Undirected Binary Fuzzy Graphs on Composition, Tensor and Normal Products

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Abstract

Undirected binary *fuzzy graphs can be obtained from two given* undirected binary *fuzzy graphs using the operations, cartesian product, composition, tensor and normal products. In this broadsheet, we find the degree of a pinnacle in* undirected binary *fuzzy graphs formed by these operations in terms of the degree of vertices in the given* undirected binary *fuzzy diagrams in some particular cases.*

Keywords-

Cartesian product, Composition, Degree of a vertex, Tensor product, Normal product.

INTRODUCTION

Fuzzy graphs introduced by Rosenfeld in 1975[1-2, 9, 10]. The operations of union, join, cartesian product and composition on two fuzzy graphs were defined by Mordeson. J. N. and Peng. C.S [3-8]. In this paper, we study about the degree of a vertex in undirected binary fuzzy graphs which are obtained from two given undirected binary fuzzy graphs using the operations cartesian product and composition of two undirected binary fuzzy graphs, tensor and normal product of undirected binary fuzzy graphs. In general, the degree of vertices in cartesian product and composition of two undirected binary fuzzy graphs, tensor and normal product of two undirected binary fuzzy graphs BG_1 and BG_2 cannot be expressed in terms of those in BG_1 and BG_2. In this paper, we find the degree of vertices in cartesian product, composition, tensor and normal product of BG_1 and BG_2 in some particular cases.

PRELIMINARIES

Definition 2.1*:*

A fuzzy subset μ on a set X is a map μ : X →[0, 1]. A map ß : $X \times X \rightarrow$ [0,1] is fuzzy relation on X if ß$(\alpha, \beta) \leq \mu(x)\Lambda\mu(y)$ for all α, β ∈X. ß is a symmetric fuzzy relation if ß$(\alpha, \beta) =$ ß(β, α) for all α, β ∈X.

Definition 2.2:

Let X be a non-empty regular set. A binary fuzzy set B in X is an objective having the practice B= {(α,$\mu^{\rho}(\alpha)$*,* $\mu^{\text{и}}(\alpha)$*)/*$\alpha \in X$*} where* $\mu^{\rho}: X$ →[0, 1] and $\mu^{\text{и}}: X$ →[0, 1] are mappings.

Definition 2.3.

A binary fuzzy graph of $BG^* = (V, E)$ *is a pair BG (A, B) where A=* $(\mu_A^{\rho}, \mu_A^{\text{и}})$ *is a binary fuzzy set in V and B=*$(\mu_B^{\rho}, \mu_B^{\text{и}})$ *is a binary fuzzy set in VXV such that* $(\mu_B^{\rho}(\alpha\beta) \leq \mu_A^{\rho}(\alpha)\Lambda\mu_A^{\rho}(\beta)$ for all $\alpha, \beta \in$ *VXV*, $(\mu_B^{\text{и}}(\alpha\beta) \geq \mu_A^{\text{и}}(\alpha)V\ \mu_A^{\text{и}}(\beta)$ for all $\alpha, \beta \in$ *VXV, and* $(\mu_B^{\text{и}}(\alpha\beta)$= $(\mu_B^{\rho}(\alpha\beta)$*=0* for all $\alpha, \beta \in$ *VXV-E.*

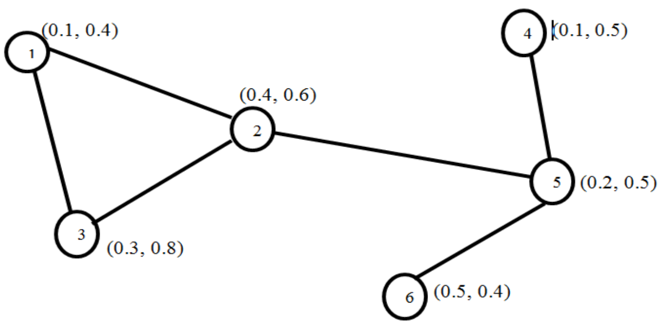

Figure 1 : Undirected Binary Graph

Definition 2.4: Let $\boldsymbol{A_1} = (\mu^{\rho}_{A_1}, \mu^{\text{и}}_{A_1})$*and* $\boldsymbol{A_2} = (\mu^{\rho}_{A_2}, \mu^{\text{и}}_{A_2})$ $\boldsymbol{B_1} = (\mu^{\rho}_{B_1}, \mu^{\text{и}}_{B_1})$*and* $\boldsymbol{B_2} = (\mu^{\rho}_{B_2}, \mu^{\text{и}}_{B_2})$ *be a binary fuzzy graph subsets of* E_1*and* E_2 *respectively*. Then the cartesian product of two BG_1 and BG_2 of graph $BG_1{}^*$and $BG_2{}^*$ by BG_1 X BG_2= $(\boldsymbol{A_1 X A_2}, \boldsymbol{B_1 X B_2})$ and defined as follows

1. $(\mu^{\rho}_{A_1} X \mu^{\rho}_{A_2})(\alpha_1, \alpha_2) = (\mu^{\rho}_{A_1}(\alpha_1) \Lambda \mu^{\rho}_{A_2}(\alpha_2))$
   $(\mu^{\text{и}}_{A_1} X \mu^{\text{и}}_{A_2})(\alpha_1, \alpha_2) = (\mu^{\text{и}}_{A_1}(\alpha_1) \Lambda \mu^{\text{и}}_{A_2}(\alpha_2))$ for all $\alpha, \beta \in V$

