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An Integrated Two-Layer Supply Chain Inventory with Declining and Price Sensitive Demand for Deteriorating Items

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Abstract: This study optimizes the retail price, the replenishment time period, initial lot size of seasonal products. Seasonal products are highly deteriorating in nature and they become useless after sales of the season. The deteriorating nature of products, not only reduces the production rate of manufacturers but also reduces retailers' stock. For this, we studied the effects of deterioration on the profits of retailers and manufacturers in the continuous two-layer supply chain and we also studied the impact of variation in the retail price, on the demand. The main aim of this article is to develop an integrated supply chain inventory model by optimizing the retail price, replenishment cycle, and initial lot size of deteriorating seasonal products, when there is no investment has been done in preservation technology. We optimized average profit by optimizing retail price for price-sensitive declining demand. The model is illustrated with sensitivity analysis and verified by numerical examples. Finally, a sensitivity analysis was performed to study the effect of changes in different parameters of the model on the optimal policy. Moreover, in order to approve the determined models, we have clarified mathematical models and examined affectability.

***Index Terms* - Inventory, Integrated Supply Chain, Price Sensitive Demand, Seasonal Product, Deterioration.**

I. INTRODUCTION

In the real life most of the products/materials, like vegetables, fruits, milk, perfumes, petroleum products, meat etc. are found deteriorating in either quantity or quality or both. Seasonal products wave demand just for a limited time period and therefore deteriorating factor is more effective on the seal products. Recently in the present scenario, most of the products are not also seasonal but also becoming of deteriorating nature simultaneously because of rapid changes in fashion and technology. For this reason, inventory management of the supply chain becomes more complex and tedious when manufacturing such products which are seasonal as well as also deteriorating in nature. Some noteworthy contributions are cited here which related to this problem. Due to the above, serious problems researchers are attracted to make models of price and stock-dependent demand patterns. In this situation, market survey shows that in the supermarket customers attract to buy products when a price discount is offered by management. Hence the demand for certain products may increase exponentially or linearly but the profit of management is always unable to increase accordingly due to crises of stock or manpower or any other circumferences.

In this situation, the problem is how to fulfill the excess demand and how to optimize the profit. Also, during the excess demand for products there may arise the problem of spacing of products and also management needs to large investments. The situation becomes more complex and tedious when the product is deteriorating in nature. Liu and Lian (1990) have explained an inventory model for those deteriorating products which had a fixed lifetime. Giri and Choudhary (1997) have developed an inventory model by considering selling and stock-dependent demand for deteriorating products. Samanta et al. (2004) designed a production inventory control model for deteriorating items with continuity and by considering shortages. Jain et al. (2007) developed an inventory model by considering the deterioration rate is z(t) with ramp type demand pattern where z(t) = αθtθ−1 and α(0 < α << 1) is the deterioration coefficient and (β > 0) is the shape parameter. Giri et al. (2012) dealt with an integrated supply chain model for a deteriorating item in which a single manufacturer and single retailer are members of the supply chain. In this model, it had assumed that demand is a function of retail price and stock. An inventory model for stock and price-dependent demand for deteriorating products is developed by Khedlekar et al. (2015). Singh and Mukherjee (2017) addressed a production inventory model for establishing a deteriorating item with continual holding cost, and a mandated rate is persistent. Singh (2019) fostered a two-distribution centre stock model for non-momentary breaking down things with stretch esteemed stock expenses and stock-subordinate interest under inflationary conditions.

Managers of any supply chain continuously improve their performance by reducing lead time and its variance. Sarkar and Majumdar (2013) have analyzed a supply chain model for vendor set up reduction cost. Hudnukar et al. (2017) gave a literature review on the supply chain of journals published between 2003 to 2015. They classified the supply chain journal in various mathematical approaches. A supply chain for the multi-purchase and the single-seller with FPR and the sub-par nature of items was created by Sarkar et al. (2018). Sett et al. (2020) presents a review that centers around a single-vendor, single-purchaser supply chain model for a single kind of item with updated administration given to the purchaser by the vendor. Singh et. al (2021) have analyzed the main objective of the study is to adopt a production model in inventory for inferring request capacities for multi-item, multi-outlet circumstances. Vendors frequently increase the benefit by providing a lower quality of a specific item. Dey et al. (2021) discussed that controllable lead time and fluctuation are fundamental issues for the executives' keen supply chain. Singh et al (2022) considered three degrees of a production inventory model for a consistent deterioration rate. This model assumes a significant part in the production of the board and assembling units. They concluded that a contractual relationship is required between the retailer and manufacturer for the smooth running of business activity.

