A Multi-channel All-Pass Filtered x Least Mean Square Algorithm for Narrowband Active Noise Control without secondary path identification

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Abstract: Now a day acoustics noise becomes the most noticeable problem in this new era. To attenuate acoustic primary noise or disturbances, Active noise control (ANC) is the most popular and most used method by generating controllable secondary sources by which output noise can be cancelled with the same amplitude but opposite sign. The primary path models the system from the reference sensor to the error sensor where the noise attenuation is to be realized, while the secondary path is the system between the output of the adaptive control filters and the error sensors. Filtered x Least Means Square is most popular algorithm used in ANC with secondary path identification. The identification requirement of the secondary path increases the complexity of the system implementation and decrease the control system performance. Therefore several new ANC algorithms have been developed, in which the identification of the second path transfer function is not required. The main focus of this paper is concerned about a multi-channel narrowband feed forward ANC system such as air intake duct system and its novelty is to introduce All-Pass Filtered x LMS algorithm which does not need to identify the secondary path. Here the first-order all pass filters with single parameter is used to improve the convergence of the LMS algorithm. The performance assessments regarding convergence speed of the proposed algorithm contrasted and other ANC adaptation algorithms are examined. Contrasted and the Filtered x LMS calculation through some PC simulation benchmark precedents, the proposed strategy is more straightforward to execute, and it accomplishes quick convergence speed. The outcomes additionally demonstrate that the proposed strategy beats different LMS calculations without secondary path identification. The proposed narrowband LMS calculation will realize the structure of productive feed forward ANC framework that can assist noise control in air intake duct application.

IndexTerms - Active noise control (ANC), Narrowband Secondary path, All-pass filtered x LMS (APFxLMS).

Introduction
ANC is considering the straightforward guideline of ruinous impedance of spreading acoustic waves [3]. It is a technique for lessening undesired noise by presenting a scratching off “anti-noise” wave through optional sources. In early 1930s ANC came from infancy to light for the first time. The essential thought was proposed in 1936 [4], in any case, genuine applications were very restricted till as of late. After that in the year 1970s there have been practical solutions of ANC when digital technologies intensely developed. The first attempt was taken at the end of 1980s including active methods for silencing interiors of all kinds of vehicles, including cars. On account of progression in the calculations for versatile sign preparing and their execution utilizing digital signal processors (DSPs); numerous fruitful utilizations of ANC have been accounted for, the most acclaimed being noise lessening headsets [5, 6]. The most well-known versatile channel and versatile calculation utilized for ANC applications are the Finite Impulse Response (FIR) and the Filtered x least mean square (FxLMS) calculation [1] which is an altered adaptation of the LMS calculation [7]. Kuo and Morgan (1999) [2] developed the basic adaptive algorithm for ANC and analyzed based on single-channel broad-band feedforward control. Zhang, Lan, and Ser (2001) [8] presented a new ANC system with online secondary path modelling. ANC system plays an important role because the convergence condition of adaptive filtering is closely related to the estimation error of the secondary path response and the real secondary path response varies with time. The most popular online identification method of secondary path transfer function is to inject an additional modeling noise into the loudspeaker. This additional noise contributes to the residual noise and this identification parts increase the control system complexity. To avoid these problems, there are some ANC algorithms that need not to identify the secondary path system. Some of them, called direction search LMS algorithm and discussed below, update direction of adaptive filter by monitoring the residual error. Zhou and DeBrunner (2007) [9] introduced an ANC algorithm for single-tone or single sinusoidal noises which does not required any secondary path identification. Based on the geometrical analysis of weight updating process in FxLMS, the algorithm proposes to change the sign in front of the step size μ appropriately by monitoring time varying error power. Wu, Chen and Qiu [10] extended Zhou's method using step size sign selection algorithm to the step size direction using four choices 180°, 0°, 90°, and -90°. Caudana, Betancourt, Cruz, Miyatake & Meana (2008) [11] presented a hybrid active noise cancelling (HANC) algorithm to overcome the acoustic feedback present in most ANC system, together with an efficient secondary path estimation scheme. Md. Moazzam et al. [12] implement a few different adaptive control algorithms in the recently developed remotely controlled Virtual Instrument Systems in Reality (VISIR) ANC/DSP remote laboratory to evaluate the performance of adaptive algorithms in 2013. Based on the strict positive real property of the FxLMS algorithm Zhang et al. [13] introduced phase shifter to an ANC, where Hilbert transform filter is used for phase compensation. Zhang and Ren (2010) [14] proposed an adaptive algorithm based on neural networks for nonlinear ANC systems, where no secondary path identification by introducing virtual primary noises to modify the ANC structure. Their controller utilizes neural networks to attenuate the effect of the primary noises and it also has simpler structure and less computation complexity. Kurczek and Paweleczyk (2014) [15] proposed an approach with no secondary path modelling, in which the adaptation stability is guaranteed by switching the sign of the step size. They also used a fuzzy inference [16] system to evaluate both sign and magnitude of the step size.
II. ADAPTIVE MECHANISM (FXLMS) WITH SECONDARY PATH IDENTIFICATION

