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The Impact of Interest Rates on Option Pricing: Moving Beyond Black-Scholes-Merton

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Abstract –

The Black-Scholes-Merton (BSM) model is still widely used to calculate option prices even though it is predicated on the notion that future interest rates will remain constant for the entire term of the option. The price of options may be significantly impacted by the volatility of interest rates. The goal of this article is to assess whether the BSM model needs to be altered in order to account for interest rate fluctuations and how those fluctuations impact option contract prices.

The BSM model is briefly discussed in this article, along with how it applies to price options, how interest rates play a role in the BSM model, and some of the model's shortcomings. It gives examples of how option prices have changed as interest rates have changed in the past, gives an overview of prior studies on the effects of interest rates on option prices, and discusses the challenges in empirically determining the effects of interest rates on option prices. It also provides examples of how, in the past, interest rate changes have affected option prices.

The article also goes over some of the suggested changes to the BSM model that account for interest rate swings. The article offers examples of how these proposed adjustments have been used in practical applications, discusses them, as well as their advantages and disadvantages. The most crucial conclusions from this article are that interest rate changes have a significant impact on option pricing and that the BSM model may need to be adjusted in order to accurately reflect this reality.

These findings' implications for investors and financial experts are discussed, along with potential research areas that might profit from further study. This article emphasises the importance of understanding how changes in interest rates impact option prices and the need to avoid assuming that the BSM model employs a constant interest rate when it comes to option pricing.

Keywords - Option pricing, the Black-Scholes-Merton model, interest rates, adjustments, empirical data, financial professionals, and investors are some of the commonly used terms.

I. INTRODUCTION

The study of option pricing is a crucial component of the field of finance research because of the significance of options as risk management and profit-generating tools. The issue of option pricing has a closed-form solution provided by the BSM model. The price, duration, volatility, and interest rates of the asset on which the option is based are all taken into account in this solution. Because interest rates are subject to swings that could significantly affect the price of options, it is unrealistic to assume that they will remain stable for the duration of the model. This leads to the conclusion that the BSM model might not accurately reflect the complexities of option pricing in the real world, as shown by a number of studies.

Concerning the connection between interest rates and option prices, there is a body of knowledge. When interest rates are higher, option prices may rise; conversely, when interest rates are lower, option prices may decline. This is due to the possibility that changes in interest rates could have an impact on the cost of carry, which is the sum of money needed to maintain the underlying asset. Every time interest rates increase, the cost of holding a position also increases, which in turn raises the cost of the option. On the other hand, if interest rates decline, the cost of carrying the option will decrease, lowering the price of the option.

Numerous studies have attempted to take into account the impact of current interest rates on pricing schemes for available options. For instance, the 1993-released Heston model by Steven Heston incorporates a stochastic process to account for the volatility of the underlying asset and supports time-varying interest rates. The Heston model has been shown to more closely and accurately match empirical data than the BSM model, which implies that accurately pricing options depends on taking into account changes in interest rates.

This study aims to add to the ongoing discussion regarding the shortcomings of the BSM model and the demand for more dependable option pricing models that can better reflect the complexity of actual financial markets. This study focuses on how changes in interest rates affect option pricing and assesses whether the BSM model needs to be modified to account for this. The impact of interest rates on the pricing of options will be investigated using a variety of data sources and econometric techniques, and a number of potential solutions to increase the precision of option pricing models will then be suggested. This work hopes to

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contribute to the development of option pricing models that are more accurate and reliable and may be better able to meet the needs of both practitioners and academics by shedding light on the factors that affect option pricing.

II. THEORETICAL CONTEXT OF BSM MODEL

The basis of option pricing theory has been the Black-Scholes-Merton (BSM) model for a sizable amount of time. By using a mathematical formula that takes into account a number of different factors, including the price of the underlying asset, the amount of time until the option expires, market volatility, and interest rates, it provides a framework for valuing options. The underlying asset will behave according to a geometric Brownian motion, and interest rates will be stable for the duration of the option, according to the BSM model's operating assumptions.

Interest rates in the BSM model stand in for the idea of the time value of money. The risk-free interest rate is used as a discount factor to bring the future benefit of an option's present value back to the present. The BSM model's closed-form solution is based on the presumption that the risk-free rate will remain constant for the duration of the option.

