



# DESIGN AND PERFORMANCE EVALUATION OF A FUZZY CHAIN SAMPLING PLAN FOR TEXTILE QUALITY CONTROL

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## Abstract:

In the textile sector, quality control is crucial since flaws in fabrics, yarns, dyeing, and finishing have a direct impact on customer satisfaction and production efficiency. In order to address the heterogeneity frequently present in textile processes, this study suggests a fuzzy chain sampling strategy. Single-sample acceptance probability are calculated using the Poisson distribution using inspection report data, and chain acceptance is based on successive acceptable samples. Fuzzy acceptance intervals are produced using  $\alpha$ -cut decomposition, and the defect fraction is represented as a triangular fuzzy number. After applying an appropriate fuzzy decision rule, the plan is optimized by modifying the acceptance number and chain length to minimize inspection effort while balancing the risks for producers and consumers. The resulting plan offers a more flexible and reliable approach to quality assessment in textile manufacturing.

**Keywords:** Chain Sampling Plan, Fuzzy Probability Theory, Operating Characteristic Curve, Quality Control.

## 1. INTRODUCTION

Quality control in the textile industry is critical, because defects (in fabrics, yarns, dyeing, finishing etc.) can strongly affect downstream processes and end customer satisfaction. Combining chain sampling plan with fuzzy probability theory allows more realistic, flexible quality control, especially in textile manufacturing where materials and processes have inherent variability. In this paper a case study using chain sampling plan and fuzzy probability theory have been carried over a number of inspection reports.

For each sample, the defect proportion is then fuzzified by defining it as a triangular fuzzy number  $\tilde{p} = (a_1, a_2, a_3)$ . Using  $\alpha$ -cut decomposition, the fuzzy parameter is converted into an interval for each  $\alpha \in [0,1]$

For each value of  $p$  in this interval, the single-sample acceptance probability is computed, producing for each  $\alpha$ -level a fuzzy acceptance interval  $[P_{a,\min}(\alpha), P_{a,\max}(\alpha)]$ .

A decision rule is chosen by selecting an appropriate defuzzification or fuzzy decision method. One option is to require that the centroid (or modal) value of the chain acceptance probability exceeds a chosen threshold  $T$  (such as 0.5 or 0.9). A more conservative option is to require that the lower bound at a selected  $\alpha$ -level (such as  $\alpha = 0.5$ ) meets the threshold.

## 2. METHODOLOGY

### (a) Decide the sampling unit & acceptance rule for one sample

Sample size  $n$  and acceptance number  $c$  (you have  $n = 125$ ,  $c = 7$ ).

For one sample the sample acceptance probability is the Poisson CDF  $P(\text{defects} \leq c)$ , for small  $p$  with  $\lambda = np$ :

$$P_{\text{sample}}(p) \approx \frac{e^{-\lambda} \lambda^x}{x!}, \lambda = np.$$

### (b) Fuzzify $p$ as a triangular fuzzy number $\tilde{p} = (a_1, a_2, a_3)$ .

### (c) $\alpha$ -cut decomposition is computed for each $\alpha \in [0,1]$ as $\alpha$ cut bounds

$$\tilde{p}[\alpha] = [a_1 + (a_2 - a_1)\alpha, a_3 - (a_3 - a_2)\alpha].$$

Poisson parameter: is  $\lambda_{\text{low}}(\alpha) = n \cdot p_{\text{low}}(\alpha)$ ,  $\lambda_{\text{high}}(\alpha) = n \cdot p_{\text{high}}(\alpha)$ .