This section derives the conventional FxLMS algorithm as a gradient-based adaptation algorithm. A detailed block diagram for single channel FxLMS feedforward ANC is illustrated in Figure 1, where \( P(z) \), \( S(z) \), and \( \tilde{S}(z) \) represent the primary path, the secondary path, and an estimated secondary path model respectively. The output \( y(n) \) of the FIR adaptive filter \( W(z) \) is processed as:

\[
y(n) = w^T(n)x(n) = \sum_{i=0}^{N-1} w_i x(n-i)
\]  

(1)

where \( w^T(n) = [w_0(n), w_1(n), \ldots, w_{N-1}(n)] \) is the coefficient or weight vector of \( W(z) \) at time \( n \) and \( x(n) = [x(n), x(n-1), \ldots, x(n-N+1)]^T \) is the reference input noise vector at time \( n \). If the secondary path \( S(z) \) can be modelled by \( Q \)-th order FIR filter with weights \( s_i \), the output \( \hat{d}(n) \) of the secondary path is given as:

\[
\hat{d}(n) = \sum_{i=1}^{Q} s_i y(n-i)
\]  

(2)

![Figure 1](image1.png)  

**Figure 1** Block diagram of ANC using FxLMS with secondary path identification [2]

A block diagram of a multiple-channel ANC system for a three-dimensional application is shown in Figure 2.

![Figure 2](image2.png)  

**Figure 2** Multiple-Channel ANC System for a 3-D Enclosure

The placement of the secondary-path transfer function following the digital filters \( W(z) \) is controlled by the LMS algorithm. The residual signal is expressed as

\[
e(n) = d(n) - \hat{d}(n)
\]  

(3)

where \( n \) is the time index, \( d(n) \) is the primary signal and it is represented as \( d(n) = h(n) \ast x(n) \), where \( h(n) \) is the impulse response vector of the primary path \( P(z) \). In FxLMS algorithm, the filter coefficient vector is updated in the negative gradient direction with step size \( \mu \) using mean square error in Equation 4:

\[
w(n + 1) = w(n) - (\mu/2)\nabla J(n)
\]  

(4)

where scalar parameter \( \mu \) is the adaptation step size, and \( \nabla J(n) \) is the mean square-error (MSE) gradient vector at time \( n \), and the cost function \( J(n) \) is expressed as \( J(n) = e^2(n) \).

From Equation 2 and Equation 3, we have in Equation 5

\[
e(n) = d(n) - s(n) \ast y(n) = d(n) - \sum_{i=1}^{Q} s_i y(n-i) = d(n) - \sum_{i=0}^{Q-1} s_i w^T(n-i)
\]  

(5)

By differentiating Equation 5, \( \nabla e(n) \) can be expressed by:

\[
\nabla e(n) = \frac{\partial e(n)}{\partial w(n)} = -\sum_{i=0}^{Q-1} s_i \frac{\partial w^T(n-i)}{\partial w(n)} x(n-i)
\]  