2. $(\mu^{\rho}_{B_1} X \mu^{\rho}_{B_2})(\alpha_1, \alpha_2)(\alpha_1, \beta_2) = (\mu^{\rho}_{A_1}(\alpha_1) \Lambda \mu^{\rho}_{B_2}(\alpha_2, \beta_2))$
   $(\mu^{\text{и}}_{B_1} X \mu^{\text{и}}_{B_2})(\alpha_1, \alpha_2)(\alpha_1, \beta_2) = (\mu^{\text{и}}_{A_1}(\alpha_1) V \mu^{\text{и}}_{B}(\alpha_2, \beta_2))$ for all $\alpha_1 \in V_2$ *and* $\alpha_2, \beta_2 \in E_2$
3. $(\mu^{\rho}_{B_1} X \mu^{\rho}_{B_2})(\alpha_1, \gamma)(\alpha_1, \gamma) = (\mu^{\rho}_{B}(\alpha_2, \beta_2) \Lambda \mu^{\rho}_{A_2}(\gamma))$
   $(\mu^{\text{и}}_{B_1} X \mu^{\text{и}}_{B_2})(\alpha_1, \gamma)(\alpha_1, \gamma) = (\mu^{\rho}_{A_1}(\alpha_1, \beta_1) V \mu^{\rho}_{B}(\gamma))$ for all $\gamma \in V_2$ *and* $\alpha_1, \beta_1 \in E_1$

Definition 2.5: Let $\boldsymbol{A_1} = (\mu^{\rho}_{A_1}, \mu^{\text{и}}_{A_1})$*and* $\boldsymbol{A_2} = (\mu^{\rho}_{A_2}, \mu^{\text{и}}_{A_2})$ *be an undirected binary fuzzy subgraph of* V_1 *and* V_2 *let* $\boldsymbol{B_1} = (\mu^{\rho}_{B_1}, \mu^{\text{и}}_{B_1})$*and* $\boldsymbol{B_2} = (\mu^{\rho}_{B_2}, \mu^{\text{и}}_{B_2})$ *be binary fuzzy graph subsets of* E_1*and* E_2 *respectively*. Then the composition of two binary *fuzzy graphs* BG_1 and BG_2 of graphs $BG_1{}^*$and $BG_2{}^*$ by $BG_1 \circ BG_2$= $(\boldsymbol{A_1 \circ A_2}, \boldsymbol{B_1 \circ B_2})$ and defined as follows

1. $(\mu^{\rho}_{A_1} \circ \mu^{\rho}_{A_2})(\alpha_1, \alpha_2) = (\mu^{\rho}_{A_1}(\alpha_1) \Lambda \mu^{\rho}_{A_2}(\alpha_2))$
   $(\mu^{\text{и}}_{A_1} \circ \mu^{\text{и}}_{A_2})(\alpha_1, \alpha_2) = (\mu^{\text{и}}_{A_1}(\alpha_1) \Lambda \mu^{\text{и}}_{A_2}(\alpha_2))$ for all $\alpha_1, \alpha_2 \in V$
2. $(\mu^{\rho}_{B_1} \circ \mu^{\rho}_{B_2})(\alpha_1, \alpha_2)(\alpha_1, \beta_2) = (\mu^{\rho}_{A_1}(\alpha_1) \Lambda \mu^{\rho}_{B_2}(\alpha_2, \beta_2))$
   $(\mu^{\text{и}}_{B_1} \circ \mu^{\text{и}}_{B_2})(\alpha_1, \alpha_2)(\alpha_1, \beta_2) = (\mu^{\text{и}}_{A_1}(\alpha_1) V \mu^{\text{и}}_{B}(\alpha_2, \beta_2))$ for all $\alpha_1 \in V_2$ *and* $\alpha_2, \beta_2 \in E_2$
3. $(\mu^{\rho}_{B_1} \circ \mu^{\rho}_{B_2})(\alpha_1, \gamma)(\alpha_1, \gamma) = (\mu^{\rho}_{B}(\alpha_2, \beta_2) \Lambda \mu^{\rho}_{A_2}(\gamma))$
   $(\mu^{\text{и}}_{B_1} \circ \mu^{\text{и}}_{B_2})(\alpha_1, \gamma)(\beta_1, \gamma) = (\mu^{\rho}_{A_1}(\alpha_1, \beta_1) V \mu^{\rho}_{B}(\gamma))$ for all $\gamma \in V_2$ *and* $\alpha_1, \beta_1 \in E_1$
4. $(\mu^{\rho}_{B_1} \circ \mu^{\rho}_{B_2})(\alpha_1, \gamma)(\beta_1, \gamma) = (\mu^{\rho}_{B}(\alpha_2, \beta_2) \Lambda \mu^{\rho}_{A_2}(\gamma))$
   $(\mu^{\text{и}}_{B_1} \circ \mu^{\text{и}}_{B_2})(\alpha_1, \alpha_2)(\beta_1, \beta_2) = (\mu^{\rho}_{A_1}(\alpha_2) \Lambda \mu^{\rho}_{A_2}(\beta_2) \Lambda \mu^{\rho}_{B_1}(\alpha_1, \alpha_2))$ for all$(\alpha_1, \alpha_2)(\beta_1, \beta_2) \in E^{\circ} - E$
5. $(\mu^{\text{и}}_{B_1} \circ \mu^{\text{и}}_{B_2})(\alpha_1, \alpha_2)(\beta_1, \beta_2) = (\mu^{\text{и}}_{A_1}(\alpha_2) V \mu^{\text{и}}_{A_2}(\beta_2) V \mu^{\text{и}}_{B_1}(\alpha_1, \alpha_2))$ for all$(\alpha_1, \alpha_2)(\beta_1, \beta_2) \in E^{\circ} - E$

