Analysis of conceptual understanding of Kinematics-Critique on mathematical Visualization of concepts

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Abstract
It is observed that students have conceptual difficulty with major kinematics and related graphs. Especially students have major difficulty in relating kinematics graphs of vector quantities like displacement, velocity and acceleration with each other. Problems based on the concepts of integration and differentiation is easy to solve in mathematics, but students have difficulty applying the concept while solving kinematics graphs. They have difficulty interpreting the graphs. We developed a statistically validated diagnostic tool of multiple choice items called Kinematics Concepts Test (KCT) covering all concepts of kinematics to identify and characterize student alternate conceptions. The data of the test conducted on 196 students was interpreted using exploratory factor analysis to identify the major concepts in which students have conceptual difficulty. Further investigation of the results of the factors analysis was done to identify how student understanding of a concept affected his responses to the questions based on another different concept and to identify student group sizes that understood each concept and answered correctly.

Keywords: Factor Analysis, Misconceptions, Mathematical Visualization

Introduction

Physics is different unlike every other science field, in the sense that students often enter the classroom with a set of preconceived notions due to personal experiences with various physical phenomena. We know the fact that objects which goes up must come back down that is the gravity prevents us from flying and that to move the heavy things we must push them hard enough. The problem is, some of these ideas possessed by students are wrong. This understanding that students possess before entering the class and receiving formal scientific instruction is usually loaded with inconsistencies due to fragmented and incomplete knowledge. When these preconceptions are not addressed while building the foundation for the subject, the concepts remain unclear and the students cannot comprehend what is taught to them. Hence, understanding the misconceptions held by students has become an extremely important aspect of research in physics education and an instrument for teaching physics effectively.

When the concepts are not clear to the student, the same process is repeated once again. Such passive learning on the part of the students causes them to misinterpret the gained knowledge in context with their previously held preconceived notions. It leaves room for errors and is inconsistent with the intended scientific knowledge meant to be transmitted.

Kinematics – the science of relationship between position of an object and its velocity and acceleration forms the basis of most of physics concepts. A strong understanding of laws of motion is critical to understanding various physics principles. However, we find that most students try to reduce this understanding to solving algebraic equations mechanically and do not develop the intuitive and conceptual graphical understanding of these phenomena. Thus it is extremely important to find ways of effectively teaching kinematics to first year undergraduate students. Researchers reported that when graphical analysis of kinematics concepts is introduced prior to algebraic problem solving techniques, the students gain a deeper understanding of the concepts [5,6].

During this study it was observed that the meaning and calculation of area under the curve was the most difficult concept for students to understand. They do not understand the meaning of area under the curve. They could not find out whether the displacement is increasing or decreasing from velocity time graph. This requires the ability to understand first level integration graphically. When students were supposed to find change in velocity from acceleration time graph, instead of finding out area under the curve they considered the change in acceleration over the time interval. Students could not relate the sign of area under the curve on velocity time graph with the sign of displacement.

Methodology

The Basic Mechanics course is taught in first year of undergraduate studies at Pune University. In the present study we have designed the test with the purpose of identifying the student's misconceptions in kinematics and their understanding of kinematics graphs at the undergraduate level [3]. This paper explains the factor analysis methodology used. Specifically, the authors wanted to examine the extent to which students could:

(1) Differentiate average and instantaneous kinematical quantity,
(2) Relate graph with corresponding real time motion and
(3) Draw position-time graph from acceleration-time and velocity-time graphs and vice-versa using concepts learnt from their introductory classes dealing with kinematics.

Materials and Methods (*Instruments)*

This study employed a multiple choice 21 items kinematics concept test (KCT). The methodology is explained detail in the author's earlier paper [3]. The test included a combination of qualitative and quantitative questions from the calculus-based undergraduate physics mechanics curriculum, which probed for:

(1) Student's ability to interpret verbal representations in kinematics.
(2) Student's ability to interpret equations in kinematics.
(3) Student's ability to interpret graphical representations

The test was based on the following list of kinematics concepts:

C1: Velocity and Acceleration as rate.
C2: Velocity and Acceleration as vector in one dimension (i.e. direction of the velocity and acceleration)
C3: Change in position as area under the velocity-time-graph and change in velocity as area under the acceleration-time-graph .
C4: Acceleration from position time graph.
C5: Position from acceleration time graph.
C6: Concept of average and instantaneous values

This list of concepts is in good agreement with the concepts identified in other studies (e.g. Andreas Lichtenberger, Andreas Vaterlaus and Clemens Wagner 04Feb2014, Hestenes, Wells & Swackhamer, 1992).

