A Study On Fuzzy $\sigma$ - Baire Space

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Abstract
In this paper, the concepts of fuzzy $\sigma$-Baire spaces are introduced and characterizations of fuzzy $\sigma$-Baire spaces are studied. Several examples are given to illustrate the concepts introduced in this paper.

Keywords: Fuzzy $F_{\sigma}$-set, fuzzy $G_{\delta}$-set, fuzzy nowhere dense set, Fuzzy $\sigma$- nowhere dense set, Fuzzy $\sigma$-first category, Fuzzy $\sigma$-second category and Fuzzy $\sigma$-Baire spaces.

1. INTRODUCTION

The fuzzy concept has invaded almost all branches of Mathematics ever since the introduction of fuzzy set by L. A. ZADEH. The theory of fuzzy topological spaces was introduced and developed by C. L. CHANG. Since then much attention has been paid to generalize the basic concepts of General Topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. In recent years, fuzzy topology has been found to be very useful in solving many practical problems. In El.Naschie showed that the notion of fuzzy topology might be relevant to Quantum Particle Physics in connection with String Theory. It has been shown that the fuzzy Kähler manifolds which are based on a topology, play the important rule in theory. In this paper we introduce the concepts of fuzzy $\sigma$-Baire spaces. Also we discuss several characterizations of fuzzy $\sigma$-Baire spaces.

Several examples are given to illustrate the concepts introduced in this paper.

2. PRELIMINARIES

By a fuzzy topological space we shall means non-empty set $X$ together with a fuzzy topology $T$ (in the sense of Chang) and denote it by $(X, T)$.

DEFINITION: 2.1

Let $\lambda$ and $\mu$ be any two fuzzy sets in $(X, T)$. Then we define $(\lambda \vee \mu) : X \rightarrow [0, 1]$ as follows: $(\lambda \vee \mu) (x) = \max \{ \lambda (x), \mu (x) \}$. Also we define $(\lambda \wedge \mu) : X \rightarrow [0, 1]$ as follows: $(\lambda \wedge \mu) (x) = \min \{ \lambda (x), \mu (x) \}$.

DEFINITION: 2.2

Let $(X, T)$ be any fuzzy topological space and $\lambda$ be any fuzzy set in $(X, T)$. We define $\text{Cl}(\lambda) = \{ \mu / \lambda \leq \mu, 1-\mu \in T \}$ and $\text{int}(\lambda) = \{ \mu / \mu \leq \lambda, \mu \in T \}$.

For any fuzzy set in a fuzzy topological space $(X, T)$, it is easy to see that $1 - \text{cl}(\lambda) = \text{int}(1 - \lambda)$ and $1 - \text{int}(\lambda) = \text{cl}(1 - \lambda)$.
DEFINITION : 2.3

Let \((X, T)\) and \((Y, S)\) be any two fuzzy topological spaces. Let \(f\) be a function from the fuzzy topological space \((X, T)\) to the fuzzy topological space \((Y, S)\).

Let \(\lambda\) be a fuzzy set in \((Y, S)\). The inverse image of \(\lambda\) under \(f\) written as \(f^{-1}(\lambda)\) is the fuzzy set in \((X, T)\) defined by:

\[
f(\lambda)(y) = \sup \{\lambda(x) \mid x \in f^{-1}(y)\} \quad \text{if } f^{-1}(y) \text{ is non-empty ;for each } y \in Y.
\]

\[
0 \quad \text{otherwise.}
\]

DEFINITION : 2.4

A fuzzy set \(\lambda\) in a fuzzy topological space \((X, T)\) is called fuzzy dense if there exists no fuzzy closed set \(\mu\) in \((X, T)\) such that \(\lambda < \mu < 1\).

DEFINITION : 2.5

A fuzzy set \(\lambda\) in a fuzzy topological space \((X, T)\) is called a fuzzy \(F_\sigma\)-set in \((X, T)\) if \(\lambda = \bigvee_{i=1}^{\infty} \lambda_i\), where \(\lambda_i \in T\) for \(i \in I\).

DEFINITION : 2.6

A fuzzy set \(\lambda\) in a fuzzy topological space \((X, T)\) is called a fuzzy \(G_\delta\)-set in \((X, T)\) if \(\lambda = \bigwedge_{i=1}^{\infty} \lambda_i\), where \(\lambda_i \in T\) for \(i \in I\).