Definition 2.6: Let $\boldsymbol{A_1} = (\mu^{\rho}_{A_1}, \mu^{\text{и}}_{A_1})$*and* $\boldsymbol{A_2} = (\mu^{\rho}_{A_2}, \mu^{\text{и}}_{A_2})$ *be an undirected binary fuzzy subgraph of* V_1 *and* V_2 *let* $\boldsymbol{B_1} = (\mu^{\rho}_{B_1}, \mu^{\text{и}}_{B_1})$*and* $\boldsymbol{B_2} = (\mu^{\rho}_{B_2}, \mu^{\text{и}}_{B_2})$ *be binary fuzzy graph subsets of* E_1*and* E_2 *respectively*. Then the normal product of two binary *fuzzy graphs* BG_1 and BG_2 of graphs $BG_1{}^*$and $BG_2{}^*$ by $BG_1 * BG_2$= $(\boldsymbol{A_1} * \boldsymbol{A_2}, \boldsymbol{B_1} * \boldsymbol{B_2})$ and defined as follows

1. $(\mu^{\rho}_{A_1} * \mu^{\rho}_{A_2})(\alpha_1, \alpha_2) = (\mu^{\rho}_{A_1}(\alpha_1) \Lambda \mu^{\rho}_{A_2}(\alpha_2))$
   $(\mu^{\text{и}}_{A_1} * \mu^{\text{и}}_{A_2})(\alpha_1, \alpha_2) = (\mu^{\text{и}}_{A_1}(\alpha_1) V \mu^{\text{и}}_{A_2}(\alpha_2))$ for all $\alpha_1, \alpha_2 \in V$
2. $(\mu^{\rho}_{B_1} * \mu^{\rho}_{B_2})(\alpha_1, \alpha_2)(\beta_1, \beta_2) = (\mu^{\rho}_{A_1}(\alpha_1) \Lambda \mu^{\rho}_{B_2}(\alpha_2, \beta_2))$
   $(\mu^{\text{и}}_{B_1} * \mu^{\text{и}}_{B_2})(\alpha_1, \alpha_2)(\alpha_1, \beta_2) = (\mu^{\text{и}}_{A_1}(\alpha_1) V \mu^{\text{и}}_{B_2}(\alpha_2, \beta_2))$ for all $\alpha_1 \in V_2$ *and* $\alpha_2, \beta_2 \in E_2$
3. $(\mu^{\rho}_{B_1} * \mu^{\rho}_{B_2})(\alpha_1, \gamma)(\beta_1, \gamma) = (\mu^{\rho}_{B}(\alpha_2, \beta_2) \Lambda \mu^{\rho}_{A_2}(\gamma))$
   $(\mu^{\text{и}}_{B_1} * \mu^{\text{и}}_{B_2})(\alpha_1, \gamma)(\beta_1, \gamma) = (\mu^{\rho}_{A_1}(\alpha_1, \beta_1) V \mu^{\rho}_{B_2}(\gamma))$ for all $\gamma \in V_2$ *and* $\alpha_1, \beta_1 \in E_1$
4. $(\mu^{\rho}_{B_1} * \mu^{\rho}_{B_2})(\alpha_1, \gamma)(\beta_1, \gamma) = (\mu^{\rho}_{B_1}(\alpha_2, \beta_2) \Lambda \mu^{\rho}_{A_2}(\gamma))$
   $(\mu^{\text{и}}_{B_1} * \mu^{\text{и}}_{B_2})(\alpha_1, \beta_2)(\alpha_2, \beta_2) = (\mu^{\rho}_{B_1}(\alpha_1, \beta_1) V \mu^{\rho}_{B_2}(\alpha_1, \beta_1))$ for all$\alpha_1, \beta_1 \in E_1, (\alpha_2, \beta_2) \in E_2$ $E^{\circ} - E$

Definition 2.7: Let $\boldsymbol{A_1} = (\mu^{\rho}_{A_1}, \mu^{\text{и}}_{A_1})$*and* $\boldsymbol{A_2} = (\mu^{\rho}_{A_2}, \mu^{\text{и}}_{A_2})$ *be an undirected binary fuzzy subgraph of* V_1 *and* V_2 *let* $\boldsymbol{B_1} = (\mu^{\rho}_{B_1}, \mu^{\text{и}}_{B_1})$*and* $\boldsymbol{B_2} = (\mu^{\rho}_{B_2}, \mu^{\text{и}}_{B_2})$ *be binary fuzzy graph subsets of* E_1*and* E_2 *respectively*. Then the tensor product of two binary *fuzzy graphs* BG_1 and BG_2 of graphs $BG_1{}^*$and $BG_2{}^*$ by $BG_1 \otimes BG_2$= $(\boldsymbol{A_1} \otimes \boldsymbol{A_2}, \boldsymbol{B_1} \otimes \boldsymbol{B_2})$ and defined as follows

