A Study of Linear Programming and Fixed Point Techniques for Transportation Problem

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Abstract*—The most critical and successful applications in the optimization refers to Transportation Problem (TP), which is a particular class of linear programming (lp) in the operation research (OR). The transportation problem is considered a vitally important aspect that has been studied in a wide range of operations, including research domains. As such, it has been used in the simulation of several real-life problems. The main objective of transportation problem solution methods is to minimize the cost or the time of transportation. An Initial Basic Feasible Solution (IBFS) for the transportation problem can be obtained by using the North-West corner rule, Minimum Cost Method, Vogel's Approximation Method and fixed point method. In this paper,we define the several linear programming and fixed point techniques for transportation problem . Thus, optimizing the transportation problem of variables has remarkably been significant to various disciplines In this paper comparison between transportation technique and linear programming technique for any problem is presented. Though the transportation problems are the particular form of the linear programming problem, therefore the same can be solved by the simplex method. Still, when the number of unknown quantities x_{ij} is more significant, then the procedure of solution becomes very lengthy and cumbersome; therefore, we shall use the convenient method to solve the transportation problem. First of all,the initial basic feasible solution is obtained for the problem, and then it is improved by iteration. We also analyze the sensitivity of transportation problems in this paper.

***Keywords*—Transportation, Linear programming problem, Basic feasible solution fixed point, Constraints.**

I. Introduction

Distribution in a company is one of the routine operational activities carried out for the sustainability of the company. Generally, every company performs the distribution process; for example, companies engaged in the field of production, construction services, expedition, suppliers, distributors, and other companies. The distribution also can be said part of marketing wherein distribution marketing processes delivery of product or service from company to consumer. Distributing products or services, the company must issue distribution costs such as transportation costs. Several factors, among others, influence the size of transportation costs in the distribution process—the distance from one source to the intended locations, the type of transportation used, and so forth. Transportation costs can be minimized using optimization techniques contained in operations research. Operational research became known in World War 2, where war strategists were required to consider how to win with limited resources such as limited troop numbers or

limited ammunition. War strategy experts must be able to generate strategies to optimize these resources by performing calculations that produce a formula or formula. Until now,the plan was developed and adopted for the optimization of various fields, such as cost optimization in the company's operations [1].The transportation model is used in the following: To decide the transportation of new materials from various centers to different manufacturing plants. In the case of a multi-plant company, this is highly useful. To determine the transportation of finished goods from different manufacturing plants to various distribution centers. For a multi-plant-multi-market company, this is useful.

Transportation Problem

The problem of transportation in a company occurs due to limited resources or limited capacity of a company in the process of distribution from several places of origin to several destinations. The settlement of transportation problems is intended to optimize transportation costs[2].The question of transportation (TP) is an unusual kind of problem with linear programming. This issue includes the transportation of the commodity from m to n terms, with a 1.a2 ... am and b1 .. b2 ..... bn capacities, respectively. There is also a penalty CIJ for the transport of the unit from Origin I to designations j. This penalty can be either cost, delivery time, delivery protection, etc. The unknown volume of x_{ij} from Origin to Target is the vector x_i. The problem of transportation (TP) is a special kind of problem with linear programming. This issue includes the transportation of the commodity from m to n terms, with a 1.a2 ... am and b1 .. b2 .... bn capacities, respectively. There is also a penalty CIJ for the transport of the unit from origin to designations.The unknown volume of xij from Origin I to Target j is the vector x_i.Optimization Transportation costs can be done by allocating the number of products/goods using the method of the settlement includes.