Deterioration is observed continuously in everything. It may be low or high for different items. Inventory models for breaking down things are varied, the more significant part of which considers a consistent deterioration rate over the long-haul of things practically speaking, similar to unstable fluids, horticultural items, etc. Yong He et al. (2013) developed an inventory model for deteriorating and seasonal products in which they assumed deterioration is controlled by preservation technology investment. Sarkar et al. (2014) developed vendor buyer supply chain model in which they designed two cases (i)first one is probability distribution dependent demand with lead time case (ii) second one distribution free demand with considering lead time case. Zhang et al. (2014) studied that what should be the design of contract in closed loop supply chain when the collection cost effort level is personal information of the retailer. They developed two cases (i) a two-part nonlinear contract and (ii) Collection effort level requirement contract. Mahata el al. (2019) Suggested an inventory model for deteriorating items for price inflation-dependent demand and permissible delay in payment. This paper presents a solution to the problem of dynamic decision-making, for this they adopt an iso-elastic and selling price-dependent demand. Calikan (2021) Developed an Economic Order Quantity (EOQ) model for exponentially deteriorating products in which they consider the product's deteriorating rate is proportional to the inventory level per unit time. The study is also analyzed for back ordering policy. Paul et al. (2021) Presented a production inventory model for deteriorating products in which they assumed a product demand is Weibull demand and the replenishment rate is uniform with and without shortages. They minimized the total expenditure costs.

Pal and Chakraborty (2020) presented an EOQ model developed for a linearly dependent demand of non-instantaneous deteriorating items under shortages. A comparative study is done under crisps and neuromorphic situations. the model gives better results under the later situations. Sundararajan el al. (2021) presented an EOQ-based inventory model for non-instantaneous deteriorating items under the linear decreasing demand under permissible delay in payment allowing shortages. The main purpose of this model is that what should be the rate of production for the manufacturer, lot size for the retailer, retail price, and replenishment cycle time of highly deteriorating seasonal products so that profit for both members should maximum. Shah and Patel (2022) Developed an EOQ model with a constant rate of deterioration rate of product and it is solved under crisp, fuzzy and cloud-fuzzy environments. They minimized the total inventory cost and compare the results in crisp, fuzzy and cloud-fuzzy environments. This article presents an integrated supply chain coordination model by incorporating holding cost, setup cost and deterioration cost, etc. In this model, the demand rate has been taken as an exponentially decreasing function of t as well as price and production rate taken as a linear function of t.

The rest of the paper is systematic. Section 2 discusses the notation/ symbolizations and the suppositions to frame the proposed inventory model. Section 3 have explained the mathematical model for retailers and section 4 drives the mathematical model for manufacturers. Section 5 discusses the cost calculation of the retailer and manufacturer. Section 6 presents the sensitivity analysis of the essential parameter. Section 6 discusses the result with the help of a numerical guide to illustrate the planned model. and section 8 shows the graphical representation. Section the final section concludes the overall findings with suggestions and guidance for future research work.

2. The following notations and assumptions are used in this proposed model.

2.1 Notations: The following notations are used

(1)	c	Retailer's per unit purchase cost and manufacturers per unit selling cost.
(2)	p	Retail price per unit products, $p \geq c$.
(3)	Sc	Setup cost per lot of products for the manufacturer.
(4)	Cp	Manufacturing cost per unit products.
(5)	Oc	Retailer's ordering cost per order.
(6)	Dc	Deteriorating cost per replenishment cycle of deteriorated products per unit,
(7)	h	Inventory holding cost per unit time per unit.
(8)	θ	The deterioration rate,
(9)	D(p, t)	Demand rate at time t; $D(p, t) = \alpha e{-}at - \beta p$ where α demand sensitive parameter, and β price coefficient Parameter.
(10)	q	Production is a linear function of time t, we consider q=rt; where r is the production scale parameter.
(11)	t_s	Production starting time for next cycle time.
(12)	T	Finite time horizon.
(13)	Q	Initial lot-size.
(14)	$I_m(t)$	Manufacturer's inventory level at time t, $t \in [0, T]$.
(15)	$I_r(t)$	Retailer's inventory level at time t for the retailer, $t \in [0, T]$.
(16)	T_r	Average profit per unit time for the retailer.
(17)	T_m	Average profit per unit time for the manufacturer.
(18)	TC	Average total profit per unit time under the integrated system.