The respondents for this study were first year undergraduate students (aged 18 to 20) from three different colleges affiliated to Pune University. The total number of students selected from these colleges was 196.

Each question had one right answer and three wrong alternatives some of which could be chosen because the student had not understood the underlying concept or because of a prevailing misconception. The kinematic motion of objects is represented by the means of equations of motion, verbal descriptions, tabular data or by using graphs. All test items were intended to assess students' understanding of kinematics graphs and basic concepts of kinematics. The factor analysis of results is shown below.

Table 1: Explain results of 8 factor Promax in terms of Concepts and skill sets required

Item	Factor								Corresponding Concept
	1	2	3	4	5	6	7	8	
Q2	0.780								Factor 1 Concept of integration
Q5	0.775								
Q17	0.687							Low P	
Q18	0.683								
Q13	0.545								
Q22	0.446	0.349							
Q19		0.708							Factor 2 Concept of velocity and acceleration as vector
Q11		0.615					0.312	High P	
Q12		0.592		0.455					
Q23			0.691						Factor 3 Concept of velocity and acceleration as rate
Q15			0.491					High to medium	
Q6			0.482						
Q8			0.438						
Q10				0.729				0.336	Factor 4 Concept of double integration
Q3				0.676					
Q21					0.748				Factor 5 Dual input graph and tabular data C8
Q16					0.748				
Q20			0.331		0.391				
Q14						0.813			Factor 6 Verbal description
Q4						0.475		0.335	
Q7							0.665		Factor 7 Calculation of acc from position time graph C4
Q9							0.649		
Q1								0.892	Factor 8 Kinematics equations
Q4								0.335	

Thus, we could identify 8 major factors which explained the majority of behaviour as given below.

The results of the test were interpreted using exploratory factor analysis to identify the major areas in which students h conceptual difficulty. The result of data analysis shows that the students have serious difficulties in qualitative a quantitative understanding of kinematics concepts and interpretation of kinematics graphs especially in relating positi velocity and acceleration graphs when taught by traditional method. The inter-correlation between items in KCT exceed 0. suggesting that there is enough communality and the data exhibits factorability (Tabachnick & Fidell, 2001).

Factor 1 Area under the curve - mathematical concept of first order integration

Factor 2 Direction concept describing velocity and acceleration as a vector.

Factor 3 Concept of velocity and acceleration as rate - calculation and interpretation of slope.

Factor 4 Qualitative description of motion involving double integration - relating acceleration velocity and displacement together considering their magnitude as well as direction

Factor 5 Concept of velocity and acceleration as rate and vector

Factor 6 Concept of integration involving verbal description

Factor 7 Drawing acceleration time graph from position time graph

Factor 8 Concept of integration involving verbal description

The attempt in this paper is to identify how students have responded to these factors (questions) and if there are any concepts they find especially difficult tom understand.

We also tried to understand how those who answered a concept completely wrong answered for other concepts

We further tried to investigate the results of the factors analysis with a view to identify student group sizes that understood each concept (factor) and answered

1. Correctly to all questions about that concept
2. Could not answer even one question about that concept correctly, i.e answered all wrong and
3. Could answer at least one, but not all questions correctly about that concept

It was an eye-opener to realize that a significant group sizes did not understand a particular concept and answered ALL questions constituting that factor wrong – perhaps a case of widespread misunderstanding of fundamentals of kinematics.
Majority of students do not get acceleration OR graphical representation and integration, major problem with 2nd level differentiation/integration, - many do not understand velocity as rate and direction! There seem to be groups at different levels as the difficulty and distribution indices are reasonable for the study
We also tried to understand how those who answered a concept completely wrong answered for other concepts.