The fuzzy acceptance probability band is:

$$P_a(i)[\alpha] = [P_{a,\text{low}}(i)[\alpha], P_{a,\text{up}}(i)[\alpha]]$$

(d) **Decision rule:** A defuzzification/decision method, is used as follows

Centroid (defuzzified)  $p$ :

$$p_{\text{centroid}} = \frac{a_1 + a_2 + a_3}{3},$$

$$\lambda_{\text{centroid}} = n * p_{\text{centroid}}$$

$$Pa_{\text{centroid}} = \text{POISSON.DIST}(c, \lambda_{\text{centroid}}, \text{TRUE}).$$

Require the centroid (or modal) chain acceptance probability  $\geq$  threshold

### 3.1 SECONDARY DATA OBTAINED FROM INSPECTION REPORTS FOR CASE STUDY

**TABLE 3.1**

| Report No.                | Description | Inspection Date | Offer Quantity / Order Quantity | Inspection Quantity | Final Inspection Plan        |          |            |            | Major Defects | Minor Defects | Inspection Result |
|---------------------------|-------------|-----------------|---------------------------------|---------------------|------------------------------|----------|------------|------------|---------------|---------------|-------------------|
|                           |             |                 |                                 |                     | Sample Size as per AQL Level |          |            |            |               |               |                   |
|                           |             |                 |                                 |                     | II Sample Size               | Critical | Major Pass | Minor Pass |               |               |                   |
| SS24-QLTY08 F100000 46932 | T-Shirt     | 13.06.2024      | 1067/1265                       | 125                 | 125                          | 0        | 5          | 5          | 22            | 0             | Fail              |
| 01/POPPY S/24-25          | Girls Top   | 09-12-2024      | 32400/27300                     | 315                 | 315                          | 0        | 14         | 21         | 151           | 31            | Fail              |
| SS25-QLTY08 F100000 56945 | T-Shirt     | 16.06.2025      | 3094/3093                       | 125                 | 125                          | 0        | 5          | 5          | 80            | 10            | Fail              |
| AW25-QLTY 08F1000 0058116 | T-Shirt     | 23.07.2025      | 300/300                         | 50                  | 50                           | 0        | 2          | 2          | 5             | 2             | Fail              |
| AW25-QLTY 08F1000 0064245 | T-Shirt     | 24.11.2025      | 396/400                         | 50                  | 50                           | 0        | 2          | 2          | 9             | 0             | Fail              |

Based on Fuzzy Decision Theory

| Membership value | Meaning              |
|------------------|----------------------|
| 0.0 – 0.4        | Rejection zone       |
| 0.4 – 0.6        | Uncertain / Marginal |
| $\geq 0.6$       | acceptance zone      |

