ANALYTIC FUNCTION IN COMPLEX VARIABLES

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ABSTRACT
This study has been undertaken complex variable theory. Analytic functions are defined; it is shown that they satisfy the Cauchy-Riemann equations. Branch point singularities and branch cuts are defined; their effect on values of a multiple-valued function is analyzed. Poles and essential singularities are identified as additional singularities. Taylor and Laurent series are introduced and their respective regions of convergence are described. Contour integrals are defined; Cauchy’s theorem and Cauchy’s integral formula are stated and proved.

KEY WORDS

1. INTRODUCTION

In this Paper, We have study about the complex variables which is the theory of analytic functions of a complex variable. The polynomial equation whose highest degree is two is called a quadratic equation or sometimes just quadratics. It is expressed in the form of:

ax² + bx + c = 0

Where x is the unknown variable and a, b and c are the constant terms.

If you’ve done any quadratic equations, you’ll know that there is a nice formula for the solution of the quadratic equation ax²+bx+c=0, given by:

X=−b±√b²−4ac2a

However, you’ll also know that if b²−4ac< 0 then there is no solution to the quadratic equation.

DEFINITION 1.1
Complex numbers are numbers that consist of two parts a real number and an imaginary number. Complex numbers are the numbers that are expressed in the form of a+ib where, a,b are real numbers and ‘i’ is an imaginary number called “iota”. The value of i= (√-1).

DEFINITION 1.2
The complex conjugate, or simply the conjugate, of a number z= x+iy is defined as the complex number x−iy and is denoted by ̅

ie) ̅=x−iy.

DEFINITION 1.3
A function f is though to f in this way, it is often referred to as a mapping, or transformation.

The image of a point z in the domain of definition S is the point w=f(z), and these to f images of all points in a set T that is contained in S is called the image of T.
DEFINITION 1.4
The image of the entire domain of definition S is called the range of f.

DEFINITION 1.5
Let a function f be defined at all points z in some deleted neighborhood (Sec. 11) of z_0.

The statement that the limit of f(z) as z approaches z_0 is a number w_0, or that
Lim f(z)= w_0, 
Z→z_0

Means that the point w=f(z) can be made arbitrarily close to w_0 if we choose the point z close enough to z_0 but distinct from it. We now express the definition of limit in a precise and usable form.

Statement means that for each positive number ε, there is a positive number δ such that

|f(z)−w_0|<ε, whenever 0<|z−z_0|<δ.

DEFINITION 1.6
A function f is continuous at a point z_0 if all three of the following conditions are satisfied

a. Lim f(z) exists,
Z→z_0

b. f(z_0) exists,
c. lim f(z)=f(z_0).
Z→z_0

DEFINITION 1.7
A real-valued function H of two real variables x and y is said to be harmonic in a given domain of the xy plane if, throughout that domain, it has continuous partial derivatives of the first and second order and satisfies the partial differential equation
H_{xx}(x,y)+H_{yy}(x,y)=0,
Known as Laplace’s equation

2. BASIC PROPERTIES ON INTRODUCE IN COMPLEX VARIABLES

PROPERTIES 2.1
The inverse image of a point w is these to f all points z in the domain of definition of f that have w as their image.

The commutative laws

z_1+z_2=z_2+z_1,
z_1z_2=z_2z_1

Associative laws

(z_1+z_2)+z_3=z_1+(z_2+z_3),
(z_1z_2)z_3=z_1(z_2z_3)

Follow easily from the definitions of addition and multiplication of complex numbers and the fact that real numbers obey these laws. For example, if
\[ z_1 = (x_1, y_1) \quad \text{and} \quad z_2 = (x_2, y_2). \]

\[ z_1 + z_2 = (x_1 + x_2, y_1 + y_2) \]
\[ = (x_2 + x_1, y_2 + y_1) \]
\[ = z_2 + z_1. \]

Verification of the rest of the above laws, as well as the distributive law

\[ z(z_1 + z_2) = zz_1 + zz_2 \]

Also, because of the associative laws, a sum \( z_1 + z_2 + z_3 \) or a product \( z_1 z_2 z_3 \) is well defined without parentheses, as is the case with real numbers.

3. BASIC THEOREMS ON ANALYTIC FUNCTION IN COMPLEX VARIABLES

THEOREM 3.1

If a function \( f(z) = u(x, y) + iv(x, y) \) is analytic in a domain \( D \), then its component functions \( u \) and \( v \) are harmonic in \( D \).