2.2 Assumptions

The following assumption is made in this model:

(1) Demand of product is D(p, t) at unit time t; we assumed that demand function $D(p, t) = (\alpha e\text{-}at\text{–} \beta t)$, is the exponential decreasing function of t and a linear function of price, where α is the initial demand and β is price sensitive parameter, and $\alpha > 0$, $a \geq 0$, $\beta > 0$,

(2) Deterioration cost and inventory holding cost are constant.

(3) Manufacturing rate is a linear function qm = rt according to demand rate.

(4) There is no cost of partly or wholly deteriorated products and there is no holding cost for them.

(5) The deterioration cost as well as inventory holding cost for both manufacturer and retailer, are the same.

(6) Shortages are not allowed and lead time is zero.

(7) Replenishment rate is finite.

(8) Initially at the business starting time, the manufacturer prepares an initial lot size of the product and supplies the product at time t = o.

(9) For obtaining the concavity condition, $e^{\theta T}$ and e^{-aT} are approximated two terms of T, because θ and a are less than 1.

(10) Deterioration rate is taken as a constant rate per unit time.

(11) We assumed that the manufacturer and retailer both form a continuous supply chain and work together as a single unit.

3. Mathematical Model for Retailer

As per the assumptions, the retailer obtains the initial stock from the manufacturer, at time t, where t ∈[0, T] , Due to demand and deterioration, the inventory level of the retailer is governed by the following first-order nonlinear differential equation at any time t:

$$\frac{dI_r(t)}{dt}+\theta I_r(t)=-D(p,t)\ ;\ 0\le t\le T \tag{1}$$

$$D(p,t)=(\alpha e^{-at}\text{-}\beta t)$$

with the boundary condition $I_r(t) = Q$, at t = 0 and $I_r(t) = 0$, at $t = T$ (2)

Components of the retailer's average profit, in the cycle time [0, T] are as follows from equation (1) with the help of boundary condition (2).

$$I_r(t)=\frac{\beta p}{\theta}(1-e^{\theta(T-t)})-\frac{\alpha}{(\theta-\alpha)}\left(e^{-\alpha t}-e^{aT}e^{\theta(T-t)}\right) \tag{3}$$

The retailer's initial lot size for at time t = 0, where t ∈ [0, T] is

$$I_r(0)=Q=\frac{\beta p}{\theta}(1-e^{\theta T})-\frac{\alpha}{(\theta-\alpha)}\left(1-e^{(\theta-\alpha)T}\right) \tag{4}$$

1. The revenue from sales in the cycle time [0, T]

$$S_r=p\int_0^T D(p,t)dt=\left[\frac{p\alpha}{a}(1-e^{aT})-\beta p^2T\right] \tag{5}$$

2. Purchasing cost for the retailer is

$$P_{cr}=cQ=\frac{\beta pc}{\theta}(1-e^{\theta T})-\frac{c\alpha}{(\theta-\alpha)}\left(1-e^{(\theta-\alpha)T}\right) \tag{6}$$

3. The holding cost *HCr* is

$$H_{cr}=h\int_0^T I_r(t)dt=h\left[\frac{\beta pT}{\theta}+\left(\frac{\alpha e^{-aT}}{\theta(\theta-a)}-\frac{\beta p}{\theta^2}\right)\left(e^{\theta T}-1\right)+\frac{\alpha}{a(\theta-a)}(e^{-aT}-1)\right] \tag{7}$$

4. Deterioration cost *DCr* in the interval of length *[0, T]* is

$$D_{cr}=D_c\theta\int_0^T I_r(t)dt$$

$$D_{cr}=D_c\theta\left[\frac{\beta pT}{\theta}+\left(\frac{\alpha e^{-aT}}{\theta(\theta-a)}-\frac{\beta p}{\theta^2}\right)\left(e^{\theta T}-1\right)+\frac{\alpha}{a(\theta-a)}(e^{-aT}-1)\right] \tag{8}$$

