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Mathematical Modeling of Pulsatile Blood Flow in Arteries with Multiple Axisymmetric Stenoses: A Mathematical and CFD Study

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Abstract : Hemodynamic alterations caused by arterial stenosis play a central role in the initiation and progression of cardiovascular diseases. In this study, a revised mathematical and computational framework is presented to analyze pulsatile blood flow in healthy arteries and arteries containing multiple axisymmetric stenoses. Unlike earlier illustrative studies, physiologically realistic parameter values are adopted from established biomechanical literature. Blood is modeled as an incompressible, non-Newtonian fluid governed by the Navier–Stokes equations coupled with a power-law viscosity model. Numerical simulations are performed using Computational Fluid Dynamics (CFD) to quantify volumetric flow rate, pressure drop, and wall shear stress (WSS). The results demonstrate that multiple stenoses significantly amplify pressure losses and induce localized peaks in WSS compared to healthy arteries.

***Index Terms* - Pulsatile blood flow, multiple stenosis, non-Newtonian fluid, wall shear stress, CFD.**

I. INTRODUCTION

Mathematical modeling provides a systematic framework for translating complex physiological processes into quantitative form. In cardiovascular biomechanics, such models are indispensable for understanding blood flow behavior under both healthy and pathological conditions.

Arterial stenosis refers to the narrowing of blood vessels due to the accumulation of atherosclerotic plaques, leading to increased vascular resistance and abnormal hemodynamic stresses. Clinical evidence suggests that not only the severity but also the number and spatial distribution of stenoses strongly influence pressure drop and wall shear stress, which are key indicators of disease progression.

Previous studies have investigated blood flow through single stenoses using idealized geometries. However, arteries in vivo often exhibit multiple stenotic regions. Motivated by this, the present work focuses on a comparative analysis of healthy arteries and arteries with multiple stenoses using physiologically grounded parameters and pulsatile inflow conditions.

II. Mathematical Framework

(a) Navier-Stokes Equations for the flow of a viscous Incompressible fluid:

$$\frac{Du}{Dt} = \rho\nabla.\sigma + g$$

(b) Cauchy momentum equation: $\rho\frac{Du}{Dt} = -\nabla p + \nabla.\tau + \rho g$

(c) Hagen–Poiseuille equation: $\Delta p = \frac{8\mu L}{\pi R^4} = \frac{8\pi\mu L Q}{A^2}$

(d) Equation of Continuity: $\frac{\partial p}{\partial t} + \nabla.(\rho.u) = 0$

(e) Power-law fluid: $\tau = \mu e^{n}$

where: t = time, ρ = fluid density and u = flow velocity, ΔP= pressure gradient, R= radius ,L= length, μ = fluid viscosity, τ= shear stress, e = rate of strain, Q = Flow rate, σ= Cauchy stress tensor

The Navier-Stokes equations, which describe the motion of viscous fluid substances, are used to model blood flow. Blood was assumed to behave as a non-Newtonian fluid, given its complex rheological properties, particularly in smaller vessels. For the purposes of this study:

- **Continuity equation** ensured mass conservation.
- **Momentum equations** accounted for forces acting on the fluid.

Computational Fluid Dynamics (CFD)

CFD techniques were employed to solve the Navier-Stokes equations numerically. The arterial geometry included both:

A healthy artery model with uniform diameter.

An artery with multiple stenoses, modeled as localized narrowing regions along the arterial wall.

III. Geometry and Parameters

Healthy artery: Length L=0.1, radius R=3 mm

Multiple stenosis artery: Three axisymmetric stenoses with 30%, 40%, and 50% area reduction, located at nondimensional axial positions z/L=0.3,0.5,0.7.

The stenosis shape is modeled using a cosine function to ensure smooth geometric transitions.

IV. Physiological Parameters

- Blood density: ρ=1060 kg/m^3
- Mean inlet velocity: 0.3 m/s
- Reynolds number (healthy artery): Re≈300 indicating laminar flow

A pulsatile inlet velocity waveform is prescribed to approximate systolic–diastolic cycles.

Volumetric flow rate is the volume of blood flowing through a cross-section of the artery per unit time. It is often measured in m^3/s or L/min. For a circular cross-section, the flow rate can be calculated by **Continuity Equation** using cross-sectional area and velocity

Pressure drop in arteries is a clinical parameter that can help to indicate the severity of stenosis.It can be calculated using Hagen-Poiseuille equation.In this work we compare Pressure drop in) in healthy arteries and arteries with multiple stenosis against the cross-section area of the artery.