1. $(\mu^{\rho}_{A_1} \otimes \mu^{\rho}_{A_2})(\alpha_1, \alpha_2) = (\mu^{\rho}_{A_1}(\alpha_1) \Lambda \mu^{\rho}_{A_2}(\alpha_2))$
   $(\mu^{\text{и}}_{A_1} \otimes \mu^{\text{и}}_{A_2})(\alpha_1, \alpha_2) = (\mu^{\text{и}}_{A_1}(\alpha_1) V \mu^{\text{и}}_{A_2}(\alpha_2))$ for all $\alpha_1, \alpha_2 \in V$
2. $(\mu^{\rho}_{B_1} \otimes \mu^{\rho}_{B_2})(\alpha_1, \alpha_2)(\beta_1, \beta_2) = (\mu^{\rho}_{B_1}(\alpha_1) \Lambda \mu^{\rho}_{B_2}(\alpha_2, \beta_2))$
   $(\mu^{\text{и}}_{B_1} \otimes \mu^{\text{и}}_{B_2})(\alpha_1, \alpha_2)(\alpha_1, \beta_2) = (\mu^{\text{и}}_{A_1}(\alpha_1, \beta_1) V \mu^{\text{и}}_{B_2}(\alpha_2, \beta_2))$ for all $\alpha_1, \beta_2 \in E_1$ *and* $\alpha_2, \beta_2 \in E_2$

Degree of a vertex in the cartesian product

In above defination, for any vertex $(\alpha_1, \beta_1) \in V$

$$d_{BG_1\,X\,BG_2}(\alpha_1, \alpha_2) = \sum_{(\alpha_1,\alpha_2)(\alpha_1,\beta_2)\in E} \left[(\mu^{\rho}_{A_1} X \mu^{\rho}_{A_2})(\alpha_1,\alpha_2)(\beta_1,\beta_2), (\mu^{и}_{A_1}, \mu^{и}_{A_2})(\alpha_1,\alpha_2)(\beta_1,\beta_2)\right]$$

$$=\sum_{(\alpha_1=\beta_1)(\alpha_2,\beta_2)\in E_2}\left[(\mu^{\rho}_{A_1}(\alpha_1)\Lambda\mu^{\rho}_{B_2})(\alpha_2,\beta_2), (\mu^{и}_{A_1}(\alpha_1)V\mu^{и}_{B_2}(\alpha_2,\beta_2)\right]+$$
$$\sum_{(\alpha_2=\beta_2)(\alpha_1,\beta_1)\in E_1}\left[(\mu^{\rho}_{B_1}(\alpha_1,\beta_1)\Lambda\mu^{\rho}_{A_2})(\beta_2), (\mu^{и}_{B_1}(\alpha_1,\beta_1)V\mu^{и}_{A_2}(\beta_2))\right]$$

In the following theorems, we define the degree of (α_1, α_2) in $BG_1\ X\ BG_2$ in terms of those in some particular cases.

Theorem 1: Let BG_1 *and* BG_2 be two undircted binary fuzzy graphs. If $\mu^{\rho}_{A_1} \geq \mu^{\rho}_{B_2}, \mu^{и}_{A_1} \leq \mu^{и}_{B_2}$ and $\mu^{\rho}_{A_2} \geq \mu^{\rho}_{B_1}, \mu^{и}_{A_2} \leq \mu^{и}_{B_1}$ then $d_{BG_1\,X\,BG_2}(\alpha_1, \alpha_2) = d_{BG_1} + d_{BG_2}$

Proof : By deffination of degree of a vertex in cartesian product

$$d_{BG_1\,X\,BG_2}(\alpha_1,\alpha_2)=\sum_{(\alpha_1=\beta_1)(\alpha_2,\beta_2)\in E_2}\left[(\mu^{\rho}_{A_1}(\alpha_1)\Lambda\mu^{\rho}_{B_2}(\alpha_2,\beta_2), (\mu^{и}_{A_1}(\alpha_1)V\mu^{и}_{B_2}(\alpha_2,\beta_2)\right]+$$
$$\sum_{(\alpha_2=\beta_2)(\alpha_1,\beta_1)\in E_1}\left[(\mu^{\rho}_{B_1}(\alpha_1,\beta_1)\Lambda\mu^{\rho}_{A_2}(\beta_2), (\mu^{и}_{B_1}(\alpha_1,\beta_1)V\mu^{и}_{A_2}(\beta_2)\right]$$
$$=\sum_{(\alpha_2,\beta_2)\in E_2}\left[(\mu^{\rho}_{B_2}(\alpha_2,\beta_2), \mu^{и}_{B_2}(\alpha_2,\beta_2)\right] + \sum_{(\alpha_1,\beta_1)\in E_1}\left[(\mu^{\rho}_{B_1}(\alpha_1,\beta_1), (\mu^{и}_{B_1}(\alpha_1,\beta_1)\right]$$
$$=d_{BG_2}(\alpha_2) + d_{BG_1}(\alpha_1)$$