(6)
If the adaptation process slow, we may assume \( \mathbf{w}(n) \approx \mathbf{w}(n - 1) \approx \cdots \approx \mathbf{w}(n - Q + 1) \). This assumption leads the result as:
\[
\forall e(n) = -\sum_{i=0}^{Q-1} s_i x(n - i) \triangleq -f(n)
\]
(7)
Combining the relationship of \( \forall f(n) = \forall [e^2(n)] = 2e(n)\forall e(n) \) and Equation 7, we also have in Equation 8
\[
\forall f(n) = -2e(n)f(n)
\]
(8)
Consequently, in Equation 9 the LMS update equation can be obtained as:
\[
\mathbf{w}(n + 1) = \mathbf{w}(n) + \mu e(n)f(n)
\]
(9)
In practical ANC applications, \( f(n) \) is not available because the system function \( S(z) \) of the secondary path is known and must be estimated. Therefore, the implementation of Equation 3.9 requires estimating \( f(n) \) by identifying \( S(z) \). As expressed in Equation 3.7, \( f(n) \) can be estimated by filtering \( x(n) \) using the identified \( S(z) \). Now, the estimated \( f(n) \), denoted by \( \hat{f}(n) \), is defined as:
\[
\hat{f}(n) = \sum_{i=0}^{Q-1} s_i x(n - i)
\]
(10)
where \( \hat{s}_i \) is an estimated weight of the secondary path which is denoted by \( \hat{S}(z) \), and \( \hat{f}(n) \) is called the filtered reference vector.
Replacing \( f(n) \) in Equation 9 by \( \hat{f}(n) \) in Equation 10 gives the following updating equation as.
\[
\mathbf{w}(n + 1) = \mathbf{w}(n) + \mu e(n)\hat{f}(n)
\]
(11)
Equations 10 and Equation 11 are the formulation of the FxLMS algorithm, which can be practically implemented.
According to [9], the convergence (stability) analysis for FxLMS with secondary path identification error is summarized as follow: Assume \( x(n) \) is a pure sinusoidal wave with angular frequency \( \omega \). The functions of \( P(z) \), \( S(z) \), \( \hat{S}(z) \), and \( W(z) \) can be represented by the complex values \( P_\omega \triangleq P(e^{j\omega}) \), \( S_\omega \triangleq S(e^{j\omega}) \), \( \hat{S}_\omega \triangleq \hat{S}(e^{j\omega}) \), and \( W_\omega \triangleq W(e^{j\omega}) \). We also assume that the estimation of the secondary path complex gain is given by in Equation 12
\[
\bar{S}_\omega = c_\omega S_\omega e^{j\theta_\omega} \quad \text{or} \quad \frac{\bar{S}_\omega}{\bar{S}_\omega} = c_\omega e^{j\theta_\omega}
\]
(12)
where \( c_\omega \) is the magnitude estimation error, and \( \theta_\omega \) is the phase estimation error in the secondary path identification. The geometric analysis derives the following convergence conditions in Equations 13 and Equation 14. [9]
\[
\mu < \frac{2 \cos \theta_\omega}{c_\omega P_\omega(\omega)|\bar{S}_\omega|^2},
\]
(13)
\[
|\theta_\omega| < \pi / 2
\]
(14)
where \( P_\omega(\omega) \) represents the power of the reference signal \( x(n) \) at the frequency \( \omega \).
From these results, if \( |\theta_\omega| > \pi / 2 \), the FxLMS adaptive filter does not converge even if we choose sufficiently small step size \( \mu \). This means that the adaptive filter weights converge if the phase of the estimated secondary path is within -90°-90° of that of the actual secondary path.

III. CONVERGENCE ANALYSIS OF ANC WITHOUT SECONDARY PATH IDENTIFICATION

In this section, the convergence analysis of the direction search LMS approaches is described. The discussion here is restricted within ANC system for narrowband noise cancellation. Figure 3 illustrates a block diagram of ANC without secondary path identification based on the direction search LMS algorithm [17]. Unlike the FxLMS, the reference signal \( x(n) \) is directly used for adaptive control without pre-filtering of an estimated secondary path transfer function. The most important part is the update direction search portion in this configuration, and some approaches were proposed.
For ANC without estimated secondary path as shown in Figure 3, the update of the adaptive filter coefficients \( w(n) \) based on the LMS algorithm is written as in Equation 15

\[
w(n) = w(n - 1) + \mu e(n)x(n)
\]  