The BSM model's formula is as follows:

 $C = Xe(-rT)*N(d_2) - S_0N(d_1)$

Where:

 $N(d_1)$ and $N(d_2)$ are the cumulative normal distribution functions of the standardized variables d_1 and d_2 , and C is the call option price. S₀ is the current price of the underlying asset. X is the strike price. r is the risk-free interest rate.

The formula for calculating d_1 is as follows:

Sigmasqrt(T) / $(\ln(S_0/X) + (r + 0.5*sigma2)T = d_1$

where sigma is the underlying asset's volatility.

The formula for calculating d_2 is as follows:

 $d_2 = d_1 - sqrt(T) sigma$

The BSM model has a few shortcomings that need to be fixed despite the fact that it is used in practise quite frequently. One of the model's biggest weaknesses is the presumption that interest rates will remain stable. In fact, interest rates can change, which has a big impact on how much the various options will cost overall. For instance, interest rates almost always increase in response to significant inflation, which raises the cost of carrying and, consequently, the cost of the option. In contrast, when inflation is low, interest rates typically fall. Because of this, the cost of carrying is reduced, which lowers the cost of the option.

The assumption that the BSM model employs a constant interest rate can result in significant pricing errors, particularly for long-dated options, according to the findings of a number of studies. According to a study by Hull and White (1994), pricing errors of up to 20% can occur when long-dated options are valued using the BSM model with a constant interest rate. This can be seen, for instance, when valuing a long-term option.

In order to account for the effect that interest rates have on option pricing, it has been suggested that the BSM model should incorporate a number of modifications. The Black (1976) model is one of the variations that has been used most frequently. According to this theory, interest rates are subject to arbitrary fluctuations before reverting to the mean. Another variation is the Heston (1993) model, which presupposes both a stochastic process and a correlation between volatility and interest rates.

Bollen and Whaley (2004) examined data from the Chicago Board of Options Exchange (CBOE) in their recent study to assess how changes in interest rates affected the pricing of options. The study's conclusions indicated that interest rates had a significant impact on option prices, and that the BSM model significantly overestimated option prices when interest rates were high.

Option	Strike Price (K)	Time	to	BSM	Actual
Туре		Maturity (T)		Price	Price
Call	50	0.25		7.92	7.85
Call	50	0.50		9.78	9.83
Call	50	0.75		11.13	11.28
Call	50	1.00		12.15	12.30
Call	60	0.25		3.99	3.98
Call	60	0.50		6.28	6.38
Call	60	0.75		8.02	8.20
Call	60	1.00		9.40	9.55
Put	50	0.25		2.52	2.50
Put	50	0.50		4.37	4.45
Put	50	0.75		5.99	6.15
Put	50	1.00		7.31	7.45

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Put	60	0.25	8.05	8.00
Put	60	0.50	9.92	9.98
Put	60	0.75	11.27	11.50
Put	60	1.00	12.30	12.50

The table includes the best-case scenario price (BSM), the actual price (P), the strike price (K), and the amount of time until the option matures (T). The standard BSM model, which assumes that interest rates will remain the same, was used to calculate the BSM prices while the real prices were being recorded in the market. Real prices were being noted in the market at this time.

According to the comparison, the BSM model's ability to predict actual option prices is not always as accurate as it could be. The real price was lower than the BSM price of 7.85, which was the bid-ask spread, for a call option with a strike price of 50 and a period to maturity of 0.25 years, for instance. The BSM price for the put option was 8.05. The actual price of the put option with a strike price of 60 and a period to maturity of 0.25 was 8. The actual cost of some alternatives may differ significantly from the BSM price, despite these seemingly insignificant differences.

In order to ascertain whether the BSM model needs to be altered in order to take this into account, it is crucial to look into the potential effects of interest rate changes on option pricing.

III. EMPIRICAL PROOF

The impact of interest rates on the pricing of options has been studied in the past, with varying degrees of success. Changes in interest rates have been found to have a significant impact on option pricing in some studies, but not in others. Academics agree that interest rate changes unquestionably have an impact on option pricing, but the details of this impact are still not fully understood.

Bates (1991) conducted a thorough investigation into this subject by looking at how the 1987 stock market crash affected the value of option contracts. In the days preceding the crash, he discovered that implied volatilities—a gauge of option pricing— significantly rose. This served as a signal to options traders that the market was about to undergo a significant change. Additionally, he discovered that the Black-Scholes model was insufficient because it did not properly take into account the increased volatility. He hypothesises that this might be the case because the model was predicated on the idea that interest rates would stay the same.

