# Analysis of constant amplitude modulation (CAM) of Goppa coded OFDM (G-OFDM) signal for reducing the PAPR of OFDM system

**[1]Sharmila Sengupta, [2] Dr. B. K. Lande**
[1]Associate Professor, [2]Professor
[1]Computer Engineering department
[1]Vivekanand Education Society's Institute of Technology, Chembur, Mumbai, India

***Abstract***— Portable devices, especially small in nature, are an essential part of the 5G system. With the advances of technology, they have substantial communication capabilities and fast computing abilities but the demand for high data rate services causes a drain in energy of these devices. Other than green communication and engineering, the performance of the communication network cannot be improved for the future devices. Since these devices are constantly used for voice communication and also requires good connectivity, multimedia and entertainment capabilities, the battery lifetime poses a problem to the power amplifier (PA) which consumes most of the power in the devices and their base stations. The PA should accurately scale the demand of high data rates or data bandwidth of these systems. Therefore, actual power requirement in radio engineering and the power consumption is not at tandem with each other. An efficient way of powering the radio signal is eventually required for sustainable communication systems. It is known that OFDM based systems have several applications in wireless communications due to its spectral efficiency and robustness against multipath fading. But the multicarrier aspect of OFDM signal is characterized by high peak-to-average power ratio (PAPR), which if not taken care of will render PAs inefficient and cause distortion in the transmitted signal. In this research, a constant amplitude modulation (CAM), Goppa coded OFDM (G-OFDM) system is suggested which would not only exploit the channel coding importance of Goppa codes but also the peak power would be made equal to the mean power forcing PAPR value to reduce to 0 dB by reducing the amplitude and phase fluctuations of OFDM signal prior to bandpass conversion.
Therefore, the PA can operate at saturation levels with increase in coverage of transmitted signal due to increase in average transmitted power. The increase in PA efficiency will enhance battery life of the devices.

***Index Terms*—*constant amplitude modulation (CAM); Goppa coded OFDM (G-OFDM); Goppa code; power amplifier (PA)***

## I. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) is a technology supporting 5G. The high data rate signal is divided into several low rate subcarriers in parallel thus increasing the symbol duration in OFDM multicarrier modulation and reducing the dispersive time of multipath delay spread. OFDM signal is a sum of several subcarriers linearly modulated resulting in a fluctuating envelope. Due to the constructive or destructive combination of the amplitude of its signal, average power may be low but intermittent high-power spikes may be there. This gives rise to high PAPR. PAPR depends on $N$ i.e. the number of OFDM subcarriers and $M$, the order of modulation of the subcarriers.

Analysis of constant amplitude modulation of Goppa coded OFDM (G-OFDM) signal for reducing the PAPR of OFDM system is presented in this paper. Related works done is discussed in section II. Methodology is given in section III. Observations and related graphs are given in section IV. And finally, in section V conclusion is provided.

### 1.1 RELATED WORK

When $N$ symbols of OFDM align in phase there may be high absolute peak power which is $N$ times the average power and can result in a high PAPR which would imply that high PAs in wireless system would be inefficient as it would behave nonlinearly leading to in-band distortion, increased the bit-error ratio (BER) and out-of-band radiation [1]. The PAPR reducing techniques[2], [3] used so far could be broadly classified as schemes causing distortion for example clipping, peak windowing; distortion less schemes like partial transmit sequences (PTS), selective mapping (SLM), coding, tone reservation and signal transformation methods like companding, discrete cosine transformation etc. OFDM with channel coding are less susceptible to wide spread delays of multipath propagation. As suggested in [4], [5] the use of coding scheme in multicarrier modulation also reduces peak envelope power with additional bits in the data symbol which would also be utilized for forward error correction. Several types of codes had been tried like convolutional codes, Reed Solomon codes, low density parity check codes etc. Therefore, Goppa codes had been tried earlier [6] to analyze the effect of coding with PAPR reduction. But not all data words after Goppa coding showed good PAPR reduction. Therefore, further reduction methods were tried like the impact of modulation schemes on peak power [7] but a substantial improvement was not observed. Even subcarrier mapping through zero padding showed reduction of PAPR in only selected codewords [8] leading to a search for a general solution for PAPR reduction for Goppa coded data. In this paper, the amplitude fluctuations of an OFDM signal is eliminated using constant amplitude and continuous phase modulation by combining special full wave sines and cosines instead of IFFT for orthogonality of subcarriers and passing the parallel stream of data through a phase modulator in CAM-GOFDM.