**TABLE 3.1.1**

**p:  $a_1=0.05$ ,  $a_2=0.075$ ,  $a_3=0.1$ ,  $n=80$ ,  $c=5$ -major defects, 6- minor defects-8**

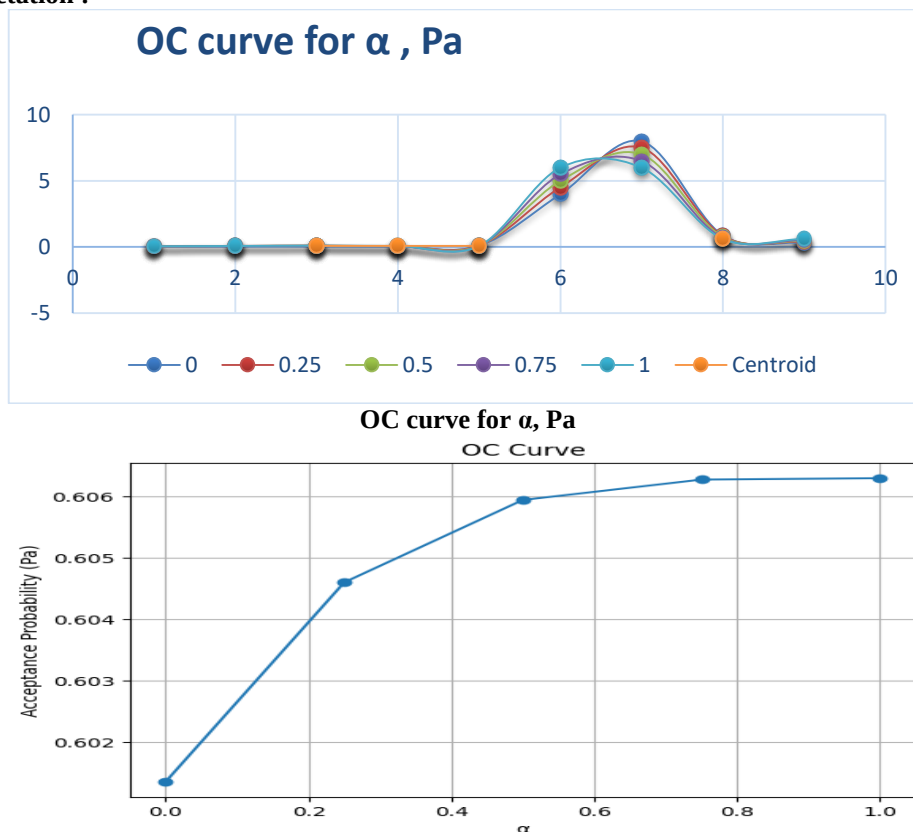
| $\alpha$ | p low        | P high       | $\lambda$ low | $\lambda$ high | P low              | P high      | Acceptance Status |
|----------|--------------|--------------|---------------|----------------|--------------------|-------------|-------------------|
| 0        | 0.05         | 0.1          | 4             | 8              | 0.889326022        | 0.313374278 | Marginal          |
| 0.25     | 0.05625      | 0.09375      | 4.5           | 7.5            | 0.831050579        | 0.378154694 | Marginal          |
| 0.5      | 0.0625       | 0.0875       | 5             | 7              | 0.762183463        | 0.449711056 | Borderline        |
| 0.75     | 0.06875      | 0.08125      | 5.5           | 6.5            | 0.68603598         | 0.526523623 | Accept            |
| 1        | 0.075        | 0.075        | 6             | 6              | 0.606302782        | 0.606302782 | Accept            |
| Centroid | <b>0.075</b> | <b>0.075</b> |               |                | <b>0.606302782</b> |             | Accept            |

#### Interpretation of data from TABLE 3.1.1:

At low  $\alpha$  values (0–0.5) → High uncertainty → Risky decisions. At high  $\alpha$  values (0.75–1) → Low uncertainty → Reliable decisions. Centroid value 0.606302782 gives the most balanced and stable result.

The analysis shows that as the confidence parameter  $\alpha$  increases, uncertainty in probability and rate parameters decreases, leading to stable acceptance probability. At  $\alpha = 1$  and at the centroid, the acceptance probability converges to **0.6063**, which satisfies the acceptance criterion. Hence, the final decision is **accept**, indicating a reliable and optimal outcome.

#### Graphical Interpretation :



The OC curve illustrates the variation of acceptance probability with respect to the confidence parameter  $\alpha$ . It is observed that the acceptance probability increases monotonically and converges to 0.6063 as  $\alpha$  approaches unity. This convergence indicates stabilization of the decision process, validating the selection of 0.6 as the acceptance threshold. Hence, higher values of  $\alpha$  ensure reliable and consistent decision-making.