5. Ordering cost of retailer

$$O_{cr}=O_c \tag{9}$$

Integrated average profit function for the retailer's considering all incomes and expenditures, in the cycle time *[0, T]* is

$$T_r(T,p)=\left[\begin{array}{l}\frac{p\alpha}{aT}(1-e^{\theta T})-\beta p^2-\frac{\beta pc}{T\theta}(1-e^{\theta T})+\frac{c\alpha}{T(\theta-\alpha)}\left(1-e^{(\theta-\alpha)T}\right)-\frac{O_c}{T}-\\(h+D_c\theta)\left[\frac{\beta pT}{\theta}+\left(\frac{\alpha e^{-aT}}{\theta(\theta-a)}-\frac{\beta p}{\theta^2}\right)\left(e^{\theta T}-1\right)+\frac{\alpha}{a(\theta-a)}(e^{-aT}-1)\right]\end{array}\right] \tag{10}$$

4. Mathematical Model for Manufacturer

As per the assumptions, the manufacturer, manufactures the products at a rate q_m per unit of time and θ is a deterioration rate per unit of time.

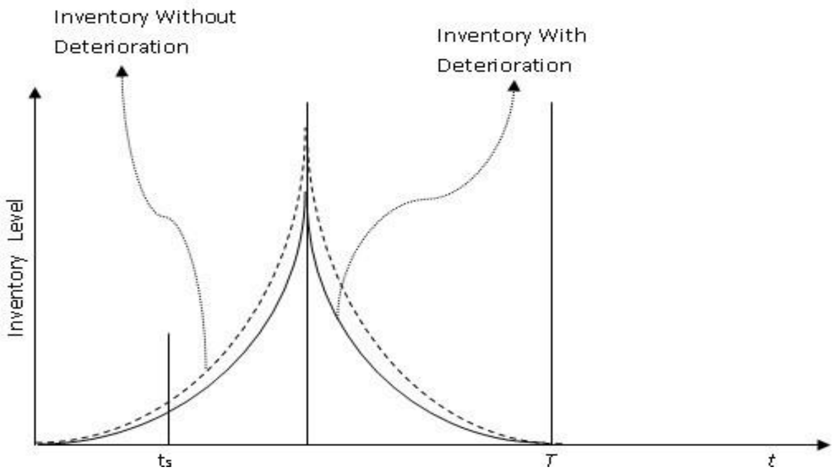

Figure 1: Inventory Level with Time

At any time, t the rate of change of the manufacturer's inventory is governed by the given ordinary differential equation. see (fig.1)

$$\frac{dI_m(t)}{dt}+\theta I_m(t)=q_m \quad ; 0\le t\le T \tag{11}$$

with boundary condition Im(ts) = 0, at t = ts and Im(t) = Q, at t = T (12)

Components of the manufacturer's average profit, in the cycle time [0, T] are as follows Equation (4.1) leads to

$$I_m(t)=\frac{rt}{\theta}-\frac{r}{\theta^2}+\frac{re^{\theta(t_s-t)}}{\theta^2}-\frac{t_s re^{\theta(t_s-t)}}{\theta} \tag{13}$$

1. Sales income in the cycle [0, T] is

$$S_m=cQ=\frac{\beta pc}{\theta}(1-e^{\theta T})-\frac{\alpha c}{(\theta-\alpha)}\left(1-e^{(\theta-\alpha)T}\right) \tag{14}$$

2. The production cost of product for manufacture is

$$PC_m=C_p\int_{t_s}^{T} r\,dt=\frac{C_p r}{2}(\mathrm{T}^2-t_s^2) \tag{15}$$

3. Raw material cost per lot for manufacture is

$$RMC=S_c \tag{16}$$

4. The deterioration cost of product per production cycle for a manufacturer is

$$DC_m=C_d\theta\int_{t_s}^{T} I_m(t)dt=C_d\theta\left[\frac{rT^2}{2\theta}-\frac{rT}{\theta^2}-\frac{re^{\theta(t_s-t)}}{\theta^2}\left(t_s-\frac{1}{\theta}\right)-\frac{rt_s^2}{2\theta}+\frac{r}{\theta^3}\right] \tag{17}$$