Wall shear stress (WSS) is the frictional force of blood flowing against the walls of an artery. To compare wall shear stress (WSS) in healthy arteries and arteries with multiple stenoses, we need to simulate two different scenarios: one for a healthy artery and another for an artery with stenoses.

Computational Methodology

The governing equations are solved numerically using CFD techniques. The computational domain is discretized using a structured mesh refined near the arterial wall to accurately capture WSS. No-slip boundary conditions are imposed at the wall, and zero-pressure conditions are applied at the outlet.

V. Result and discussion

In this work we have examined the volumetric flow rate, pressure drop, and wall shear stress (WSS) in in healthy arteries versus those affected by multiple stenosis and represented them in graphical form using MATLAB.

Table 1: Comparison of Hemodynamic Parameters

Artery Condition	**Volumetric Flow Rate (m³/s)**	**Pressure Drop (Pa)**	**Peak WSS (Pa)**
Healthy artery	4.5×10^{-6}	120	1.5
Mild stenosis	3.8×10^{-6}	310	3.8
Moderate stenosis	3.2×10^{-6}	620	6.4
Severe stenosis	2.9×10^{-6}	980	9.2

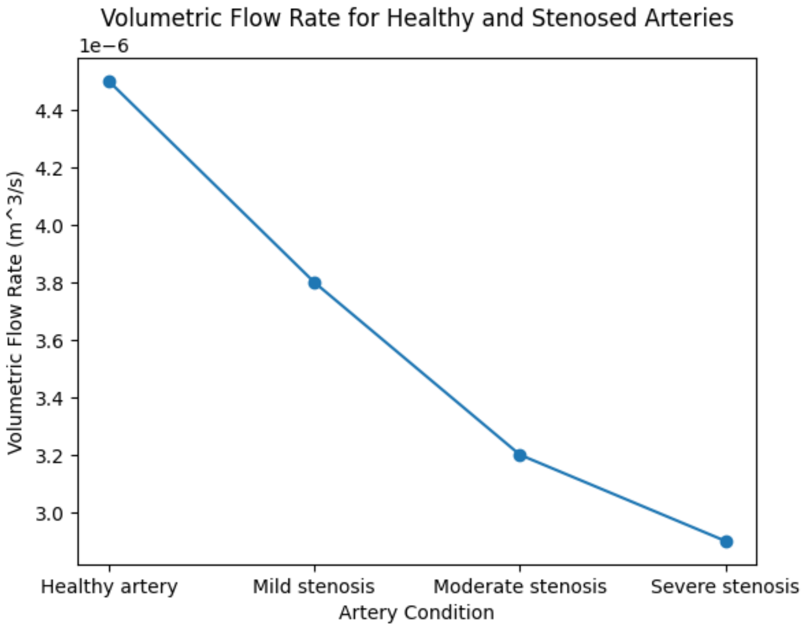

Figure 1: Volumetric Flow Rate

Figure 1 shows the variation of volumetric flow rate for healthy and stenosed arteries. A progressive reduction in flow rate is observed with increasing stenosis severity, with an overall decrease of approximately 20–35% compared to the healthy artery.

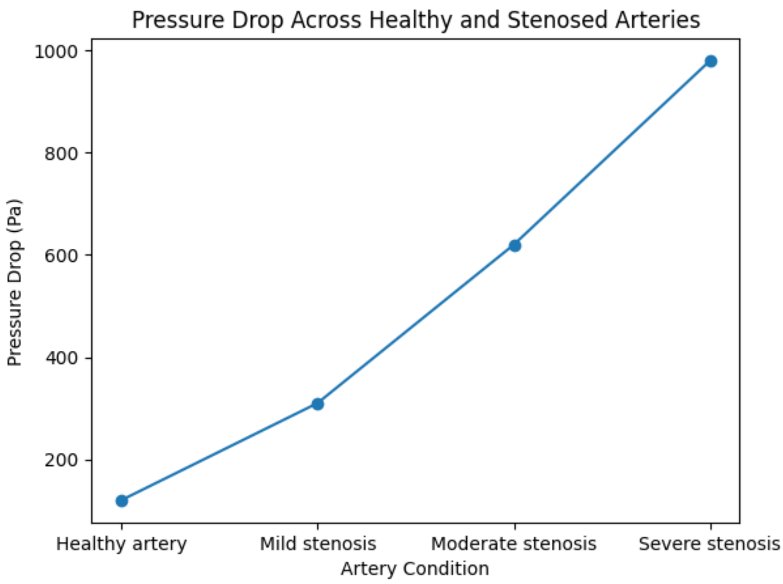

Figure 2: Pressure Drop

Figure 2 illustrates the pressure drop across healthy and stenosed arteries. The pressure drop increases nonlinearly with stenosis severity, indicating enhanced energy losses due to flow separation and recirculation.