(15)

where \( \mu \) is a sufficiently small step size. One can see that in Equation 15, the reference signal does not need to pass through the secondary path. According to [9], for a signal-tone input \( X_\omega(n) \) with angular frequency \( \omega \), Equation 15 yields in Equation 6

\[
W_\omega(n) = W_\omega(n - 1) + \mu P_\omega(\omega)|S_\omega|\left[\frac{P_\omega}{S_\omega} - W_\omega(n - 1)\right]e^{j\angle S_\omega}
\]  

(16)

where \( \angle S_\omega \) represents the angle of \( S_\omega \) and \( |S_\omega| \) represents the amplitude of \( S_\omega \). For the case without secondary path identification, the previously defined estimation errors \( \epsilon_\omega, \theta_\omega \) in 3.2 are \( \epsilon_\omega = |S_\omega|, \theta_\omega = \angle S_\omega \). Thus, from the conditions of Equations 13 and 14, LMS algorithm in Equation 15 can converge if it satisfies the below maintain Equation 17 and Equation 18.

\[
\mu < \frac{2 \cos \angle S_\omega}{P_\omega(\omega)|S_\omega|},
\]  

(17)

\[
|\angle S_\omega| < 90^\circ.
\]  

(18)

Consequently, the update of \( W_\omega(n) \) converges to the ideal value even without secondary path identification. On the other hand, if \( |\angle S_\omega| > 90^\circ \), then the adaptive update algorithm will fail to convergence so that the ANC could not cancel the reference noise. To avoid this divergence, the sign in front of the step size in Equation 3.15 is changed, i.e. shown in Equation 19

\[
w(n) = w(n - 1) - \mu e(n)x(n)
\]  

(19)

Then, it become like as in Equation 20

\[
W_\omega(n) = W_\omega(n - 1) + \mu P(\omega)|S_\omega|\left[\frac{P_\omega}{S_\omega} - W_\omega(n - 1)\right]e^{j(\angle S_\omega - 180^\circ)}
\]  

(20)

The convergence condition of Equation 20 in this updating formula is given by \( |\angle S_\omega - 180^\circ| < 90^\circ \), and this condition is satisfied for the current case \( |\angle S_\omega| > 90^\circ \). The next problem is to know when to change the sign of step size to give a converging adaptive filter weight update. In the frequency domain at the angular frequency \( \omega \) of reference signal \( x(n) \), the block wise updating scheme of adaptive filter weight vector is given as in Equation 21:

\[
W(\omega, k + N) = W(\omega, k) - \mu C(k)X_\omega(k)E_\omega(k)
\]  

(21)

where \( N \) is the length of block or FFT length, \( k \) is the time index of a block, \( C(n) \) is assumed to be +1, −1, +i, or −i, \( i: \) imaginary unit, \( X_\omega(n) \) and \( E_\omega(n) \) are the frequency components of \( x(n) \) and \( e(n) \) at the frequency \( \omega \). The update direction of \( C(n) \) is given by the angle of \( +1, -1, +i, or -i \), that is, 0°, 180°, and ±90°. Basically, the algorithm by Wu’s ANC has the same scheme as the block diagram shown in Figure 2.7. The merit of using either +i, or −i is to avoid slow convergence when the response of the secondary path is close to ±90°. In addition to the direction, \( C(n) \) can also control the step size. However, Wu’ LMS is carried out in block wise FFT domain as shown in Equation 21, so that some time delay is introduced at weight updating computation by computational complexity caused by the sub band splitting. However, the above discussed ANC without secondary path identification are operated in frequency domain. This previous approach, therefore, required Hilbert transform for converting the algorithm into the frequency domain. The main benefit to use all-pass filter is that it changes the phase shift only while magnitude of the response is not change. The proposed system need not to transform into frequency domain as performed in previous algorithms such as adopted in [10]. It also needs not to implement Hilbert transform for converting the algorithm into the frequency domain. Kurczyk and Pawełczyk [15] extended the idea of the delayed LMS algorithm where instead of an estimated secondary path model, time delay operator with variable delay time is used. Figure 4 illustrates the block diagram of delay variable LMS algorithm.