## II METHODOLOGY

### *2.1 Encoding*

A group of $N$ input bit sequence is encoded into a block of BPSK bits *s(l), (l=1,....,N)* where each bit duration is $T$ secs. These $N$ symbols are serial to parallel converted and modulated using $N$ orthogonal sub-carriers $\{e^{j2\pi f_k t},\ldots,e^{j2\pi f_{N-1+k}t}\}$, with the $l$-th subcarrier frequency $f_l = \frac{1}{NT}\ (Hz)$, where $k$ = 0 or 1. Thus the OFDM modulated signal *s(t)* for a block duration $NT$ is given by eq. 2.1 and 2.2.

$$s(t) = \frac{1}{N}\sum_{l=k}^{N-1+k} s(l)e^{j2\pi f_l t} \qquad 2.1$$

$$= \frac{1}{N}\sum_{l=k}^{N-1+k} s(l)e^{\frac{j2\pi lt}{\mathrm{NT}}} \qquad 2.2$$

### 2.1 Comparing PAPR specifications

Sampling *s(t) at* $t = nT$, where *(n=1,2,3...N),* $s(n) \triangleq s(nT) = \frac{1}{N}\sum_{l=1}^{N} s(l)e^{\frac{j2\pi ln}{N}}$ which is equivalent to *N-pt.* IDFT of the $N$ modulated symbol $s(l)$. The peak to average power (PAPR) is 10 $log_{10}\left(\frac{16}{4}\right) = 6.02\ dB$ for a 4 bit data word with no parity bit added to $s(t)$. The envelope power is calculated as $s(t)^* s(t)$ where peak $power = max\{abs(s(t) * s(t)\}$. It reduces to 10 $log_{10}\left(\frac{7.04}{4}\right) =$ 2.48 dB for a 3 bit word and 1 parity bit added with no change in data rate [4]. To find the effect of Goppa coding the information bits are first multiplied by Goppa matrix derived using Sage Math; the output of which is as mentioned below in Fig. 1 and 2. It shows the generator and parity check matrix to be used in the encoding and decoding of information bits.

*sage: load(“DataCollecctionScript.sage”)*
*Insert m value : 4*
*Enter the No of Errors (Even No) : 2*
*('modulus=',* $x^4 + x + 1$*)*
*m=4, n=16, t=2*
*('Goppa-polynomial:',* $X^2 + X + Z^3 + Z^2 + Z + 1$
*('codelocators ',* $[Z^2+1, Z, Z^3+Z, Z^2, Z^2+Z+1, Z^3, Z^3+Z^2+Z, Z+1, Z^3+Z^2+Z+1, Z^2+Z, Z^3+Z^2+1, Z^3+Z^2, Z^3+1, Z^3+Z+1, 1, 0]$*)*
*('H_goppa rank ', 8)*

$$\begin{bmatrix}
1&0&0&0&0&1&0&0&0&1&1&1&0&0&0&0\\
0&1&1&1&1&1&0&0&0&0&0&1&1&1&0&0\\
0&1&0&1&0&0&0&1&1&0&0&0&1&1&0&0\\
1&0&1&0&1&1&1&0&0&1&0&1&1&1&0&1\\
1&0&0&1&1&1&0&0&0&1&0&0&1&0&0&0\\
1&1&0&1&1&0&0&1&0&1&1&1&0&0&0&0\\
1&0&1&1&0&1&1&0&1&0&0&0&1&1&0&0\\
1&0&0&1&0&0&1&0&1&0&1&1&0&0&1&1
\end{bmatrix}$$