DEFINITION : 2.7

A fuzzy set \(\lambda\) in a fuzzy topological space \((X, T)\) is called fuzzy semi-open if \(\lambda \leq \text{cl}_{\text{int}}(\lambda)\). The complement of \(\lambda\) in \((X, T)\) is called a fuzzy semi-closed set in \((X, T)\).

DEFINITION : 2.8

A fuzzy topological space \((X, T)\) is called an fuzzy open hereditarily irresolvable space if \(\text{int} \text{cl}(\lambda)\) is not equal to 0, then \(\text{int}(\lambda)\) is not equal to 0 for any non-zero fuzzy set in \((X, T)\).

DEFINITION : 2.9

A fuzzy topological space \((X, T)\) is called fuzzy first category if the fuzzy set \(1_X\) is a fuzzy first category set in \((X, T)\). That is, \(1_X = \bigvee_{i=1}^{\infty} (\lambda_i)\) where \(\lambda_i\)'s are fuzzy nowhere dense sets in \((X, T)\). Otherwise, \((X, T)\) will called a fuzzy second category space.

3. FUZZY \(\sigma\)-NOWHERE DENSE SETS

DEFINITION : 3.1

Let \((X,T)\) be a fuzzy topological space. A fuzzy set \(\lambda\) in \((X, T)\) is called a fuzzy \(\sigma\)-nowhere dense set if \(\lambda\) is a fuzzy \(F_\sigma\)-set in \((X, T)\) such that \(\text{int}(\lambda) = 0\).

EXAMPLE : 3.1

Let \(X = \{a, b, c\}\). The fuzzy sets \(\lambda, \mu\) and are defined on \(X\) as follows:

- \(\lambda: X \rightarrow [0, 1]\) is defined as \(\lambda(a) = 0.3; \lambda(b) = 0.7; \lambda(c) = 0.4\).
- \(\mu: X \rightarrow [0, 1]\) is defined as \(\mu(a) = 0.5; \mu(b) = 0.4; \mu(c) = 0.8\).
- \(\alpha: X \rightarrow [0, 1]\) is defined as \(\alpha(a) = 0.2; \alpha(b) = 0.4; \alpha(c) = 0.6\).
Then, \( T = \{0, \lambda, \mu, \alpha, \lambda \vee \mu, \lambda \wedge \alpha, \lambda \wedge \mu, \alpha \vee (\lambda \wedge \mu), 1\} \) is clearly a fuzzy topology on \( X \). Now consider the fuzzy set \( \beta = [1-(\lambda \vee \mu)] \vee [1-(\lambda \wedge \alpha)] \) in \((X, T)\). Then \( \beta \) is a fuzzy \( \mathcal{F}\sigma \)-set in \((X, T)\) and \( \text{int}(\beta)=0 \) and hence \( \beta \) is a fuzzy \( \sigma \)-nowhere dense set in \((X, T)\). The fuzzy set \( \gamma = (1-\lambda) \vee (1-\mu) \vee (1-\alpha) \) is a fuzzy \( \mathcal{F}\sigma \)-set in \((X, T)\) and \( \text{int}(\lambda) = 0 \) and hence \( \gamma \) is not a fuzzy \( \sigma \)-nowhere dense set in \((X, T)\).

**DEFINITION : 3.2**

Let \( \lambda \) be a fuzzy \( \sigma \)-first category set in \((X, T)\). Then \( 1-\lambda \) is called a fuzzy \( \sigma \)-residual set in \((X, T)\).

**PROPOSITION : 3.1**

If \( \lambda \) is a fuzzy dense set in \((X, T)\) such that \( \mu \leq (1-\lambda) \), where \( \mu \) is a fuzzy \( \mathcal{F}\sigma \)-set in \((X, T)\), then \( \mu \) is a fuzzy \( \sigma \)-nowhere dense set in \((X, T)\).

**PROOF:**

Let \( \lambda \) be a fuzzy dense set in \((X, T)\) such that \( \mu \leq (1-\lambda) \). 