Example 1

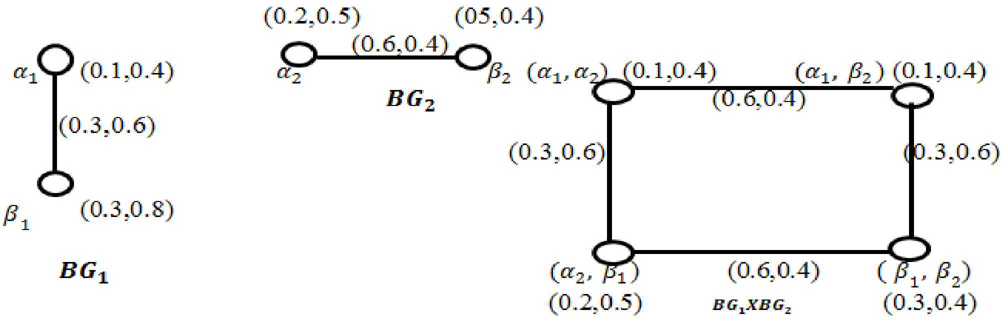

Figure 2 :Cartesian product

Here $\mu^{\rho}_{A_1} \geq \mu^{\rho}_{B_2}, \mu^{и}_{A_1} \leq \mu^{и}_{B_2}$ and $\mu^{\rho}_{A_2} \geq \mu^{\rho}_{B_1}, \mu^{и}_{A_2} \leq \mu^{и}_{B_1}$ by theorem of 2.

$d_{BG_1\,X\,BG_2}(\alpha_1, \alpha_2) = d_{BG_1} + d_{BG_2}$

= (0.6+0.4, 0.3+0.5, 0.6+0.4, 0.3+0.5)

= (1.0, 0.8, 1.0, 0.8)

Similarly we find to all vertex in BG_1XBG_2 **in figure 2**

Degree of a vertex in composition product

In above defination, for any vertex $(\alpha_1, \beta_1) \in V$

$$d_{BG_1\circ BG_2}(\alpha_1, \alpha_2) = \sum_{(\alpha_1,\alpha_2)(\beta_1,\beta_2)\in E} \left[(\mu^{\rho}_{A_1}\circ \mu^{\rho}_{A_2})(\alpha_1,\alpha_2)(\beta_1,\beta_2), (\mu^{и}_{A_1}\circ \mu^{и}_{A_2})(\alpha_1,\alpha_2)(\beta_1,\beta_2)\right]$$
$$=\sum_{(\alpha_1=\beta_1)(\alpha_2,\beta_2)\in E_2}\left[(\mu^{\rho}_{A_1}(\alpha_1)\Lambda\mu^{\rho}_{B_2})(\alpha_2,\beta_2), (\mu^{и}_{A_1}(\alpha_1)V\mu^{и}_{B_2}(\alpha_2,\beta_2))\right]$$
$$+\sum_{(\alpha_2=\beta_2)(\alpha_1,\beta_1)\in E_1}\left[(\mu^{\rho}_{A_2}(\alpha_2)\Lambda\mu^{\rho}_{B_1}(\alpha_1,\beta_1), (\mu^{и}_{A_2}(\alpha_2)V\mu^{и}_{B_1}(\alpha_1,\beta_1))\right]$$
$$+\sum_{(\alpha_2\neq\beta_2)(\alpha_1,\beta_1)\in E_1}\left[(\mu^{\rho}_{A_2}(\alpha_2)\Lambda\mu^{\rho}_{A_2}(\beta_1)\Lambda\mu^{\rho}_{B_1}(\alpha_1,\beta_1)), ((\mu^{и}_{A_1}(\alpha_2)V\mu^{и}_{A_2}(\beta_2)V\mu^{и}_{B_1}(\alpha_1,\beta_1)\right]$$

Theorem 2: Let BG_1 *and* BG_2 be two undircted binary fuzzy graphs. If $\mu^{\rho}_{A_1} \geq \mu^{\rho}_{B_2}, \mu^{и}_{A_1} \leq \mu^{и}_{B_2}$ and $\mu^{\rho}_{A_2} \geq \mu^{\rho}_{B_1}, \mu^{и}_{A_2} \leq \mu^{и}_{B_1}$ then $d_{BG_1\circ BG_2}(\alpha_1, \alpha_2) = d_{BG_1}(\alpha_1)|V_2| + d_{BG_2}(\beta_2)$

Proof : By deffination of degree of a vertex in composition product

$$d_{BG_1\circ BG_2}(\alpha_1,\alpha_2)=\sum_{(\alpha_1,\alpha_2)(\beta_1,\beta_2)\in E}\left[(\mu^{\rho}_{A_1}\circ\mu^{\rho}_{A_2}(\alpha_1,\alpha_2)(\beta_1,\beta_2), (\mu^{и}_{A_1}\circ\mu^{и}_{A_2}(\alpha_1,\alpha_2)(\alpha_2,\beta_2)\right]$$
$$=\sum_{(\alpha_1=\beta_1)(\alpha_2,\beta_2)\in E_2}\left[(\mu^{\rho}_{A_1}(\alpha_1)\Lambda\mu^{\rho}_{B_2}(\alpha_2,\beta_2), (\mu^{и}_{A_1}(\alpha_1)V\mu^{и}_{B_2}(\alpha_2,\beta_2)\right]+$$
$$\sum_{(\alpha_2=\beta_2)(\alpha_1,\beta_1)\in E_1}\left[(\mu^{\rho}_{B_1}(\alpha_1)\Lambda\mu^{\rho}_{A_2}(\alpha_1,\beta_2), (\mu^{и}_{B_1}(\alpha_1,\beta_1)V\mu^{и}_{A_2}(\beta_2)\right]$$
$$\sum_{(\alpha_2\neq\beta_2)(\alpha_1,\beta_1)\in E_1}\left[(\mu^{\rho}_{A_2}(\alpha_2)\Lambda\mu^{\rho}_{A_2}(\beta_1)\Lambda\mu^{\rho}_{B_1}(\alpha_1,\beta_1)), ((\mu^{и}_{A_1}(\alpha_2)V\mu^{и}_{A_2}(\beta_2)V\mu^{и}_{B_1}(\alpha_1,\beta_1)\right]$$