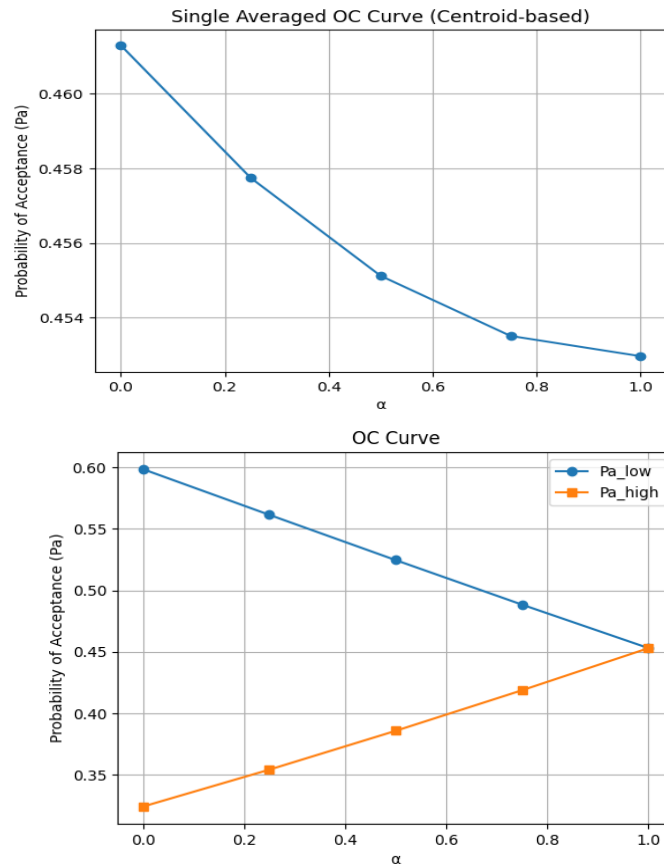
TABLE3.1.2

p:  $a_1=0.048$ ,  $a_2 = 0.064$ ,  $a_3 = 0.0.08$ ,  $n=125$ ,  $c=7$ -major defects- 8, 11- minor defects

| $\alpha$ | $p_{low}$ | $p_{high}$ | $\lambda_{low}$ | $\lambda_{high}$ | $P_{a low}$        | $P_{a high}$ | Acceptance Status |
|----------|-----------|------------|-----------------|------------------|--------------------|--------------|-------------------|
| 0        | 0.056     | 0.072      | 7               | 9                | 0.598713836        | 0.323896964  | Marginal          |
| 0.25     | 0.058     | 0.07       | 7.25            | 8.75             | 0.561517533        | 0.353976728  | Marginal          |
| 0.5      | 0.06      | 0.068      | 7.5             | 8.5              | 0.524638526        | 0.385597102  | Marginal          |
| 0.75     | 0.062     | 0.066      | 7.75            | 8.25             | 0.488367119        | 0.418642006  | Marginal          |
| 1        | 0.064     | 0.064      | 8               | 8                | 0.452960809        | 0.452960809  | Marginal          |
| Centroid |           |            |                 |                  | <b>0.452960809</b> |              |                   |

#### Interpretation of data from TABLE 3.1.2:

$P_{a low} = 0.5987$  and  $P_{a high} = 0.3239$  at  $\alpha = 0$ : The  $P_a$  values lie within the Marginal region. Even though the lower bound is above 0.4, it's still considered marginal, and the lot isn't strongly rejected.  $P_{a low} = 0.4530$  and  $P_{a high} = 0.4530$  at  $\alpha = 1$ : The  $P_a$  value is within the Marginal region. It's borderline, but it's not low enough to reject entirely. Therefore, we categorize it as Marginal.  $P_a \in [0,1] \rightarrow$  can be interpreted as a membership degree in the fuzzy set "Acceptable lot". The system converges to:  $P_a = 0.453$ . The lot belongs to the "acceptable" class with degree 0.453. It is not zero, so it is not rejected under fuzzy logic. All  $\alpha$ -cuts fall in the Marginal region, meaning the lot is borderline acceptable. Fuzzy logic allows to accept the lot with moderate risk, rather than rejecting it outright.