II. NWC. Method (North West Corner)

The minimum-cost method finds a better starting solution by concentrating on the cheapest routes. The process starts by assigning as much as possible to the cell with the smallest unit cost. Next, the satisfied row or column is crossed out, and the amounts of supply and demand are adjusted accordingly. If both a row and a column are satisfied simultaneously, only one is crossed out, the same as in the north-west –corner method. Next, look for the uncrossed-out cell with the smallest unit cost and repeat the process until precisely one row or column is left uncrossed out.The transportation method that allocates products/goods starts from the top left corner of the table or starts from the cell located in the first column and the first row. The optimization result of the method is not maximal because the allocation does not pay attention to transportation costs. The North-West Corner Rule is a method adopted to compute the initial feasible solution of the transportation problem. The North-west name corner is given to this method because the essential variables are selected from the extreme left corner [2].The significant advantage of the north-west corner rule method is that it is effortless and easy to apply. Its primary disadvantage, however, is that it is not sensitive to costs and consequently yields inadequate initial solutions

Step-1: Select the upper left corner cell of the transportation matrix and allocate min (s1, d1).

Step-2: a. Subtract this value from supply and demand of respective row and column.
b. If the supply is 0, then cross (strike) that row and move down to the next cell.
c. If the demand is 0, then pass (hit) that column and jump right to the next cell.

d. If supply and demand both are 0, then cross (strike) both row & column and move diagonally to the next cell.

Step-3: Repeat these steps until all supply and demand values are 0.

III. C Method (Least Cost)

The transportation method that allocates products/goods starts from the cell with the smallest transportation cost. The results of allocations using the least cost method will generally be more efficient and effective when compared with the North West Corner method.Least Cost Method (LC M) Steps (Rule)[3].

Step-1: Select the cell having minimum unit cost c_{ij} and allocate as much as possible, i.e., min(s_i,d_j).

Step-2: a. Subtract this min value from supply s_i and demand d_j.
b. If the supply s_i is 0, then cross (strike) that row, and If the demand d_j is 0, then pass (hit) that column.
c. If min unit cost cell is not unique, then select the cell where maximum allocation can be possible.

Step-3: Repeat these steps for all uncrossed (unstriked) rows and columns until all supply and demand values are 0.

IV. VAM Method (Vogel's Approximation Method)

VAM is an improved version of the least-cost method that generally, but not always, produces better starting solutions. VAM is based upon the concept of minimizing opportunity (or penalty) costs. The opportunity cost for a given supply row or demand column is defined as the difference between the lowest price and the next most economical cost alternative. This method is preferred over the methods discussed above because it generally yields an optimum, or close to optimum, starting solutions. Consequently, if we use the first solution obtained by VAM and proceed to solve for the optimum solution, the amount of time required to arrive at the optimum solution is significantly reduced. VogelApproach (VA) is a recursive method to compute an appropriate, feasible alternative of a transportation hitch. This approachis better than the other two approaches, i.e., North West Corner Rule (NWC) & Least cost approach (LCA), Because obtainedkey feasible answers. The optimal explanation is closer to this method. The current Vogel Approximation Method (VAM) algorithm follows:
This method is preferred over the NWCM and VAM because the initial basic feasible solution obtained by this method is either an optimal solution or very nearer to the optimal solution [4].

Vogel's Approximation Method (VAM) Steps (Rule)

Step-1: Find the cells having the smallest and next to most trivial cost in each row and write the difference (called penalty) along the side of the table in the row penalty.

Step-2: Find the cells having the smallest and next to most minor cost in each column and write the difference (called penalty) along the side of the table in each column penalty.

Step-3: Select the row or column with the maximum penalty and find a cell that has the least cost in the selected row or column. Allocate as much as possible in this cell. If there is a tie in the values of penalties, then select the cell where maximum allocation can be reasonable.

Step-4: Adjust the supply & demand and cross out (strikeout) the satisfying row or column.

Step-5: Repeatthis step until all supply and demand values are 0.

V. Russel's Approximation Method (RAM)

Solving the problem of transportation cost optimization using Russel's approximation method can be said to have the same logic or working method using the Vogel approximation method.For each source row i remaining under consideration, determine its u_i , which is the largest unit cost C_{ij} still remaining in that row. For each destination column j remaining under consideration, determine its v_j , which is the largest unit cost C_{ij} still remaining in that column. For each variable x_{ij} not previously selected in these rows and columns.Russell's Approximation Method (RAM)[5]:

Step-1: For each source row still under consideration, determine itsu_i (largest cost in row i).