Assuming that \( f \) is analytic in \( D \), we start with the observation that the first-order partial derivatives of its component functions must satisfy the Cauchy–Riemann equations throughout \( D \):

\[ u_x = v_y, \quad u_y = -v_x. \]

Differentiating both sides of these equations with respect to \( x \), we have

\[ u_{xx} = v_{yx}, \quad u_{yx} = -v_{xx}. \]

Differentiation with respect to \( y \) yields

\[ u_{xy} = v_{yy}, \quad u_{yy} = -v_{xy}. \]

Now, by a theorem in advanced calculus, the continuity of the partial derivatives of \( u \) and \( v \) ensures that

\[ u_{xx} = u_{xy} \quad \text{and} \quad v_{yx} = v_{xy}. \]

It then follows from equations

\[ u_{xx} + u_{yy} = 0 \quad \text{and} \quad v_{xx} + v_{yy} = 0. \]

That is, \( u \) and \( v \) are harmonic in \( D \).

THEOREM 3.2

A function \( f(z) = u(x, y) + iv(x, y) \) is analytic in a domain \( D \). If and only if \( v \) is a harmonic conjugate of \( u \).

THEOREM 3.3

Suppose that

(a) A function \( f \) is analytic throughout a domain \( D \);
(b) \( f(z) = 0 \) at each point \( z \) of a domain or line segment contained in \( D \). Then \( f(z) \equiv 0 \) in \( D \);

that is, \( f(z) \) is identically equal to zero throughout \( D \).

THEOREM 3.4

In this section, we obtain a pair of equations that the first-order partial derivatives of the component functions \( u \) and \( v \) of a function

\[ f(z) = u(x, y) + iv(x, y) \]

must satisfy at a point \( z_0 = (x_0, y_0) \) when the derivative of \( f \) exists there.

We also show to express \( f'(z_0) \) in terms of those partial derivatives.

We write

\[ z_0 = x_0 + iy_0, \quad \Delta z = \Delta x + i\Delta y, \]

and

\[ \Delta w = f(z_0 + \Delta z) - f(z_0) \]
\[ = [u(x_0 + \Delta x, y_0 + \Delta y) - u(x_0, y_0)] + i[v(x_0 + \Delta x, y_0 + \Delta y) - v(x_0, y_0)]. \]
THEOREM 3.5
Suppose that \( f(z) = u(x,y) + iv(x,y) \) and that \( f'(z) \) exists at a point \( z_0 = x_0 + iy_0 \). Then the first-order partial derivatives of \( u \) and \( v \) must exist at \( (x_0, y_0) \), and they must satisfy the Cauchy–Riemann equations \( u_x = v_y, \quad u_y = -v_x \) there.
Also, \( f'(z_0) \) can be written
\[
f'(z_0) = u_x + iv_x,
\]
where these partial derivatives are to be evaluated at \( (x_0, y_0) \).

4. BASIC EXAMPLES ON ANALYTIC FUNCTION IN COMPLEX VARIABLES

EXAMPLE 4.1
Example for definition 1.1
2+3i is a complex number, where 2 is a real number (Re) and 3i is an imaginary number (Im).

\[
a = \text{Re}z, \quad b = \text{Im}z.
\]

\[
2 = \text{Re}z, \quad 3 = \text{Im}z.
\]

EXAMPLE 4.2
Theorem 3.1 based problem show that the function \( f(z) = z^2 = x^2 - y^2 + i2xy \) is differentiable every where and That \( f'(z) = 2z \).

Here \( u(x,y) = x^2 - y^2 \) and \( v(x,y) = 2xy \).

\[
\begin{align*}
  u_x &= 2x, & v_x &= 2y \\
  u_y &= -2y, & v_y &= 2x
\end{align*}
\]
Cauchy–Riemann equations

\[
u_x = v_y, \quad \text{and} \quad u_y = -v_x
\]

\[
u_x = 2x = v_y, \quad \text{and} \quad u_y = -2y = -v_x
\]
The Cauchy–Riemann equations are satisfied everywhere

The derivative
\[
f'(z) = 2x + i2y
\]
\[
= 2(x + iy)
\]
\[
= 2z
\]

5. CONCLUSION
In this paper, we present function of complex number with basic concepts, definition, theorems and analytic function definition and theorems. We have discuss this concept with simple problems.
REFERENCE