Figure1 H matrix from SageMath program for (16,8) Goppa codes

('G_goppa nrows', 8)

$$\begin{bmatrix}
1&0&0&0&0&0&1&0&0&0&1&0&1&1&1&0\\
0&1&0&0&0&0&0&1&0&0&1&1&0&0&1&1\\
0&0&1&0&0&0&1&0&0&0&1&1&0&0&0&1\\
0&0&0&1&0&0&1&1&0&0&1&1&1&1&0&0\\
0&0&0&0&1&0&1&1&0&0&0&0&1&0&0&1\\
0&0&0&0&0&1&0&1&0&0&1&0&1&0&1&0\\
0&0&0&0&0&0&0&0&1&0&1&1&0&1&1&0\\
0&0&0&0&0&0&0&0&0&1&1&0&1&1&0&1
\end{bmatrix}$$

Figure2 G matrix from SageMath program for (16,8) Goppa codes

The result of Goppa coding on PAPR for 8 bit BPSK modulated data is shown in Fig. 3 and shows that reduction of PAPR after coding has happened only for few data symbols and therefore only Goppa coding would not be a solution for peak power reduction.

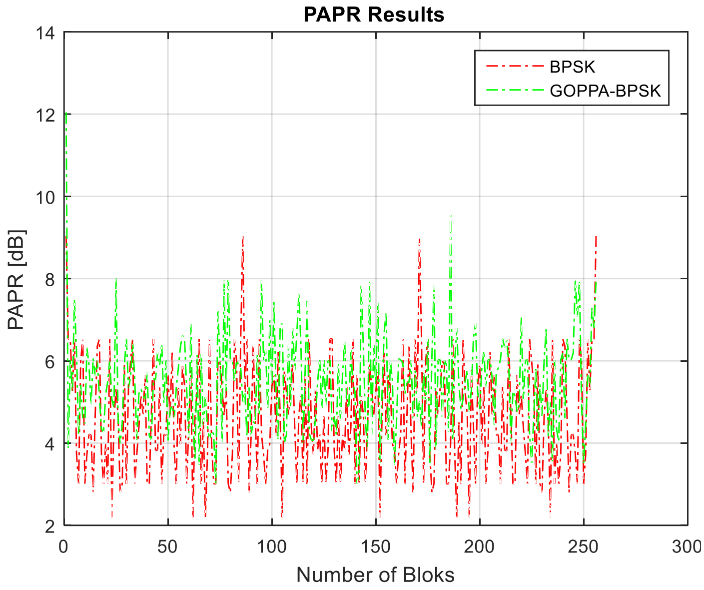

Figure3 PAPR for (16,8,5) BPSK modulated OFDM and G-OFDM

### *2.2 Modulation technique in PAPR reduction*

Modulation techniques play a role in PAPR reduction as constellation choice affects the peak absolute power of transmitted signal. Also, with M-ary modulation methods the number of carriers decrease and therefore the data rate increases with lesser transmission power but with increased complexity in transceiver design. The peak power depends on the data symbols and indirectly would depend on their constellation mapping. But it is observed that the values of PAPR are still higher for certain data symbols in all modulation techniques. So, only a choice of modulation technique would not reduce PAPR values effectively. Subcarrier mapping method was also tried where zero padding (ZP) was used to increase sampling rates for better resolution of signals. DFT is discrete and finite, therefore ZP after *N/2* samples in the frequency domain maintains spectral symmetry and gives a time domain sequence which has reduced amplitude. But this method also fails to reduce PAPR substantially.

## III **OBSERVATIONS AND GRAPHS**

As it can be seen in Fig. 4 that the orthogonal subcarriers has large power fluctuations with respect to samples which would lead to intermodulation distortions for applications requiring high power amplification in long distance communication.