$=d_{BG_2}(\alpha_2) + |V_2| d_{BG_1}(\alpha_1)$

Example 2:

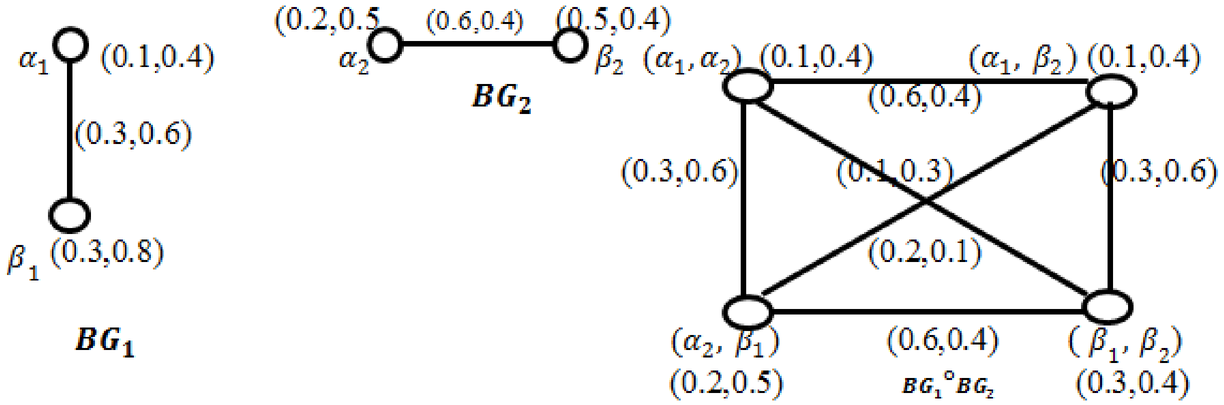

Figure 3

:Composition product

Here $d_{BG_1 \circ BG_2}(\alpha_1, \alpha_2) = d_{BG_2}(\alpha_2) + |V_2| d_{BG_1}(\alpha_1)$

$= 0.4 + 2\ (0.3) = 1.0$

Similarly we find to all vertex of $d_{BG_1 \circ BG_2}(\alpha_1, \alpha_2)$.

Degree of a vertex in tensor product

In above defination, for any vertex $(\alpha_1, \beta_1) \in V_1 X V_2$

$$d_{BG_1 \otimes BG_2}(\alpha_1, \alpha_2) = \sum_{(\alpha_1,\alpha_2)(\beta_1,\beta_2)\in E} [(\mu^{\rho}_{A_1} \otimes \mu^{\rho}_{A_2})(\alpha_1,\alpha_2)(\beta_1,\beta_2), (\mu^{\text{и}}_{A_1} \otimes \mu^{\text{и}}_{A_2})(\alpha_1,\alpha_2)(\beta_1,\beta_2)]$$

$$=\sum_{(\alpha_1=\beta_1)(\alpha_2,\beta_2)\in E_2} [(\mu^{\rho}_{B_1}(\alpha_1,\alpha_2) \Lambda \mu^{\rho}_{B_2}(\beta_1,\beta_2), (\mu^{\text{и}}_{B_1}(\alpha_1,\alpha_2) V \mu^{\text{и}}_{B_2}(\beta_1,\beta_2))]$$

Theorem 3: Let BG_1 *and* BG_2 be two undircted binary fuzzy graphs. If $\mu^{\rho}_{B_2} \geq \mu^{\rho}_{B_1}, \mu^{\text{и}}_{B_1} \leq \mu^{\text{и}}_{B_2}$ then $d_{BG_1 \otimes BG_2}(\alpha_1, \alpha_2) = d_{BG_1}(\alpha_1)$ and $\mu^{\rho}_{B_1} \geq \mu^{\rho}_{B_2}, \mu^{\text{и}}_{B_1} \leq \mu^{\text{и}}_{B_2}$ then $d_{BG_1 \otimes BG_2}(\alpha_1, \alpha_2) = d_{BG_2}(\beta_2)$

Proof : By deffination of degree of a vertex in cartesian product

$d_{BG_1 \otimes BG_2}(\alpha_1, \alpha_2) = \sum_{(\alpha_1,\beta_1)\in E_1} [(\mu^{\rho}_{B_1}(\alpha_1,\beta_1) \Lambda \mu^{\rho}_{B_2}(\alpha_2,\beta_2), (\mu^{\text{и}}_{B_1}(\alpha_1,\beta_1) V \mu^{\text{и}}_{A_2}(\alpha_2,\beta_2)]$