Step-2: For each destination column still under consideration, determine its V_j (most significant cost in column j).
Step-3: For each variable, calculate $\Delta_{ij}=c_{ij}-(\bar{U}i+\bar{V}j)$.

Step-4: Select the variable having the most negative Δ value, break ties arbitrarily.

Step-5: Allocate as much as possible. Eliminate necessary cells from consideration. Return to Step-1.

VI. Formulation of the Interval Fixed Point Transportation Problem (IFPTP)

In the following section, the first FPTP is introduced then the problem is modeled when the parameters have arrived in interval forms. When a setting is in interval form, it means that there is no preference between values in the interval for the Decision-Maker (DM).

6.1 Standard representation of FPTP and IFPTP

The FPTP can be described as a distribution problem in which m sources (warehouses or suppliers) and n destinations (demand points or consumers) are involved. The product can be transported from each m source to any of n destinations with the associated cost of c_{ij} per unit. Also, a fixed charge of f_{ij} appears in the objective function if the related variable means x_{ij} is positive. In a balanced FPTP, it is assumed that the total amount of supplies in sources is equal to the sum of demand parameters in different destinations. Still, in real system problems, this condition may not always hold, so in the current paper, we consider FPTP in its standard form, not the balanced representation where it suffices to suppose that there are enough products in the sources to satisfy the demand of each destination. The problem is to find the best strategy of shipment that minimizes the total transportation costs (i.e., variable costs plus fixed point) such that all of the constraints hold.

$$min \sum_{i=1}^{m} \sum_{j=1}^{n} (c_{ij}x_{ij} + f_{ij}y_{ij}) \qquad (1)$$

$$s,t \sum_{j=1}^{n} x_{ij<a_i} \quad i = 1, \ldots..m$$

$$\sum_{i=1}^{m} x_{ij<b_j} \quad j = 1, \ldots \ldots n$$

$$x_{ij\geq 0\ for\ all\ i=1,\ldots.m, j=1,\ldots..n}$$

$$y_{ij} = 0\ if\ x_{ij} = 0$$

$$y_{ij} = 1\ if\ x_{ij} > 0$$

6.2 Numerical examples

In this section, numerical examples are provided for both solution procedures. Consider a company with three factories in cities S1, S2, and S3, which are busy producing a specific type of product. Four other cities receive this product as consumers, namely, D1, D2, D3, and D4. The particular conditions on producing the product caused by uncertainty and vagueness in the supply and demand values in factories and destinations, respectively. Previous experiences showed that these values could be delivered in interval forms. Another company is given the responsibility of shipping the product from factories to the consumers. While sending through each rout, special cares are needed on a shipment that is done by a technician and causes a fixed charge cost for opening each rout. This is independent of the number of goods transferred through a rout. Also, variable transportation costs are available in interval forms. The producing company seeks the best strategy of shipment that satisfies all restrictions and minimizes the total cost of transportation, concurrently. Data are gathered in two tables to be solved by the two solution procedures

VII. Literature Survey

D. S. Sarode and R. Tuli[2019] presented the Optimal solution of the Planar Four Index Transportation Problem is obtained by a novel algorithm. The Modified Distribution method to achievean optimal solution is extended by introducing an extended loop with new indices as a type of commodity and modes of transport. It is observed that the proposed algorithm is not only easier to understand and implement but also requires a lesser number of iterations to arrive at the optimal solution [6].