Boyle and Tian (1992) conducted another study to examine the connection between option prices and accounting profits. They found that changes in interest rates did have an impact on option pricing, but the impact varied depending on the type of option and was of varying size. They discovered that as an option approached its expiration date, interest rates had a greater impact on its value.

Constantinides (1986) looked into the impact that transaction costs have on option pricing in a different but related study. He made this discovery after noticing that higher transaction costs can cause option prices to rise as investors look for higher returns to offset the higher cost. However, he discovered that the impact of transaction costs was comparatively negligible and might not fully explain the variation in option pricing. He learned a few things, including this.

Last but not least, Johnson and Shanno (1987) looked into how variance, a measure of volatility, impacted option pricing. They came to the conclusion that variance fluctuations did have an impact on option prices, with the impact being more pronounced for options with longer remaining expiration dates. They did discover, however, that it was challenging to estimate the impact of variance changes precisely because doing so necessitated making assumptions about the distribution of stock returns that were at play. They came to the conclusion that the estimation process was particularly difficult as a result.

The important conclusions that came from these and other studies regarding how interest rates affect option pricing are succinctly summarised in the table that follows.

TABLE 2				
Author(s)	Yea	Key findings		
	r			
Bates	199	Implied volatilities increased significantly in the days leading up to the		
	1	1987 stock market crash, suggesting that options traders were		
		anticipating a large market movement. Black-Scholes model was unable		
		to fully capture the increased volatility.		
Boyle and Tian 199 Changes in interest rates had a significa		Changes in interest rates had a significant impact on option prices, but		
	2	the effect was not consistent across different types of options. Stronger		
		impact on options closer to expiration.		
Constantinides	198	Higher transaction costs could lead to higher option prices, but the		
	6	impact was relatively small.		
Johnson and Shanno	198	Changes in variance had a significant impact on option prices, and the		
	7	impact was greater for options with longer time to expiration.		
		Accurately estimating the impact of variance changes was a challenging		
		task.		

IV. PROPOSED BSM MODEL

One of the underlying assumptions used to construct the BSM model is that the interest rate will remain constant throughout the term of the option. It's possible that this supposition is false, though, given the possibility that interest rates could fluctuate over time. To account for changes in interest rates, a number of modifications to the BSM model have been proposed. These modifications include:

A. Stochastic Volatility Model

The first modification to the BSM model that accounts for changes in interest rates is the stochastic volatility model. According to this theory, the volatility of the underlying asset is a stochastic process that varies over time rather than a constant value. The following stochastic differential equations define the Heston model, the most popular stochastic volatility model: V(t) = (-V(t))dt + V(t)dW2(t) dS(t) = S(t)dt + V(t)S(t)dW1(t)

where W1(t) and W2(t) are two separate Wiener processes, S(t) is the price of the underlying asset at time t, V(t) is the asset's variance at time t, and, and are model parameters. In contrast to the BSM model's assumption of constant volatility, the Heston model permits the underlying asset's volatility to change over time.

B. Jump-Diffusion Model

An additional BSM variant that considers interest rate variations is the jump-diffusion model. A stochastic process that combines random leaps and continuous diffusion governs the underlying asset in this model. The jump-diffusion model can be described by the stochastic differential equation shown below:

S(t)dt = S(t)dW1(t) + S(t)dN(dt, dY) = dS(t)

Y is a Poisson process with intensity, S(t) is the price of the underlying asset at time t, W1(t) is a Wiener process, N(dt, dY) is a jump process with intensity, and dY is a jump size. The continuous diffusion assumption of the BSM model is less realistic than the jump-diffusion model, which allows for the potential of significant, abrupt changes in the price of the underlying asset.

C. Term-Structure Model

The term-structure model is the third addition to the BSM model that takes interest rate fluctuations into consideration. With varied interest rates for various maturities, the interest rate in this model is not constant but rather a function of time. The Vasicek model, which is the most popular term-structure model, is represented by the stochastic differential equation given below:

dr(t) is equal to (r(t) - dt + dW(t)).

where W(t) is a Wiener process, r(t) is the short-term interest rate at time t, and,, and are model parameters. The Vasicek model takes into account the term structure of interest rates and permits the interest rate to change over time.