$= \sum_{(\alpha_1,\beta_1)\in E_1} [(\mu^{\rho}_{B_1}(\alpha_1,\beta_1), (\mu^{\text{и}}_{B_1}(\alpha_1,\beta_1)]$

$= d_{BG_1}(\alpha_1)$

Example 3:

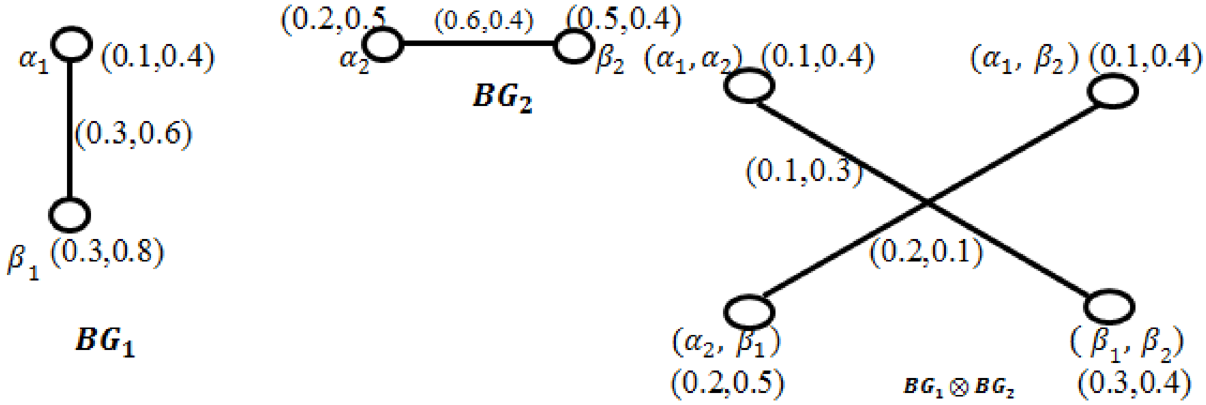

Figure 4 :

Tensor product

Here $d_{BG_1 \otimes BG_2}(\alpha_1, \alpha_2)$= (0.3, 0.6, 0.6, 0.4)= $d_{BG_1}(\alpha_1)$

$d_{BG_1 \otimes BG_2}(\alpha_1, \alpha_2)$= (0.3, 0.6, 0.6, 0.4)= $d_{BG_1}(\beta_1)$

Degree of a vertex in normal product

In above defination, for any vertex $(\alpha_1, \beta_1) \in V_1 X V_2$

$$d_{BG_1 X BG_2}(\alpha_1, \alpha_2) = \sum_{(\alpha_1,\alpha_2)(\beta_1,\beta_2)\in E} [(\mu^{\rho}_{B_1} X \mu^{\rho}_{B_2})(\alpha_1,\alpha_2)(\beta_1,\beta_2), (\mu^{\text{и}}_{B_1} X \mu^{\text{и}}_{B_2})(\alpha_1,\alpha_2)(\beta_1,\beta_2)]$$

$$=\sum_{(\alpha_1=\beta_1)(\alpha_2,\beta_2)\in E_2} [(\mu^{\rho}_{A_1}(\alpha_1) \Lambda \mu^{\rho}_{B_2}(\alpha_2,\beta_2), (\mu^{\text{и}}_{A_1}(\alpha_1) V \mu^{\text{и}}_{B_2}(\alpha_2,\beta_2)]$$

$$+\sum_{(\alpha_2=\beta_2)(\alpha_1,\beta_1)\in E_1}\left[(\mu^{\rho}_{B_1}(\alpha_1,\beta_2)\Lambda\mu^{\rho}_{A_2}(\alpha_1),(\mu^{\text{и}}_{B_1}(\alpha_1,\beta_1)V\mu^{\text{и}}_{A_2}(\beta_2)\right]$$

$$+\sum_{(\alpha_2,\beta_2)\in E_2(\alpha_1,\beta_1)\in E_1}\left[(\mu^{\rho}_{B_2}(\alpha_2,\beta_2)\Lambda\mu^{\rho}_{A_2}(\beta_1)\Lambda\mu^{\rho}_{B_1}(\alpha_1,\beta_1)),(\mu^{\text{и}}_{B_2}(\alpha_2,\beta_2)V\mu^{\text{и}}_{B_1}(\alpha_1,\beta_1))\right]$$

Theorem 4: Let BG_1 *and* BG_2 be two undircted binary fuzzy graphs. If $\mu^{\rho}_{B_2}\geq\mu^{\rho}_{B_1}$, $\mu^{\text{и}}_{B_2}\leq\mu^{\text{и}}_{B_1}$ and $\mu^{\rho}_{A_2}\geq\mu^{\rho}_{B_1}$, $\mu^{\text{и}}_{B_1}\leq\mu^{\text{и}}_{B_1}$ then $d_{BG_1 X\,BG_2}(\alpha_1,\alpha_2)=d_{BG_2}(\beta_2)+|V_2|d_{BG_1}(\alpha_1)$