Now \( \mu \leq (1-\lambda) \) 

\[ \text{int}(\mu) \leq \text{int}(1-\lambda) = 1 - \text{cl}(\lambda) = 1 - 1 = 0 \]

& hence \( \text{int}(\mu) = 0 \).

Therefore \( \mu \) is a fuzzy \( \sigma \)-nowhere dense set in \((X, T)\).

**PROPOSITION : 3.2**

If \( \lambda \) is a fuzzy \( \mathcal{F}\sigma \)-set and fuzzy nowhere dense set in \((X, T)\), then \( \lambda \) is a fuzzy \( \sigma \)-nowhere dense set in \((X, T)\).

**PROOF :**

Let \( \lambda \) is a fuzzy \( \mathcal{F}\sigma \)-set and fuzzy nowhere dense set in \((X, T)\).

Now \( \lambda \leq \text{cl}(\lambda) \) for any fuzzy set in \((X, T)\).

Then, \( \text{int}(\lambda) \leq \text{intcl}(\lambda) \).

Since \( \lambda \) is a fuzzy nowhere dense set in \((X, T)\), 

\[ \text{intcl}(\lambda) = 0 \]

& hence \( \text{int}(\lambda) = 0 \) and \( \lambda \) is a fuzzy \( \mathcal{F}\sigma \)-set 

Therefore \( \lambda \) is a fuzzy \( \sigma \)-nowhere dense set in \((X, T)\).

**4. FUZZY \( \sigma \)-BAIRE SPACE**

**DEFINITION : 4.1**

Let \((X, T)\) be a fuzzy topological space. Then \((X, T)\) is called a fuzzy \( \sigma \)-Baire Space if \( \text{int}\left(\bigvee_{i=0}^{\infty} (\lambda_i)\right) = \emptyset \) where \( \lambda_i \) are fuzzy \( \sigma \)-no where dense sets in \((X,T)\).

**PROPOSITION : 4.1**

If the fuzzy topological space \((X, T)\) is a fuzzy Baire space and if the fuzzy nowhere dense sets in \((X, T)\) are fuzzy \( \mathcal{F}\sigma \)-sets in \((X, T)\), then \((X, T)\) is a fuzzy\( \sigma \)-Baire Space.
Let \((X, T)\) be a fuzzy \(\sigma\)-Baire space such that every fuzzy nowhere dense set \(\lambda_i\) is a fuzzy \(F\sigma\)-set in \((X, T)\).

Then, \(\text{int}(\bigvee_{i=0}^{\infty} (\lambda_i)) = 0\) where \(\lambda_i\)s are fuzzy no where dense sets in \((X, T)\).

By Proposition : 3.2
\(\lambda_i\)s is a fuzzy \(\sigma\)-nowhere dense set in \((X, T)\).

Hence \(\text{int}(\bigvee_{i=1}^{\infty} (\lambda_i)) = 0\) where \(\lambda_i\)s are fuzzy \(\sigma\)-nowhere dense set in \((X, T)\).

Therefore \((X, T)\) is a fuzzy \(\sigma\)-Baire Space.

5. FUNCTIONS AND FUZZY \(\sigma\)-BAIRE SPACES

DEFINITION : 5.1

A function \(f: (X, T) \rightarrow (Y, S)\) from a fuzzy topological space \((X, T)\) into another fuzzy topological space \((Y, S)\), is said to be fuzzy open if the image of every fuzzy open set in \((X, T)\), is fuzzy open in \((Y, S)\).

Definition : 5.2

A function \(f: (X, T) \rightarrow (Y, S)\) from a fuzzy topological space \((X, T)\) into another fuzzy topological space \((Y, S)\), is called fuzzy contra-continuous if \(f^{-1}(\lambda)\) is fuzzy closed (open) in \((X, T)\), for each fuzzy open (closed) set \(\lambda\) in \((Y, S)\).

Proposition : 5.1

If \(f: (X, T) \rightarrow (Y, S)\) is an fuzzy contra-continuous and fuzzy open function from a topological space \((X, T)\) onto a fuzzy open hereditarily irresolvable space \((Y, S)\) then \((Y, S)\) is a fuzzy \(\sigma\)-Baire space.

Conclusion

In this Paper, We studied conditions under which a fuzzy topological spaces become fuzzy \(\sigma\)-Baire space.

REFERENCES