Selmair et al. [2019] propose an adapted version of VAM which reaches better solutions for non-quadratic matrices, namely Vogel's Approximation Method for non-quadratic Matrices (VAM-nq). Subsequently, VAM-nq is compared with ILP, HM and VAM by solving patterns of different sizes in computational experiments to determine the proximity to the optimal solution and the computation time. The experimental results demonstrated that both VAM and VAM-nq are five to ten times faster in computing results than HM and ILP across all tested matrix sizes. However, we proved that VAM is not able to generate optimal solutions in large quadratic matrices constantly (starting at approx. 15 × 15) or small non-quadratic models (starting at approx. 5 × 6). We show that VAM produces insufficient results, especially for non-quadratic matrices. The result deviates further from the optimum if the matrix size increases. Our proposed VAM-nq can provide similar results as the original VAM for quadratic forms but delivers much better results in non-quadratic instances, often reaching an optimum solution. This is especially important for practical use cases since quadratic matrices are rather rare [7].

Banik et al. [2018]show the result of the optimum amount of supply with the corresponding destination together in a single output. Verification of the program or 'code' has been done with the help of hand calculation. The company can determine the supply unit with a destination to minimize the transportation cost before every shifting of products from the warehouse to the destination. an online-based business company intended to spread its business over the whole Rajshahi District. As there is no cost for manufacturing, the only factor that can maximize the profit is the product transportation cost. Determination of the optimum & reliable transportation medium was the first challenge. A suitable destination for distributing the products in the selected location has been chosen carefully based on demand & minimum rent of the warehouse[8].

Ravi et al. [2018]proposed in this paper, namely DFSD (Difference form Standard Deviation) method is applied for finding the optimal solution for transportation problems. The proposed algorithm is a unique way to reach a feasible and optimal solution without or with degeneracy condition. It is directly finding the optimal solution with a minimum number of iterations compared to other existing methods [9].

Hlatká et al. [2017] deal with logistics processes in companies. Logistics aims to optimize material, finance, and information flows to optimize overall costs in companies. The first part of the article describes the methods employing which the processes in companies can be optimized, namely the Vogel approximation method. In the second part of the article, the plan is implemented in a particular manufacturing company with the result of distribution route optimization and reduction of operating costs of transport[10].

Khan et al. [2016] proposed an analysis identifying the reason of different solutions by different methods is carried out. The results represent a novelty for the current literature.The branch of Linear Programming Problem (LPP) dealing with the transportation of a single homogeneous product from several sources to numerous destinations in such a way that the total transportation cost is minimized while satisfying all supply and demand restrictions is Transportation Problem (TP). We see that for solving such type of TP, the initial basic feasible solution varies for different solution procedures and also for variousexamples [11].

Ahmed et al. [2016]Propose an algorithm "Incessant Allocation Method" to obtain an initial basic feasible solution for the transportation problems. Several numbers of numerical problems are also solved to justify the method. Obtained results show that the proposed algorithm is useful in solving transportation problems.Industries require planning in transporting their products from production centers to the user's end with minimal carrying costs to maximize profit. This process is known as the Transportation Problem, which is used to analyze and minimize transportation costs. This problem is well discussed in operation research for its full application in various fields, such as scheduling, personnel assignment, product mix problems, and many others so that this problem is not confined to transportation or distribution only. In the solution procedure of a transportation problem, finding an initial basic feasible solution is the prerequisite to obtain the optimal solution. Again, development is a continuous and endless process to find the best among the bests. The growing complexity of management calls for the development of sound methods and techniques for the solution of the problems[12].

VIII.Comparison of differentmethods:

Since the transportation problem (TP) is a special case of the Linear Programming Problem (LPP), hence, a BFS of a transportation problem has the same definition as for LPPNorth-west corner method is used when the purpose of completing demand No. 1 and then the next and is used when the purpose of completing the warehouse No. 1 and then the next. Advantages of North-West.Corner method is a quick solution because computations take a short time but yield a wronganswer because it is very far from the optimal solution. Vogel's approximation method and Minimum-cost method is used to obtain the shortest road. The advantage of Vogel's approximation method and the Minimum-cost method yields the best starting basic solution because it gives the initial solution very near to optimal solution. Still, the answerto Vogel's approximation methods is slow because computations take a long time. The cost of transportation with Vogel's approximation method and the Minimum-cost method is less than the North-West corner method. The cost of shipping is less than the North-west corner method.