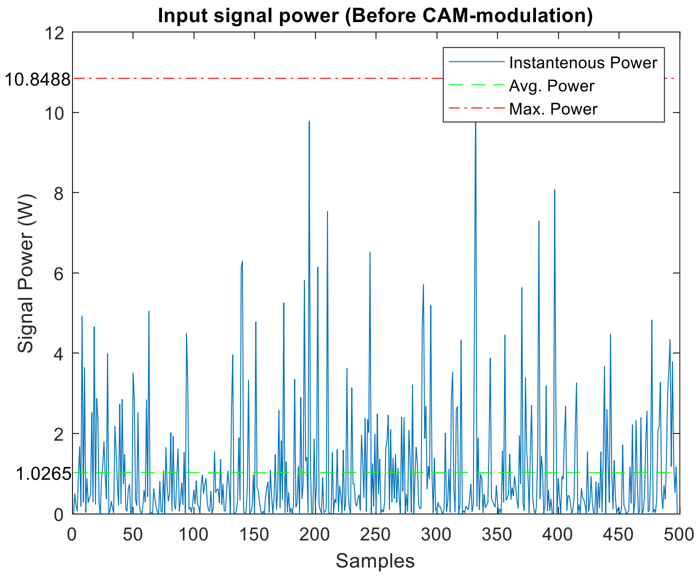

Figure 4 Relation between instantaneous, average and peak power in OFDM

As input power to the PA crosses linear dynamic range it amplifies the harmonics equally as the fundamental frequencies of the information bearing signal and leads to spectral regrowth. In this paper CAM-GOFDM is used to reduce the PAPR to 0 dB and can be thought of as a general solution [11] to power related problems in OFDM systems. It introduces correlated phase states among adjacent OFDM symbols by selecting definite values for modulation index *"h"*.

The bandpass CAM- OFDM signal can be generalised as

$$x_{bp}(t) = \sum_i \left\{ \sum_{k=0}^{N-1} |C_{i,k}| \, cos\left[2\pi\left(f_C + \frac{k}{T}\right)t + arg(C_{i,k})\right]\right\} \times g(t - iT) \qquad 3.1$$

where $f_c$ is the carrier frequency, $C_{i,k}$ are Goppa coded data symbols and T is the block period.
The baseband G-OFDM signal can therefore be expressed as

$$s(t) = \sum_i \sum_{k=0}^{N-1} C_{i,k}\, e^{j2\pi f_k t} \times g(t - i\mathrm{T}) \quad 3.2$$

The bandpass signal in eq. 3.2 can be simplified as

$$x(t) = |s(t)| \cos\big(2\pi f_c t + \varphi(t)\big) \text{ where } \varphi(t) = arg\big(s(t)\big)$$

When $\varphi(t) = 0$,

$$x(t) = |s(t)| \cos(2\pi f_c t) \cong A \cos \omega_C t \quad 3.3$$

which is similar to equation of amplitude modulation and therefore would lead to peak power at certain instants. For phase modulation the instantaneous phase of the carrier signal should vary with the instantaneous amplitude of the modulating signal and therefore $s(t)$ is modified to $s'(t)$, a phase modulated G-OFDM signal prior to bandpass conversion.
Therefore, with $\alpha$ taken as a constant
$s'(t) = e^{j\varphi(t)} = e^{j\alpha s(t)}$

$$\text{and } \varphi(t) = R\{s(t)\} = \alpha s(t)$$

Magnitude of power of $s(t) = |s'(t)|^2 = 1$

$$\therefore x(t) = R\{s'(t) e^{j2\pi f_C t}\} = R\big\{e^{j\alpha s(t)} e^{j2\pi f_C t}\big\}$$

$$= R\left\{e^{j\alpha C_{i,k} e^{j2\pi f_k t}} e^{j2\pi f_C t}\right\}$$

$$= R\big\{e^{j\alpha C_{i,k}(cos\, \varphi_k(t) + j sin\, \varphi_k(t))} e^{j2\pi f_C t}\big\}, \text{ where } \varphi_k(t) = 2\pi f_k t$$

$$= e^{-\alpha C_{i,k} sin\, \varphi_k(t)} \times cos(\alpha C_{i,k}\, cos\, \varphi_k(t) + 2\pi f_C t)$$

If *s(t)* is real, then $\cos \varphi_k(t) = 1$ & $\sin \varphi_k(t) = 0$

$$\therefore x(t) = cos(\alpha C_{i,k} + 2\pi f_C t) = cos(2\pi f_C t + \alpha C_{i,k}) \quad 3.4$$

which is a phase modulated signal as desired for getting constant magnitude of G-OFDM signals.
OFDM transmits data or information through amplitude variations of RF signal and would require linear amplification to do so successfully. As input power to the PA crosses linear dynamic range it amplifies the harmonics equally as the fundamental frequencies of the information bearing signal and leads to spectral regrowth. In this paper CAM-GOFDM is used to reduce the PAPR to 0 dB and can be thought of as a general solution [9] to power related problems in OFDM systems. It introduces correlated phase states among adjacent OFDM symbols by selecting definite values for modulation index ‘h’ or $K = 2\pi h$ radians.


