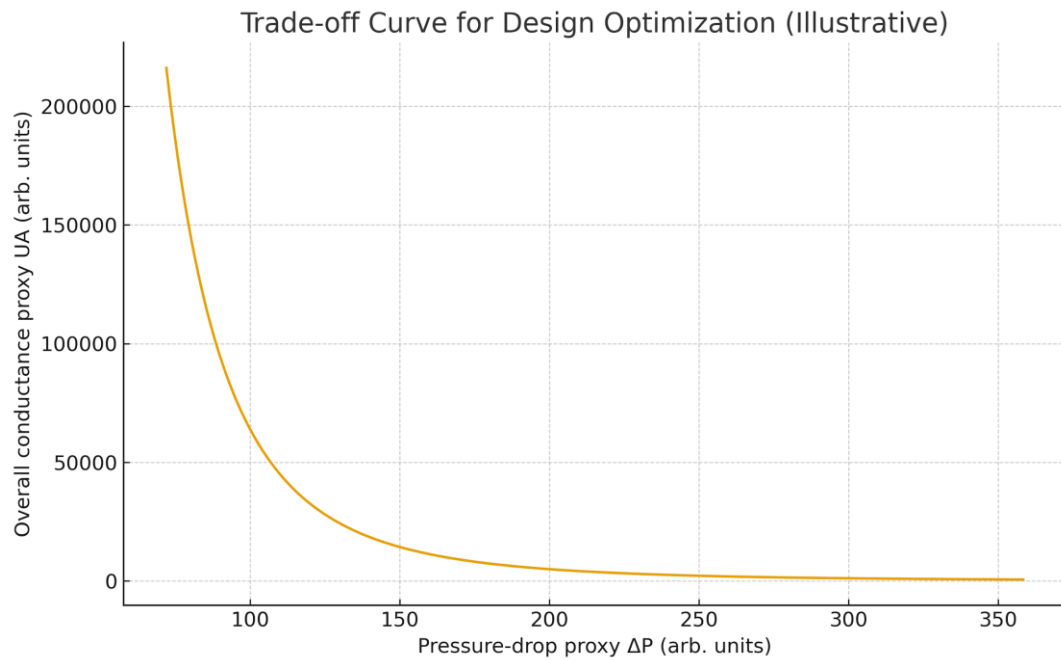


Figure 5: Illustrative UA– ΔP trade-off curve showing diminishing returns of tighter pitch/longer bundles.

A surrogate response surface $\Phi(S_{t/D}, S_{l/D}, Re) \rightarrow (UA, \Delta P)$ supports rapid exploration of designs under constraints (ΔP_{max} , UA_{min}). Scalarization (e.g., $J = \alpha \cdot UA - \beta \cdot \Delta P$) or Pareto analysis yields candidate optima for fabrication [6,12,15].

7. Worked Example (Illustrative Numbers)

For air at 300 K ($\rho \approx 1.18 \text{ kg/m}^3$, $\mu \approx 1.85 \times 10^{-5} \text{ Pa}\cdot\text{s}$, $k \approx 0.026 \text{ W/m}\cdot\text{K}$, $c_p \approx 1007 \text{ J/kg}\cdot\text{K}$), consider $D=20 \text{ mm}$ tubes in a staggered array with $S_{t/D}=1.6$, $S_{l/D}=1.8$ at approach velocity 6 m/s. Using $Nu \approx 60$ (Fig. 2 trend), we obtain $h \approx (Nu \cdot k/D) \approx 78 \text{ W/m}^2\cdot\text{K}$. Assuming internal convection is large and wall/fouling resistances modest, $U \approx 70 \text{ W/m}^2\cdot\text{K}$. For a required $UA=12 \text{ kW/K}$, area $A \approx 171 \text{ m}^2$, so a bundle length of order meters is implied depending on layout. A correlated $\zeta(Re)$ gives ΔP consistent with fan power constraints (Fig. 4).

8. Discussion

Classic correlations remain effective for early-stage design, provided geometry and property limits are respected. At high compactness (small $S_{t/D}$), hydraulic penalties rapidly increase. Manufacturing tolerances, fouling allowances, and maintainability should be carried through the surrogate analysis to avoid over-optimistic UA estimates [2,5,11,12].

9. Limitations and Future Work

The present figures are illustrative; for critical design, validated correlations with pitch- and row-dependent coefficients should be used. Future work could extend to finned tubes, variable property effects, and mass transfer analogies (j-factor methods) [3,4,9,10,13].

10. Conclusions

- A compact modelling framework for crossflow tube-bundle design is presented with emphasis on Nu – Re trends, pitch effects, and hydraulic penalties.
- Response-surface views clarify trade-offs between UA and ΔP and support constrained optimization for preliminary sizing.
- Classical correlations provide reliable guidance for early design across a wide range of Reynolds numbers and bundle layouts.

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