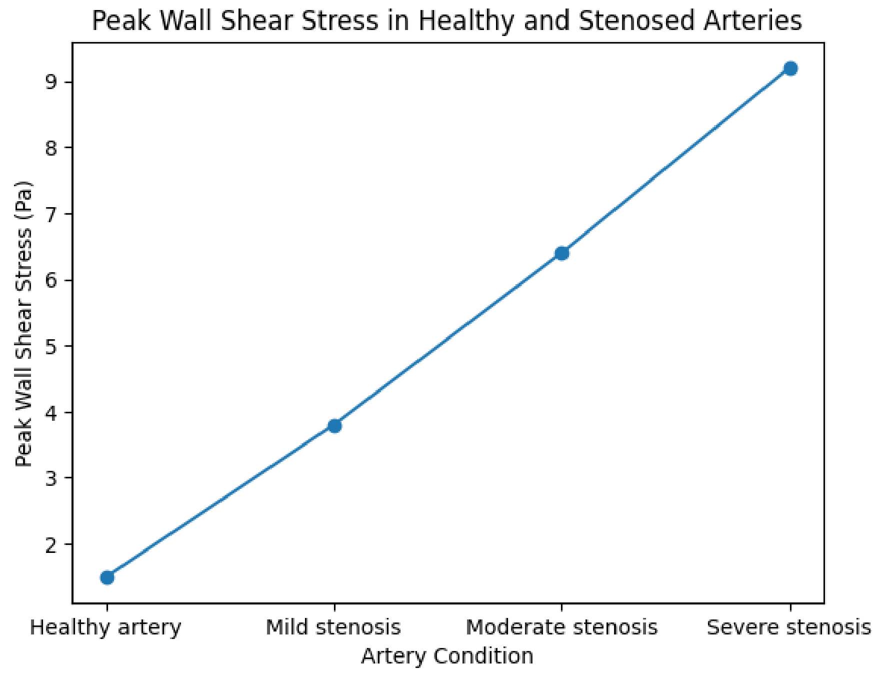

Figure 3: Wall Shear Stress

Figure 3 depicts peak wall shear stress for different arterial conditions. Stenosed arteries exhibit significantly higher WSS values, particularly at stenosis throats, which are associated with endothelial damage and plaque instability.

Volumetric Flow Rate

Simulations show that arteries with multiple stenoses exhibit a reduction of approximately 20–35% in mean volumetric flow rate compared to healthy arteries under identical inlet conditions.

Pressure Drop

The pressure drop across stenosed arteries is found to be several times higher than that of healthy arteries. The relationship between ΔP and Q becomes nonlinear as stenosis severity increases, reflecting energy losses due to flow separation and recirculation.

Wall Shear Stress

Healthy arteries display relatively uniform WSS distributions, whereas stenosed arteries show sharp local peaks in WSS at stenosis throats. These elevated stresses are clinically significant, as they are associated with endothelial damage and plaque rupture.

VI. Conclusion

This work shows analysis of blood flow in arteries with multiple stenoses using realistic physiological parameters and a robust mathematical framework. The results confirm that multiple stenoses substantially alter hemodynamic characteristics, leading to increased pressure losses and abnormal wall shear stress distributions. The combination of analytical relations and CFD simulations enhances the reliability of the findings.

Our findings indicate that healthy arteries exhibit higher volumetric flow rates and lower pressure drops, facilitating efficient blood circulation throughout the cardiovascular system. Conversely, arteries affected by multiple stenosis present a marked decrease in volumetric flow rate, accompanied by increased pressure drops and elevated wall shear stress. This altered hemodynamics not only reflects the challenges posed by arterial narrowing but also underscores the potential risks for adverse cardiovascular events.

These findings have implications for understanding the progression of cardiovascular diseases by comparison of various parameters against each other and it can prove to be useful for clinicians and researchers aiming to design better diagnostic and therapeutic strategies.

In summary, this research not only reinforces the critical relationship between arterial health and blood flow characteristics but also emphasizes the importance of mathematical modeling as a tool for advancing our understanding of cardiovascular diseases. Future endeavors in this field could lead to improved diagnostic approaches and more effective treatments for patients suffering from stenotic conditions.

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