Original Image


K (2*pi*h) =0.3
K (2*pi*h) =0.4
K (2*pi*h) =0.5
K (2*pi*h) =0.6

Figure 5 Comparison of decoded images with the original for different values of ‘K’

If the input information to OFDM block with phase modulation is taken as an image then under no channel impairments it can be decoded easily. The images in Fig. 5 shows that modulation index plays an important role in decoding of the original image. The concept of unity power is shown in Fig. 6 where the concentration of all phases are seen near the unity circle. Similar results are observed for a speech sample as shown in Fig. 7, 8 and 9 where at h=0.2, 0.7 and above this speech sample could not be decoded. Fig. 10 provides PAPR for 256 samples of CAM- G-OFDM system.

But the receiver complexity of CAM-GOFDM is much more than a normal OFDM receiver and without intensive design of filters, demodulators and detectors, the information carried in phase cannot be successfully demodulated.

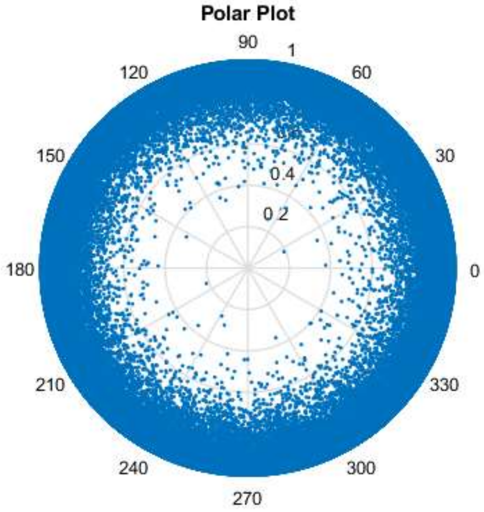

Figure 6 Polar plot of the image signal after constant amplitude modulation

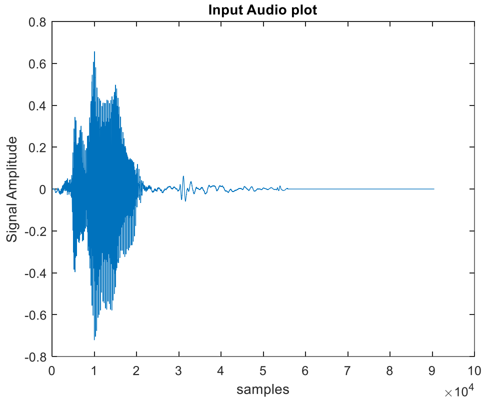

Figure 7 Original speech signal for word “hello”

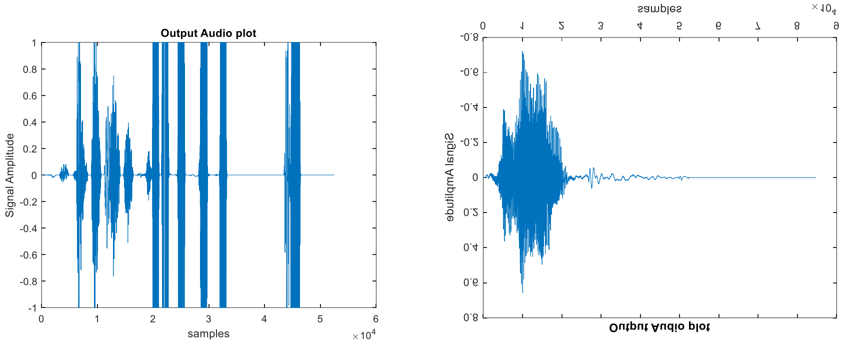

Figure 8 Decoded output at h=0.4 and 0.6 after constant amplitude modulation of speech signal