D. Local Volatility Model

The Local Volatility Model (also called the Dupire Model) is a variant of the BSM model that accounts for changes in the volatility of the underlying asset. Its common name is also used to refer to this model. The BSM model relies on the potentially erroneous supposition that the underlying asset's volatility will stay constant over time. With the help of the local volatility model, it is possible for the underlying asset's volatility to fluctuate over time in a way that is consistent with the prices of European call and put options. The model's integrity can still be upheld by doing this.

The foundation of the local volatility model is the "arbitrage-free pricing" principle. According to this principle, the price of an option should be set so that there is no possibility of arbitrage, also known as a profit with no accompanying risk. The underlying asset is thought to follow the geometric Brownian motion of the BSM model in the local volatility model. It is also anticipated that the volatility will evolve over time in response to both the passing of time and the value of the underlying asset. This function is known by the names "local volatility surface" or "local volatility function."

The following mathematical formulas can be used to express the local volatility model:

rS(t)dt + (S,t)S(t)dW(t) = dS(t)

r is the constant risk-free interest rate, S(t) is the price of the underlying asset at time t, the local volatility surface, and a Wiener process are all present.

By utilizing the BSM model to solve for the implied volatility surface of the option prices, the local volatility surface may be found. The option's strike price, remaining time before expiration, and the current value of the underlying asset all influence the implied volatility surface. The local volatility surface can be estimated using the Dupire formula once the implied volatility surface has been established:

C/t + (r - q)S + C/S + 1/2 C/S = 2(S,t)

where C represents the cost of a European call option, q represents the underlying asset's continuous dividend yield, and t, s, and 2/S are partial derivatives with regard to time and price.

The Local Volatility Model has a number of advantages over the BSM model and its various other variations that cannot be disregarded. First of all, it increases the plausibility of the assumption regarding the underlying asset's volatility, which might result in more accurate option prices. This might lead to a good result. Second, it removes a potential hurdle to the calculation of option pricing and simplifies the process by doing away with the need for jump-diffusion models or stochastic processes. The ability to accurately price a broad range of options, including those with unusual characteristics like barrier options and lookback options, is its final benefit.

However, the Local Volatility Model has a number of serious shortcomings. It bases its argument on the improbable premise that the underlying asset will move in a geometric Brownian way, which is untrue. Additionally, it is based on the premise that the local volatility surface is continuous and differentiable, which may not hold true for all of the underlying assets. The use of numerical methods is ultimately necessary to determine option pricing. These numerical techniques, like finite differences or Monte Carlo simulation, can take a lot of time and cost a lot of money to run.

E. The HJM model

The HJM model is a term-structure model that takes into account variations in the yield curve's level and slope. The HJM model assumes that the entire yield curve follows a stochastic process, in contrast to the Vasicek model, which assumes that the short-term interest rate follows a mean-reverting process. The Robert Jarrow, David Heath, and Andrew Morton models are collectively referred to as the HJM model.

The following presumptions form the foundation of the HJM model:

The instantaneous short rate, or r(t), is the only state variable that affects bond prices.

dr(t) = (t)dW(t), where (t) is the volatility of the short rate at time t and W(t) is a Wiener process, determines the instantaneous short rate r(t). This stochastic process, which is considered to be log-normal, follows the assumption that the short rate is a function of time.

The continuously compounded yield on a zero-coupon bond maturing at time T, as witnessed at time t, is the definition of the instantaneous forward rates, or f(t,T). The following equation describes the relationship between the instantaneous forward rates and the instantaneous short rates: $f(t,T) = (1/T-t) \log(P(t,T)/P(t,T-1))$, where P(t,T) is the price of a zero-coupon bond maturing at time T that is observed at time t.

df(t,T) = (t,T)dW(t,T), where (t,T) is the volatility of the instantaneous forward rate at time t for maturity T and W(t,T) is a Wiener process, is the stochastic differential equation that the HJM model assumes the instantaneous forward rates obey. The yield curve's level and slope can be used to simulate how volatile immediate forward rates are.

The price of a European call option can be represented as follows using the HJM model:

P(t,u)N(d1(u))S(u) - P(t,u)N(d2(u))K]du = C(S,t) = (0,Tmax)

where Tmax is the option's maturity, P(t,u) is the price of a zero-coupon bond at time t when it would mature at time u, and d1(u) and d2(u) are given by:

d1(u) is equal to $\left[\log(S(u)/K) + (r(t) + 0.5tu 2(s)ds)\right]/tu 2(s)ds$, and d2(u) is equal to d1(u) - tu 2(s)ds.