5. The holding cost of the product per production cycle for the manufacturer is

$$HC_m=h\int_{t_s}^{T} I_m(t)dt=h\left[\frac{rT^2}{2\theta}-\frac{rT}{\theta^2}-\frac{re^{\theta(t_s-t)}}{\theta^2}\left(t_s-\frac{1}{\theta}\right)-\frac{rt_s^2}{2\theta}+\frac{r}{\theta^3}\right] \tag{18}$$

Hence integrated average cost function for the manufacturer considering all incomes and expenditures, in the cycle time [0, T] is

$$T_m(T,p)=\left[\begin{array}{l}\frac{1}{T}\left(\frac{\beta pc}{\theta}(1-e^{\theta T})-\frac{\alpha c}{(\theta-\alpha)}\left(1-e^{(\theta-\alpha)T}\right)\frac{C_p r}{2}(T^2-t_s^2)\right)-\\ S_c-\frac{(h+C_d\,\theta)}{T}\left(\frac{rT^2}{2\theta}-\frac{rT}{\theta^2}-\frac{re^{\theta(t_s-t)}}{\theta^2}\left(t_s-\frac{1}{\theta}\right)-\frac{rt_s^2}{2\theta}+\frac{r}{\theta^3}\right)\end{array}\right] \quad (19)$$

5. Total Profit Function or Total Cost:

In this article we assumed that the manufacturer and retailer both are form a continuous two-layer supply chain inventory system and both two members are work together as a single unit. The profit of manufacturer and retailer both are affected simultaneously by retailing price, replenishment cycle time and initial lot size of product and therefore we can consider retail price, replenishment cycle time and initial lot size of product as the decision variables of whole supply chain inventory system. To analyzed the effects of decision variables we consider the following total average profit function of whole inventory supply chain system

$$TC=T_r(T,p)+T_m(T,p)$$

$$TC(T,p)=\left[\begin{array}{l}\left\{\begin{array}{l}\frac{p\alpha}{aT}(1-e^{\theta T})-\beta p^2-\frac{\beta pc}{T\theta}(1-e^{\theta T})+\frac{c\alpha}{T(\theta-\alpha)}\left(1-e^{(\theta-\alpha)T}\right)-\frac{O_c}{T}-\\ (h+D_c\theta)\left[\frac{\beta pT}{\theta}+\left(\frac{\alpha e^{-aT}}{\theta(\theta-a)}-\frac{\beta p}{\theta^2}\right)\left(e^{\theta T}-1\right)+\frac{\alpha}{a(\theta-a)}(e^{-aT}-1)\right]\end{array}\right\}\\ +\left\{\begin{array}{l}\frac{1}{T}\left(\frac{\beta pc}{\theta}(1-e^{\theta T})-\frac{\alpha c}{(\theta-\alpha)}\left(1-e^{(\theta-\alpha)T}\right)\frac{C_p r}{2}(T^2-t_s^2)\right)-\\ S_c-\frac{(h+C_d\,\theta)}{T}\left(\frac{rT^2}{2\theta}-\frac{rT}{\theta^2}-\frac{re^{\theta(t_s-t)}}{\theta^2}\left(t_s-\frac{1}{\theta}\right)-\frac{rt_s^2}{2\theta}+\frac{r}{\theta^3}\right)\end{array}\right\}\end{array}\right] \quad (20)$$

Proposition 5.1 The total supply chain profit function follows the jointly concavity properties with respect to retail price p and time horizon T.

Proof; The first order partial derivative of profit function *TC(T, p)* with respect to *T* and *p* respectively are given as

$$\frac{\partial TC(T,p)}{\partial T}=\left[\begin{array}{l}-\frac{C_p r}{2}\left(1+\frac{t_s^2}{T^2}\right)+\frac{(h+C_d\theta)}{T}\left(\frac{rT^2}{2\theta}-\frac{rT}{\theta^2}-\frac{re^{\theta(t_s-T)}}{\theta^2}\left(t_s-\frac{1}{\theta}\right)-\frac{rt_s^2}{2\theta}+\frac{r}{\theta^3}\right)\\ +\frac{O_c}{T^2}+\frac{(h+C_d\theta)}{T^2}\left\{\frac{rT^2}{2\theta}-\frac{rT}{\theta^2}-\frac{re^{\theta(t_s-T)}}{\theta^2}\left(t_s-\frac{1}{\theta}\right)-\frac{rt_s^2}{2\theta}+\frac{r}{\theta^3}\right\}\end{array}\right] \quad (21)$$