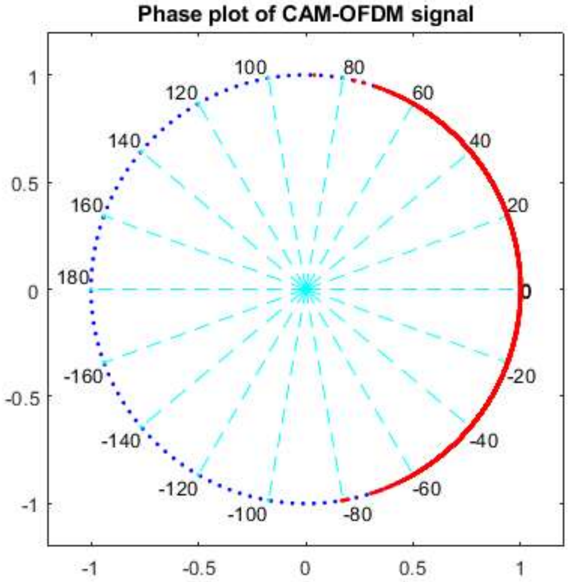

Figure 9 Phasor diagram of sound signal showing the samples lying on unity circle after phase modulation

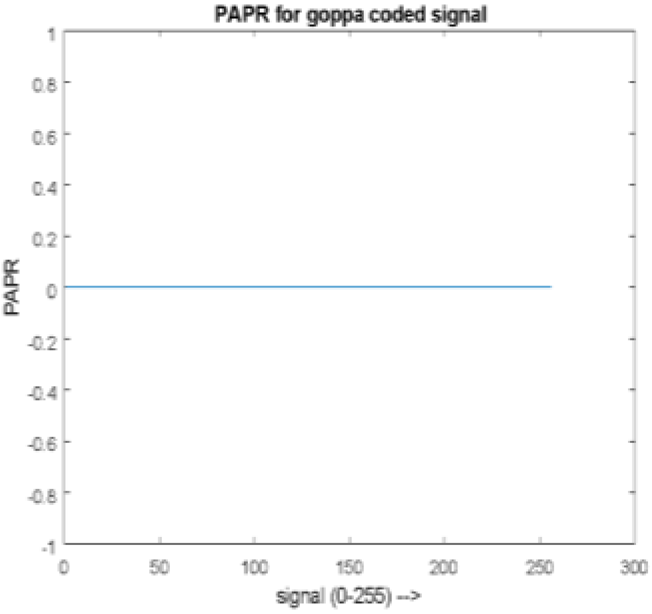

Figure 10 The graph shows a constant value of 0dB PAPR for all symbols of G-OFDM signal phase modulated before transmission

The spectral efficiency of OFDM signals is almost same as CAM-OFDM signals but only in the baseband form and is given by $= \frac{bits/s}{bandwidth} = \frac{Nlog_2\frac{M}{T_B}}{\frac{N}{T_B}} = log_2M \quad b/s/Hz$ for OFDM and the $=\frac{2\times 0.5(\frac{N}{2})log_2\frac{M}{T_B}}{(\frac{N}{2})/T_B} = log_2M \quad b/s/Hz$ for CAM-OFDM signals.

The modulation index of CAM-OFDM controls the spectral spreading and detection performance. It achieves low peak to average power ratio at the cost of the higher complexity equalization under multipath channel. The effect of phase noise on the system would certainly change the decoded output but would not alter PAPR behaviour. The required bandwidth increases with the increased modulation index or increased data variance. Therefore a summary on the performance could be given based on following parameters.

**High capability of PAPR reduction**: The zero dB PAPR of CAM-OFDM over conventional OFDM increases transmitter efficiency.

**Average power**: The average power is equal to instantaneous power and is equal to 1.
**Implementation complexity**: Though the receiver part of CAM-OFDM is not fully exploited, it is understood that phase noise, spectral efficiency and choice of modulation index will make the receiver design complex.
**Bandwidth expansion**: With channel coding that is redundant bits used in data there would be loss in data rate which will affect the bandwidth. Also since phase modulation is used bandpass signal will have more bandwidth than base band and would require some bandwidth saving schemes for efficient communication
**BER performance**: BER will depend on modulation index (h). Since smaller h is spectrally efficient, choice of h would be a trade-off between performance and BER.
**Additional power needed**: Since it is a constant amplitude modulation, it is a power efficient system.
**Spectral spillage**: It may happen if proper filters are not employed.
**Other factors**: Since it is a nonlinear wideband modulation scheme it should affect the nonlinear devices in the receiver.