![Figure 4](https://www.ijrar.org/images/ijrar1amp019/images/ijrar1amp019_124.jpg)

**Figure 4** ANC with the shifted-delay LMS algorithm [15]
Figure 5 Subband implementation ANC without secondary path identification based on: (a) Morgan’s method and (b) DeBrunner’s method (The dashed line designates optional performance monitoring stage.) [9]

The delay variable $k(n)$ is an integer which can control of direction search and it is tuned in conjunction with the step size change. Their filter weight update equation, called varying-delay LMS algorithm with normalization step size is given by in Equation 22:

$$\mathbf{w}(n) = \mathbf{w}(n-1) - \frac{\mu}{\|r(n)\|^2} e(n) x(n-k(n))$$ (22)

where $\mu_N$ is normalized step size defined by the user, and $r(n)$ is the output of delay operator.

This adaptation algorithm repeats the following two stages:

**Phase adjustment** — Selecting the parameters ($\mu$ and $k$) that improve the convergence speed by testing four sets. 1) $\mu$ is positive, and $k=0$; 2) $\mu$ is negative, and $k=0$; 3) $\mu$ is positive, and $k>0$; 4) $\mu$ is negative, and $k>0$;

**Divergence direction** — Estimate the error signal energy, if it is over the predefined threshold, then the algorithm returns to the phase adjustment stage.

In the phase adjustment stage, two cases with $k=0$ are the same adaptation processes used in Zhou’s ANC system. In the tonal noise reduction which is addressed in this thesis, the delay integer $k$ can be defined as the following form in Equation 23. [15]

$$k = \text{round} \left( \frac{f_s}{f_s} \right)$$ (23)

where $f$ stands for signal frequency, $f_s$ is the sampling frequency, and round means the rounding to the nearest integer.

IV. **ALL-PASS FXLMS ALGORITHM**

The block diagram of the proposed all-pass filtered x LMS (APFxLMS) is shown in Figure 6. In the figure, the block of $A(z, \alpha)$ is realized by the block diagram as shown in Figure 7, where the parameter $\alpha$ will be controlled by monitoring error behavior.

Figure 6 All pass-filtered x LMS without secondary path identification
The function of the updating of the parameter $\alpha$ and step size is divided into four stages as shown in Figure 8.

4.1 Convergence Analysis of APFxLMS

This section describes the convergence analysis of the proposed APFxLMS [18] filter weight’s updating algorithm for single tone noise cancellation. The basic theory applied to this issue is frequency analysis developed in [9]. At first, reference noise signal is pure sinusoidal noise with normalized frequency $\Omega$ ($0 < \Omega < \pi$). In the frequency domain, the transfer functions $P(z), S(z), A(z),$ and $W(z)$ are represented by the complex numbers of $P_\Omega \triangleq P(e^{j\Omega}), S_\Omega \triangleq S(e^{j\Omega}), A_\Omega \triangleq A(e^{j\Omega}),$ and $W_\Omega \triangleq W(e^{j\Omega})$. Since the complex numbers of $S_\Omega$ and $A_\Omega$ can be represented by their polar form as given in Equation 31:

$$S_\Omega = |S_\Omega|e^{j\theta_S}, \quad A_\Omega = e^{j\theta_A(\alpha)}$$  \hspace{1cm} (31)

where $\theta_S = \angle S(e^{j\Omega})$, and $\theta_A(\alpha) = \angle A(e^{j\Omega})$, the following expression, which is equivalent form of Equation 11, is obtained in the next Equation 32.

$$\frac{A_\Omega}{S_\Omega} = \frac{1}{|S_\Omega|} e^{j(\theta_A(\alpha) - \theta_S)}$$  \hspace{1cm} (32)

Substituting $c_\omega = \frac{1}{|S_\Omega|}, \theta_\omega = \theta_A(\alpha) - \theta_S$ into the convergence conditions Equation 12 and Equation 13 yields the convergence conditions of APFxLMS as in the following Equation 33 and Equation 34.

$$\mu < \frac{2\cos(\theta_A(\alpha) - \theta_S)}{P_x(\Omega)|S_\Omega|},$$  \hspace{1cm} (33)  

$$|\theta_A(\alpha) - \theta_S| < \pi/2$$  \hspace{1cm} (34)

where $P_x(\Omega)$ is the power spectrum of the reference noise $x(n)$.