$$\frac{\partial TC(T,p)}{\partial p}=\left[\begin{array}{l}\frac{\alpha}{aT}(1-e^{\theta T})+(h+D_c\theta)\left\{\frac{\beta}{\theta}-\frac{\beta p}{T\theta^2}\left(e^{\theta T}-1\right)\right\}\\ -2\beta p-\frac{\beta c}{T\theta}\left(1-e^{\theta T}\right)+\frac{1}{T}\left\{\frac{\beta c}{\theta}\left(1-e^{\theta T}\right)\right\}\end{array}\right] \quad (22)$$

For surety that profit function *TC(T, p)* follows the joint concavity property with respect to *T*, and *p*, the Hessian matrix of profit *TC(T, p),* must be negative semi-definite

The hessian matrix of the given function is as follows

$$TC=\begin{bmatrix}\frac{\partial^2 TC(T,p)}{\partial T^2} & \frac{\partial^2 TC(T,p)}{\partial T\partial p}\\ \frac{\partial^2 TC(T,p)}{\partial p\partial T} & \frac{\partial^2 TC(T,p)}{\partial p^2}\end{bmatrix}=\begin{bmatrix}-\eta & 0\\ 0 & -2\beta\end{bmatrix} \quad (23)$$

If $\beta > 0$, and when $\eta > 0$, the Hessian matrix of profit function *TC(T, p),* is negative semi definite and hence profit πm follows the jointly concavity property with respect to T and p. Where η is

$$\eta=\left[\begin{array}{l}\frac{2(h+C_d\theta)}{T^2}\left(\frac{rT}{\theta}-\frac{r}{\theta^2}-\frac{re^{\theta(t_s-T)}}{\theta^2}\left(t_s-\frac{1}{\theta}\right)\right)-\frac{(h+C_d\theta)}{T}\left(\frac{r}{\theta}+re^{\theta(t_s-T)}\left(t_s-\frac{1}{\theta}\right)\right)\\+\frac{3C_prt_s^2}{T^3}-\frac{2O_c}{T^3}+\frac{2(h+C_d\theta)}{T^3}\left\{\frac{rT^2}{2\theta}-\frac{rT}{\theta^2}-\frac{re^{\theta(t_s-T)}}{\theta^2}\left(t_s-\frac{1}{\theta}\right)-\frac{rt_s^2}{2\theta}+\frac{r}{\theta^3}\right\}\end{array}\right]$$

Proposition 5.2 For optimum value of T, an unique optimum price p is given by p∗ where p∗ is

$$p^*=\frac{\left\{\frac{\alpha}{aT}(1-e^{\theta T})-(h+D_c\theta)\left(\frac{\beta}{\theta}-\frac{\beta p}{T\theta^2}\left(e^{\theta T}-1\right)\right)\right\}}{2\beta}$$

Proof: From equation (22)

If $p^*$is an optimal value of p, then we have

$$\frac{\partial TC(T,p)}{\partial p}=0$$

$$\frac{\partial TC(T,p)}{\partial p}=\left[\frac{\alpha}{aT}(1-e^{\theta T})+(h+D_c\theta)\left\{\frac{\beta}{\theta}-\frac{\beta p}{T\theta^2}\left(e^{\theta T}-1\right)\right\}\right]=0$$

After solving this we

$$p^*=\frac{\left\{\frac{\alpha}{aT}(1-e^{\theta T})-(h+D_c\theta)\left(\frac{\beta}{\theta}-\frac{\beta p}{T\theta^2}\left(e^{\theta T}-1\right)\right)\right\}}{2\beta}$$

if $\beta>0$

6. Sensitivity Analysis of Given Model:

For this model we analysis here two tables of data set are given**: (i)** Variation of total profit, when deterioration rate, replenishment cycle time and retail price are variable

(ii) Variation of total profit, when deterioration rate, replenishment cycle time are variable and retail price is fixed.