Table 1:Comparision of different approaches

Sr. No.	Name of the Method	Initial Basic Feasible Solution				
		Problem 1	Problem 2	Problem 3	Problem 4	Problem 5
1	North-West Corner	Rs. 88	Rs. 97	Rs. 162	Rs. 635	Rs. 730
2	Least cost Method	Rs. 92	Rs. 88	Rs. 61	Rs. 510	Rs. 555
5	**VAM.**	**Rs. 76**	**Rs. 73**	**Rs. 56**	**Rs. 510**	**Rs. 555**
6	**RAM**	**Rs. 76**	**Rs. 73**	**Rs. 56**	**Rs. 510**	**Rs. 555**

IX. Sensitivity Analysis of TP.

This involves the development of understanding how the information in the final tableau can be given managerial interpretations. This will be done by examining the application of sensitivity analysis to linear programming problems. One of the right-hand-side values or coefficients of the objective function is changed. Then, the changes in the optimal solution and optimal value are examined. The harmonious relation between supply and demand in transportation problem makes it difficult to use traditional sensitivity analysis methods. Therefore, in the process of changing supply or demand resources, at least one more support needs to be changed to make the balanced relation possible. In this study, utilizing the concept of full differential of changes for sensitivity analysis of the right-hand-side parameter in the transportation problem, a method is set forth. This method examines simultaneous and related changes of supply and demand without making any change in the basis. The mentioned method utilizes Arasham and Kahn "s simplex algorithm to obtain a fundamental inverse matrix.

X. . Testing for Optimality

When we are solving a transportation problem, the initial basic feasible solution (IBFS) cannot be considered as the optimal solution because there may exist a better solution (which is called Basic Feasible Solution FBS) that optimizes the objective function better. To evaluate whether an answer is the Basic Feasible Solution (BFS) for a particular problem, we use the optimality test to assess and improve the solution if there is a need for improvement. The optimality test was used for all the initial basic feasible solution obtained from the various methods employed above, to find the optimal solution for each technique. The stepping stone method was applied to test for optimality is given below:

10.1 Optimality Test for North-west Corner Method

The stepping stone method was used, evaluating the empty cells (unallocated cells) and reallocating the battery with the highest negative. And there was a decrease in the total cost of distribution. However, the optimality test still indicates that there is a need for further improvement, and thus we proceed to the second iteration. Similarly, the same sequence procedures and analyses were carried out after the second iteration. There wasa more decrease in the total cost of distribution in comparison to the first iteration. However, the test of optimality still signifies that there is a better solution to be found even though there was a decrease in the total cost of distribution.

10.2 Optimality Test for Least Cost Method

The optimality test after the third iteration still indicates that there is a need for optimization of the solution. Hence, we proceeded to the fourth iteration, as given in Table 9. Table 9 shows that the optimal solution (Basic Feasible Solution) is being reached, and there was a noticeable drop in the total cost of distribution in comparison to the previous iterations for the least square method.

10.3 Optimality Test for Vogel Approximation Method

In the same way that an optimality test was carried out for the North-West corner method. Least cost method we applied on the Vogel Approximation method to determine
the Basic Feasible Solution.

XI. Conclusion

The transportation cost is an essential element of the total cost structure for any business. The transportation problem was formulated as a Linear Programming and solved with the standard LP solvers such as the Management scientist module to obtain the optimal solution. The number of variables and constraints is more in the given problem then the solution by Linear Programming will be very lengthy. A, however,a simple method known as transportation technique exists by which the answer is found ina smaller number of steps only a few and straightforwardly and quickly. In this paper,the comparison in four methods is different. The decision-maker may choose the optimal result of the running of the three programs (minimum) and determined the number of units transported from source I to destination j.We also analyze the sensitivity analysis and testing for optimality with the different techniques of transportation problems.

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