If the step size $\mu$ is set as sufficiently small value, the condition in Equation 3.33 can be satisfied. On the other hand, if the phase of the secondary path system $\theta_S$ is negative, a proper parameter $\alpha$ can be chosen so that $\theta_A(\alpha)$ comes $\theta_S$ as closer as possible.
V. RESULTS AND DISCUSSION

In order to show the effectiveness of the proposed APFxLMS algorithm, several computer simulation experiments are conducted. Figure 9 shows the result of noise signal, primary and secondary signal for popular FxLMS method with secondary path identification (1\textsuperscript{st}), Standard LMS method without secondary path identification (2\textsuperscript{nd}), proposed APFxLMS (3\textsuperscript{rd}) and the delay LMS (4\textsuperscript{th}) for multi-channel narrowband feed forward system. Here dual-channel are used. The input reference noise wave (red), the middle one shows the primary signal (blue) and the secondary path signal (green), and the residual error signal (magenta) is shown at the bottom in Figure 9 and it is clearly observed that the proposed APFxLMS system gives the better residual noise signal compared to other ANC systems.

![Figure 9](image)

Figure 9   Results of Noise, Primary, Secondary for FxLMS (1\textsuperscript{st}), Standard LMS (2\textsuperscript{nd}), Proposed APFxLMS (3\textsuperscript{rd}), Delay LMS (4\textsuperscript{th})

Figure 10 (left), it observed that RNPL and NRS [18] of FxLMS, Standard LMS and Delay are -13.5dB, -4.5dB and -17.2 dB and 55, 53, 52 respectively, whereas RNPL and NRS of the proposed APFxLMS are -18.5 dB and 47, which reach the best result among all ANCs even after using multiple noise source and using the dual APFxLMS get the best result among other ANC systems. Alternative features of adaptation mechanism can be observed by illustrating the tap weight trajectory as well as the residual time series of simulation results as shown in Figure 10 (middle) and 10 (right).
Comparing the results shown in Figure 10 (right) shows that the Standard LMS, FxLMS and Delay LMS algorithms have slow residual noise decreasing speed, whereas the proposed method in this study present quite fast convergence speed, which is generally wanted in ANC. Table 1 shows the comparison of the proposed APFxLMS with ANCs in terms of RNPL and NRS.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>RNPL[dB]</th>
<th>NRS</th>
</tr>
</thead>
<tbody>
<tr>
<td>FxLMS</td>
<td>-13.5</td>
<td>55</td>
</tr>
<tr>
<td>Standard LMS</td>
<td>-4.5</td>
<td>53</td>
</tr>
<tr>
<td>APFxLMS(Single)</td>
<td>-18.5</td>
<td>47</td>
</tr>
<tr>
<td>Delay LMS</td>
<td>-17.2</td>
<td>52</td>
</tr>
</tbody>
</table>

From the table 1, it clearly shows that APFxLMS demonstrates the best performance then others ANCs with/without secondary path identification.

VI CONCLUSIONS

The major work can be catalogued into three parts. At first, a theoretical explanation of ANC system without secondary path identification was discussed and it improves adaptation stability as well as its convergence speed by a 1st order all-pass filter to change the phase shift of the ANC system. Consequently, the proposed algorithm achieves a better noise reduction system. Second, a systematic approach for ANC systems in state of estimation of the secondary path is verified and validated with a standard and recent benchmark problems. The Performance of the convergent analysis is obtained by computer simulation using MATLAB. The proposed system get the better results other than the standard and delay LMS algorithm. At the end, the proposed ANC system compare with the existing ANC with and without secondary path identification through computer simulations. For evaluating the noise reduction ability of each algorithm, in terms of RNPL and NRS, APFxLMS demonstrates the best performance then other ANCs with/without secondary path identification.
Recommendations of Future Work

Several future issues of this study are as follows:

i. Develop a continuous determination scheme of $\alpha$ by monitoring the power of residual error signal.

ii. Extend the proposed APFxLMS algorithm to broadband noise cancelling ANC.

iii. For physical system dynamics that changes continuously, system identification should be done online instead of off-line. This will ensure that the model is truly adaptive to the dynamics of the physical system. For this reason, the online modelling techniques should be studied and implemented.

References