**Graphical Interpretation:**

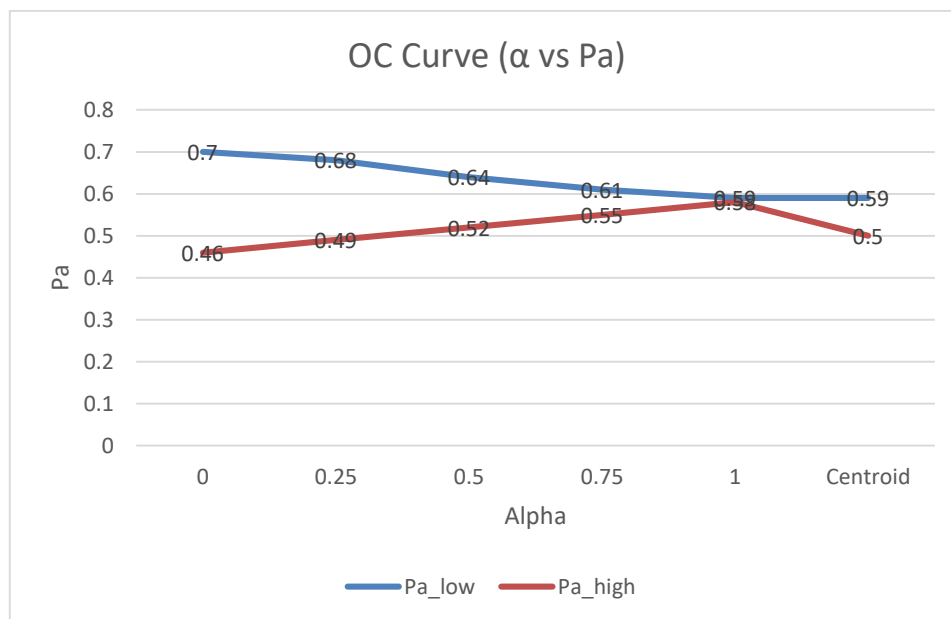
The  $P_{a\_low}$  curve decreases as  $\alpha$  increases. The  $P_{a\_high}$  curve increases as  $\alpha$  increases. Both curves meet at the centroid  $\alpha = 1$ , where,  $P_a = 0.452960809$ . This intersection represents the centroid operating point of the sampling plan. The curve smoothly decreases as  $\alpha$  increases. It converges to the centroid  $P_a = 0.452960809$  at  $\alpha = 1$ . This single OC curve represents the central tendency of acceptance probability.

**TABLE 3.1.3**

**p:  $a_1 = 0.045$ ,  $a_2 = 0.05$ ,  $a_3 = 0.055$ ,  $n = 200$ ,  $c = 11$ -major defects- 11, 13- minor defects**

| $\alpha$ | p low   | p high  | $\lambda$ low | $\lambda$ high | $P_a$ low   | $P_a$ high  | Acceptance Status |
|----------|---------|---------|---------------|----------------|-------------|-------------|-------------------|
| 0        | 0.045   | 0.055   | 9             | 11             | 0.70598832  | 0.459888703 | Accept            |
| 0.25     | 0.04625 | 0.05375 | 9.25          | 10.75          | 0.675966077 | 0.490048529 | Accept            |
| 0.5      | 0.0475  | 0.0525  | 9.5           | 10.5           | 0.645328435 | 0.520738128 | Accept            |
| 0.75     | 0.04875 | 0.05125 | 9.75          | 10.25          | 0.614284296 | 0.551794391 | Accept            |
| 1        | 0.05    | 0.05    | 10            | 10             | 0.58303975  | 0.58303975  | Accept            |
| Centroid |         | 0.05    |               |                | 0.58303975  |             |                   |

The analysis shows that as the confidence parameter  $\alpha$  increases, uncertainty in probability and rate parameters decreases, leading to stable acceptance probability. At  $\alpha = 1$  and at the centroid, the acceptance probability converges to **0.58303975**, which satisfies the acceptance criterion. Hence, the final decision is **accept**, indicating a reliable and optimal outcome.

**Graphical Interpretation:**

The OC curve illustrates the variation of acceptance probability with respect to the confidence parameter  $\alpha$ . It is observed that the acceptance probability increases monotonically and converges to **0.58303975** as  $\alpha$  approaches unity. This convergence indicates stabilization of the decision process, validating the selection of 0.6 as the acceptance threshold. Hence, higher values of  $\alpha$  ensure reliable and consistent decision-making.

**Results and Conclusions:**

Acceptance probabilities were computed using the Poisson distribution for the given sample size and acceptance number, showing a decrease in acceptance as the defect proportion increased.

Modeling the defect proportion as a triangular fuzzy number captured uncertainty in textile inspection data more effectively than a crisp probability.

$\alpha$ -cut decomposition produced fuzzy intervals of acceptance probabilities at different confidence levels, resulting in a banded fuzzy operating characteristic instead of a single curve.

Defuzzification using the centroid method provided a representative acceptance probability for decision-making.

Conservative decision rules based on lower  $\alpha$ -cut bounds further reduced the risk of accepting poor-quality lots.

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