Table 1: Optimal outputs when the price is variable

Rate of θ per 100 units	Q	D(p, t)	TC	p	T	TCr	TCm
0.09	41.79	40.09	1166.41	16.91	0.94	479.81	686.60
0.11	43.42	41.20	1131.65	16.96	0.97	440.47	691.18
0.13	43.14	40.57	1151.03	17.02	0.96	455.16	695.88
0.15	43.29	40.34	1157.97	17.08	0.96	456.98	700.99
0.17	43.54	40.19	1162.23	17.12	0.96	455.86	706.37
0.19	43.80	40.06	1165.98	17.16	0.95	454.07	711.90
0.21	44.10	39.95	1169.05	17.20	0.95	451.46	717.59
0.23	44.40	39.84	1171.89	17.24	0.95	448.50	723.39
0.25	44.70	39.74	1174.53	17.27	0.95	445.24	729.28

Table 2: Optimal outputs when the price is fixed

Rate of θ per 100 units	Q	D(p, t)	TC	p	T	TCr	TCm
0.09	41.80	40.10	1166.38	16.91	0.94	479.76	686.62
0.11	43.59	41.36	1126.00	16.91	0.98	433.23	692.78
0.13	43.53	40.94	1138.08	16.91	0.96	438.29	699.79
0.15	43.86	40.85	1139.25	16.91	0.96	432.52	706.73
0.17	44.25	40.83	1138.84	16.91	0.96	425.09	713.75

0.19	44.67	40.82	1137.92	16.91	0.96	417.07	720.85
0.21	45.10	40.81	1136.61	16.91	0.96	408.56	728.05
0.23	45.54	40.80	1135.28	16.91	0.96	399.94	735.33
0.25	45.98	40.80	1133.93	16.91	0.96	391.212	742.72

1. By the observation of data table-1 shows, when price p is fixed, it is clear that increment of deterioration rate θ, increases the lot size, retail price of the product, and the profit of manufacturer as well as total profit, while decreases the retailer's profit and replenishment cycle time in the interval 0.13 to 0.25. Furthermore, fluctuate the demand of products, profit of retailers and total profit in the interval 0.09 to 0.13.

2. Similarly, observation of data table-2 shows, the increment of deterioration rate, increases the lot size of the product, the profit of the manufacturer, and the total profit, while decreasing the demand and the profit of the retailer in the interval 0.13 to 0.25, moreover all outputs fluctuate in the interval 0.09 to 0.13. Analysis shows that only the retailer's profit is affected by the deterioration rate of the product in this type of market situation.

Table 3: Optimal outputs when the price is variable

% variation of α	Q	D(p, t)	TCr	TCm	TC	T	p
- 20 %	38.95	37.17	-15.33	558.17	542.84	1.07	13.01
-15%	39.68	37.91	95.66	589.75	685.41	1.04	13.98
-10%	40.39	38.65	215.2	621.33	836.53	1.00	14.96
-5%	41.09	39.37	343.47	652.90	996.37	0.97	15.93
0%	41.80	40.10	479.81	684.48	1164.30	0.94	16.91
5%	42.48	40.80	625.13	716.05	1341.19	0.92	17.89
10%	43.17	41.50	778.11	747.63	1525.75	0.89	18.86
15%	43.85	42.19	939.88	779.21	1719.09	0.87	19.84
20%	44.51	42.88	1109.7	810.79	1920.50	0.85	20.82

Observation of table 3. shows that increment of the base parameter α increases the initial lot size of the product, the profits of manufacturer and retailer, and retail price, while decreasing the time horizon T. Furthermore, increment of base demand is comparatively more beneficial for the retailer because due to increases manufacturing expenditure profit of manufacturer not increase comparatively.

Table 4: Optimal outputs when the price is variable

% variation of β	Q	D(p, t)	TC	T	TCr	TCm	p
- 20 %	36.14	34.83	1669.10	0.83	1000.52	668.58	21.80
-15%	37.56	36.16	1525.56	0.86	852.93	672.63	20.36
-10%	38.99	37.50	1394.50	0.89	717.87	676.63	19.08
-5%	40.40	38.80	1274.72	0.92	594.14	680.58	17.9
0%	41.80	40.09	1164.30	0.94	479.81	684.48	16.91
5%	43.19	41.38	1061.96	0.97	373.61	688.35	15.98
10%	44.56	42.65	966.68	0.99	274.50	692.18	15.14
15%	45.95	43.93	876.98	1.02	180.99	695.99	14.37
20%	47.30	45.18	793.42	1.047	93.66	699.76	13.66

Increment of price sensitive parameter β increases the initial lot size and time horizon T, however, decreases the total profit and retail price p. Analysis of data table 4 shows that for present market situations, increasing the retail price is harmful to the retailer's profit but beneficial to the manufacturer's profit.