## III CONCLUSION

It is understood from the above discussions that Goppa codes were successful in scrambling the data symbols in an OFDM system, but it could provide a uniform reduction of PAPR for all information bits. Even change of modulation techniques also could not provide any substantial reduction in PAPR. So, a zero padded G-OFDM was tried with no outcomes for certain code-words. A study OFDM signal has reflected that its unpredictable amplitude variations in time domain and reconstructive sum of its carriers in the frequency domain are the main reasons because of which power spikes might occur at instants not known beforehand for N carrier system. The complexity of cross correlation products thus developed after OFDM modulation increases with increase in N. Therefore, the message signal carrying information bits is phase modulated in CAM-OFDM in such a way that instantaneous complex signal amplitude may vary but the magnitude will remain constant and therefore OFDM signal could be recovered from the phase information of the received signal. This in other words solves the PAPR problem and directly and indirectly makes other blocks dependent on it in an OFDM system power efficient, reliable and economic in the long run.

## IV ACKNOWLEDGMENT

I am using this opportunity to express my gratitude to thank all the people who contributed in some way to the work described in this paper. My deepest thanks to my project guide for giving timely inputs and giving me intellectual freedom of work.

## REFERENCES

[1] VAN NEE R J, PRASARD R. OFDM FOR WIRELESS MULTIMEDIA COMMUNICATIONS. ARTECH HOUSE. BOSTON, 1999.
[2] TAO JIANG, YIYAN WU, “AN OVERVIEW: PEAK-TO-AVERAGE POWER RATIO REDUCTION TECHNIQUES FOR OFDM SIGNALS”, IEEE TRANSACTIONS ON BROADCASTING, VOL. 54, NO. 2, JUNE 2008
[3] MD. IBRAHIM ABDULLAH ET. AL.,“COMPARATIVE STUDY OF PAPR REDUCTION TECHNIQUES IN OFDM”, ARPN JOURNAL OF SYSTEMS AND SOFTWARE, VOL. 1, NO. 8, NOVEMBER 2011
[4] JONES, T.A. WILKINSON AND S.K. BARTON, “BLOCK CODING SCHEME FOR REDUCTION OF PEAK TO MEAN ENVELOPE POWER RATIO OF MULTICARRIER TRANSMISSION SCHEMES”, ELECTRON. LETTERS, 1994, PP.2098-2099
[5] D. WULICH, “REDUCTION OF PEAK TO MEAN RATIO OF MULTICARRIER MODULATION USING CYCLIC CODING”, ELECTRONIC LETTERS, VOL. 32 PP. 432-433, 1996
[6] SHARMILA SENGUPTA, B.K. LANDE, “PAPR REDUCTION IN OFDM USING GOPPA CODES”, IEEE INTERNATIONAL WIE CONFERENCE ON ELECTRICAL AND COMPUTER ENGINEERING (WIECON-ECE) PP.104-107 DOI: 10.1109/WIECON-ECE.2016.8009096
[7] SHARMILA SENGUPTA, B.K. LANDE, “PERFORMANCE ANALYSIS OF PAPR IN G-OFDM WITH DIFFERENT DIGITAL MODULATION METHODS”, JOURNAL OF COMMUNICATIONS TECHNOLOGY, ELECTRONICS AND COMPUTER SCIENCE, ISSUE 17, 2018 ISSN 2457-905X, PP. 8-13
[8] P. VAN EETVELT, G. WADE, AND M. TOMLINSON, “PEAK TO AVERAGE POWER REDUCTION FOR OFDM SCHEMES BY SELECTIVE SCRAMBLING,” ELECT. LETT.,VOL. 32, NO. 21, PP. 1963–1964, OCT. 1996.
[9] STEVE C. THOMPSON, AHSEN U. AHMEDT, JOHN *G.* PROAKIS, JAMES R. ZEIDLER, “CONSTANT ENVELOPE OFDM PHASE MODULATION: SPECTRAL CONTAINMENT, SIGNAL SPACE PROPERTIES AND PERFORMANCE”, MILCOM 2004 LEEE MILITARY COMMUNICATIONS CONFERENCE