Proof : By deffination of degree of a vertex in cartesian product

$$d_{BG_1 X\,BG_2}(\alpha_1,\alpha_2)=\sum_{(\alpha_1,\alpha_2)(\beta_1,\beta_2)\in E}\left[(\mu^{\rho}_{B_1}X\mu^{\rho}_{B_2}(\alpha_1,\alpha_2)(\beta_1,\beta_2),(\mu^{\text{и}}_{B_1}X\mu^{\text{и}}_{B_2}(\alpha_1,\alpha_2)(\alpha_2,\beta_2)\right]$$

$$=\sum_{(\alpha_1=\beta_1)(\alpha_2,\beta_2)\in E_2}\left[(\mu^{\rho}_{A_1}(\alpha_1)\Lambda\mu^{\rho}_{B_2}(\alpha_2,\beta_2),(\mu^{\text{и}}_{A_1}(\alpha_1)V\mu^{\text{и}}_{B_2}(\alpha_2,\beta_2)\right]$$

$$+\sum_{(\alpha_2=\beta_2)(\alpha_1,\beta_1)\in E_1}\left[(\mu^{\rho}_{A_1}(\alpha_1)\Lambda\mu^{\rho}_{B_2}(\alpha_2,\beta_2),(\mu^{\text{и}}_{A_1}(\alpha_1)V\mu^{\text{и}}_{B_2}(\beta_1,\beta_2)\right]$$

$$+\sum_{(\alpha_2=\beta_2)(\alpha_1,\beta_1)\in E_1}\left[(\mu^{\rho}_{A_1}(\alpha_2)\Lambda\mu^{\rho}_{B_1}(\alpha_1,\beta_1),(\mu^{\text{и}}_{A_2}(\alpha_2)V\mu^{\text{и}}_{B_1}(\alpha_1,\beta_2)\right]$$

$$+\sum_{(\alpha_1,\alpha_2)(\beta_1,\beta_2)\in E}\left[(\mu^{\rho}_{B_1}(\alpha_1,\beta_1)X\,\mu^{\rho}_{B_2}(\alpha_2,\beta_2),(\mu^{\text{и}}_{B_1}(\alpha_1,\beta_1)X\,\mu^{\text{и}}_{B_2}(\alpha_2,\beta_2)\right]$$

$$=\sum_{(\alpha_1=\beta_1)(\alpha_2,\beta_2)\in E_2}\left[\mu^{\rho}_{B_2}(\alpha_2,\beta_2),\mu^{\text{и}}_{B_2}(\alpha_2,\beta_2)\right]$$

$$+\sum_{(\alpha_2=\beta_2)(\alpha_1,\beta_1)\in E_1}\left[(\mu^{\rho}_{B_1}(\alpha_1,\beta_2),(\mu^{\text{и}}_{B_1}(\alpha_1,\beta_1)\right]$$

$$+\sum_{(\alpha_1,\beta_1)\in E_1}\left[\mu^{\rho}_{B_1}(\alpha_1,\beta_1),\mu^{\text{и}}_{B_1}(\alpha_1,\beta_1)\right]$$

$$=d_{BG_2}(\alpha_2)+|V_2|d_{BG_1}(\alpha_1)$$

Example 4:

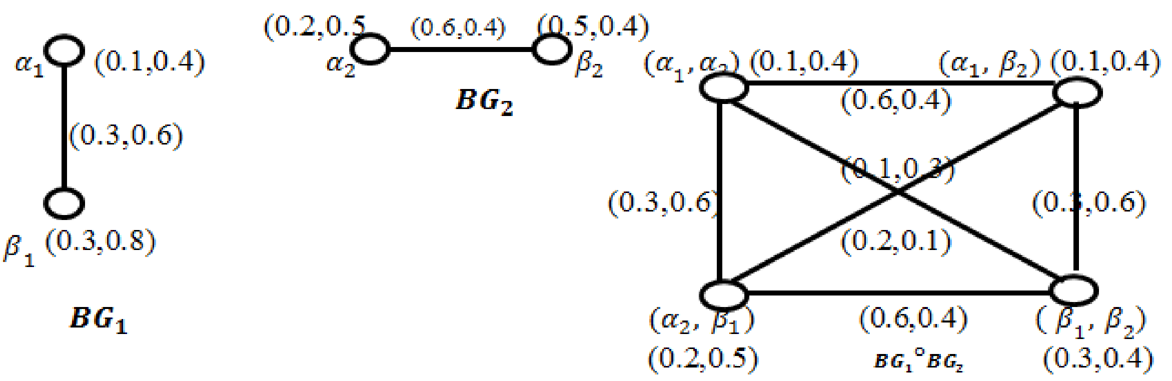

Figure 5 :Normal product

$$d_{BG_1 X\,BG_2}(\alpha_1,\alpha_2)=(0.3+0.3+0.3,0.6+0.6+0.4,0.4+0.4+0.4,1.8+1.8+1.2)$$

$$=(0.9,\ 1.6,\ 1.2,4.2)$$

$$d_{BG_1 X\,BG_2}(\alpha_1,\alpha_2)=0.3,0.4,0.4)+2(0.3,0.6,0.4,1.8)$$

$$=(0.9,\ 1.6,\ 1.2,4.2)$$

CONCLUSION

In this paper, we have found the degrees of vertices in $BG_1 * BG_2$, $BG_1\ X\ BG_2$, $BG_1 \circ BG_2$, $BG_1 \otimes BG_2$in terms of degree of vertices in and under some conditions and illustrated them through examples. This will be helpful when the graphs are very large. Also they will be very useful in studying various properties of cartesian product, composition, tensor product, normal product of two bipolar fuzzy graphs.

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