Table 5: Optimal outputs when the price is variable

% variation of a	Q	D(p, t)	TC	p	T	TCr	TCm
- 20 %	43.10	41.30	1164.31	17.08	0.96	473.39	690.92
-15%	43.72	41.87	1137.52	17.02	0.98	447.96	689.56
-10%	37.57	36.20	1297.37	17.06	0.84	611.61	685.76
-5%	41.27	39.61	1188.21	16.97	0.93	502.36	685.84
0%	41.80	40.10	1164.42	16.91	0.94	479.94	684.48
5%	41.97	40.25	1150.47	16.86	0.95	467.46	683.00
10%	42.00	40.29	1140.29	16.81	0.95	458.79	681.5
15%	42.03	40.30	1130.63	16.77	0.95	450.64	679.99
20%	42.04	40.32	1121.12	16.72	0.96	442.65	678.48

7. Numerical Analysis:

The numerical example is as follows by giving the input value of a used parameter; $\alpha = 80$, $\beta = 2$, $h = 0.05$, $\theta = 0.09$, $a = 0.1$, $r = 1$, $c = 16$, $Oc = 125$, $Cp = 12$, $Sc = 20$, $Cd = 4$, $ts = 0.1$ then

Table: 6 Optimal values of the following parameters

p*	T*	Q	Average profit of the retailer	Average profit of the manufacturer	Average Total Profit
16.91	0.94	41.80	479.81	648.48	1164.30

8. Graphical Analysis of the proposed Model:

Retailer’s profit decreases marginally if shape parameter ’a’ increases while all other outputs show no significant changes, however total profit of the supply chain shows the maximum value at ’a’=0.095 has been shown in figure 2.

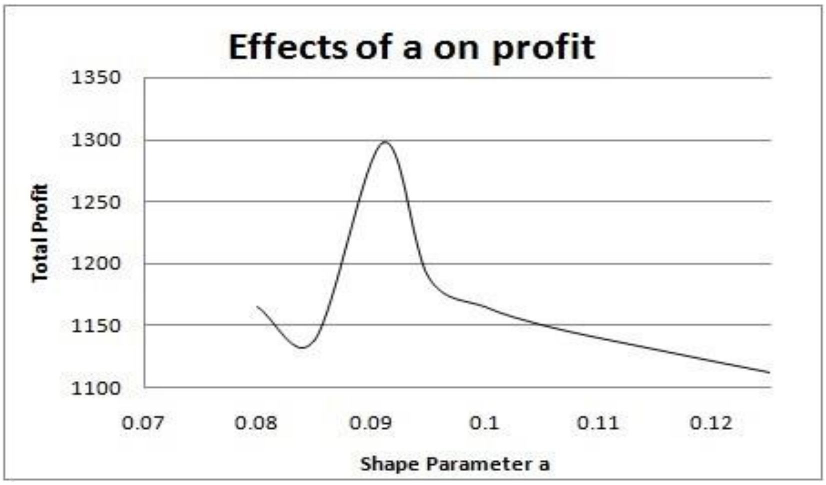

Figure 2: Effect of ‘**a**’ on Total Profit

9. Conclusion

Worldwide markets offer sell openings and posture production the executive’s challenges for makers of deteriorating things. Grasping the advantage of the distinction in planning the selling period of the deteriorating things at various markets offers a unique chance to work on the benefit of a deteriorating producer. The article presents an integrated supply chain model for deteriorating seasonal products and we have found the optimum retail price, initial lot size, and time cycle for obtaining maximum average total profit. By sensitive analysis we have remarked that the deterioration property of the product is more effective on the profit of the retailer as compared to the manufacturer in the present market situations, therefore it is advised to retailers to order small lot sizes of highly deteriorating seasonal products. For production management, it is also advised to efficiently coordinate with the retailer for obtaining more profit, because fixed price-dependent outputs have values better than variable price-dependent outputs values. One can extend this model by considering a multilayer supply chain or using preservation technology. The model can also be extended in a fuzzy environment. Future analysts can conduct research about multi-thing, retail swelling rate, perishability, or discount markdown. One can also consider a multi-echelon supply chain model with setting up cost reduction.

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