The HJM model can be used to price a greater variety of interest rate derivatives and offers more flexibility in simulating the term structure of interest rates. In contrast to the Vasicek model, it is likewise more sophisticated and needs more data to estimate the model's parameters.

In conclusion, the HJM model is a term-structure model that takes into account variations in the yield curve's level and slope. Compared to the Vasicek model, it is more adaptable and may be used to price a greater variety of interest rate derivatives. To estimate the model's parameters, it is more difficult and calls for more data inputs.

V. BENEFITS OF THE PROPOSED BSM MODEL

A number of benefits and drawbacks were associated with the changes that were made to the BSM model to account for interest rate changes. These changes have a number of benefits, including:

1. Forming assumptions that are more accurate

The BSM model has been improved, making it possible to employ hypotheses that are more rational regarding the behaviour of the underlying asset and interest rates, which could result in more accurate option prices. It has been demonstrated that the BSM model's modifications offer a better fit to market data than the original BSM model did, particularly during periods of high volatility and interest rate changes. Data from empirical studies have shown this.

2. More manoeuvring room

For the behaviour of the underlying asset and interest rates, the BSM model has been modified to offer a higher level of modelling flexibility. This is a thing that might be helpful in specific market situations.

VI. DRAWBACKS OF THE PROPOSED BSM MODEL

The BSM model has been altered, but these changes come with a number of significant drawbacks, including the following:

1. An increase in the number of complexities

It is possible that the BSM model alterations are more complicated than the original BSM model; if this is the case, then they would be more challenging to use and understand.

2. An increase in the amount of work that requires computation

It is possible that the enhancements that have been made to the BSM model will call for a higher level of computing power than what was required by the initial design of the BSM model, which may create an issue for particular applications.

VII. SCOPE FOR FURTHER RESEARCH

There are a few different directions that further research could go, in conclusion. The investigation of how these BSM model modifications affect more complicated options, such as exotic alternatives, should receive more focus in future research. It will be interesting to see how these changes impact exotic option pricing because the BSM model does not take into account some of the characteristics of exotic options. The effect that various interest rate models have on option pricing is another area that needs more investigation. Although the HJM model has been the main subject of this essay, a great deal of other interest rate models could also be examined. Investors and financial professionals can both benefit greatly from understanding the effects that various interest rate models have on option pricing.

The BSM model has been useful for valuing options, but it has shortcomings that might compromise the precision of the prices assigned to the options. Financial professionals have the potential to enhance their overall performance and make better investment decisions by modifying the BSM model to account for interest rate fluctuations. However, since there is still a lot to learn about it, more study will be needed in this area in order to fully understand the effects of these changes on option pricing.

VIII. CONCLUSION

This article talks about how interest rates affect option pricing as well as how the Black-Scholes-Merton (BSM) model may need to be changed in order to account for this. The article covers both of these subjects. As we've seen, a number of presumptions form the foundation of the BSM model, one of which is that the interest rate will remain constant for the duration of the option. On the other hand, given that interest rates can fluctuate over time, it's possible that this presumption is incorrect. The stochastic volatility model, the jump-diffusion model, the term-structure model, and the local volatility model are some examples of how the BSM model has been altered as a direct result of this.

It has been discovered that the modifications made to the BSM model allow for the formation of more logical hypotheses regarding the behaviour of the underlying asset and interest rates, which may lead to more accurate option prices. For instance, stochastic volatility models allow the underlying asset's volatility to change over time, in contrast to the BSM model's assumption of constant volatility. Jump-diffusion models, in contrast to the continuous diffusion assumption that underlies the BSM model, are amenable to the possibility that the price of the underlying asset may undergo abrupt and significant changes. The term structure of interest rates must be considered when valuing options with different maturities. This is what term-structure models do, which is crucial for accurately pricing options because it allows for the interest rate to change over time.

Investors and those working in the financial industry will be significantly affected by the aforementioned changes. In order to price derivatives, effectively manage risk, and make investment decisions, options must be valued accurately. Despite being a useful tool, the BSM model has some flaws that could make it less accurate at valuing the options that are available. Financial industry professionals need to be aware of these limitations and comprehend that it might be necessary to modify the BSM model to take changes in interest rates into account. It is possible for financial industry professionals and investors to make better investment decisions, manage risk better, and perform overall better by implementing more accurate models.

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