The Table 2 below provides the summary of overall responses, providing counts and percentages for students who answered:

1. All questions related to the individual factors 1-8 correctly
2. All questions related to the individual factors 1-8 wrongly
3. At least one of the questions for individual factors 1-8 corrcetly, but not all (i.e. at least 1 correct and at least 1 wrong)

Thus out of a total of 196 students answering a total of 21 questions distributed among 8 factors 1-8 as (6,2,2,2,3,2,2,2 – Total 21), the distribution for the above was as per Table 2 below.:

Table 2: Summary for Distribution of 8 Factors Solution answered

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	Factor 1	**Factor 2**	**Factor 3**	**Factor 4**	**Factor 5**	**Factor 6**	**Factor 7**	**Factor 8**
Total # Questions	6	2	2	2	3	2	2	2
Base (Total number of Respondents)	196	196	196	196	196	196	196	196
1. ALL Questions Understood (All correct)	12	58	7	27	38	37	43	35
Percentage of students	6.12	29.59	3.57	13.78	19.39	18.88	21.94	17.86
2. Not even one Questions Understood (All wrong)	71	32	45	90	37	76	73	82
Percentage of students	36.22	16.33	22.96	45.92	18.88	38.78	37.24	41.84
3. At Least One but not All Questions Understood	113	106	144	79	121	83	80	79
Percentage of students	57.65	54.08	73.47	40.31	61.73	42.35	40.82	40.31
Total (Sigma)	196	196	196	196	196	196	196	196
Total %	100	100	100	100	100	100	100	100

results in table 2 observed that students had confusion about basic relationships between speed, velocity and acceleration. This can be seen from the comparatively low scores where all the responses in the category were correct. Also, the percentages where at least one, but not all responses were correct were higher than those with all correct indicating confusion in students mind. This was confirmed by the fact that Factor 3, concept of velocity and acceleration as rate, the basic tenet of laws of motion had a very low score for All correct and a very high score for at least one but not all as correct (or wrong). It was observed that about 50% students had at least one response incorrect.

Factor 2, the concept of velocity and acceleration as vector was better understood as shown by high score on al correct and low score on all wrong.

Factor 4, the qualitative description of motion relating acceleration velocity and displacement together considering their magnitude as well as direction was least understood as majority (46%) students got both the related questions for this factor wrong.

Factors related to integration (1, 6 and 8) had very high scores for all wrong, showing that the concept of integration is not well understood among students. Thus, the students were able to derive velocity/acceleration from speed/velocity better (first differential) than the other way around (first integral).

We can also check how those who answered individual factors (set of questions) correctly, also answered for other factors correctly (Table 3 below). The base gives counts for each factor answered correctly while the counts below provide how many out of the students answering correctly for a factor, answered correctly for another factor.

Table 3: All Answered Correct

	All Answered Correct							
	Factor 1	**Factor 2**	**Factor 3**	**Factor 4**	**Factor 5**	**Factor 6**	**Factor 7**	**Factor 8**
Total # Questions	6	2	2	2	3	2	2	2
Base (All Answered Correct)	12	58	7	27	38	37	43	35
Factor 1 (All Answered Wrong)	0	21	2	10	12	15	15	14
Percent	0	36.21	28.57	37.04	31.58	40.54	34.88	40
Factor 2	1	0	1	2	4	1	2	5
Percent	8.33	0	14.29	7.41	10.53	2.7	4.65	14.29
Factor 3	1	11	0	4	9	3	6	8
Percent	8.33	18.97	0	14.81	23.68	8.11	13.95	22.86
Factor 4	3	23	2	0	15	17	20	10
Percent	25	39.66	28.57	0	39.47	45.95	46.51	28.57
Factor 5	1	5	1	2	0	3	7	4
Percent	8.33	8.62	14.29	7.41	0	8.11	16.28	11.43
Factor 6	3	16	3	9	12	0	16	0
Percent	25	27.59	42.86	33.33	31.58	0	37.21	0
Factor 7	2	19	1	5	6	15	0	11
Percent	16.67	32.76	14.29	18.52	15.79	40.54	0	31.43
Factor 8 (All Answered Wrong)	2	20	5	10	10	0	17	0
Percent	16.67	34.48	71.43	37.04	26.32	0	39.53	0
Anwered At least one correct/wrong in ALL Factors	7	7	1	7	8	9	6	10
Percent	58.33	12.07	14.29	25.93	21.05	24.32	13.95	28.57

Table 3 results indicates that, among students who answered correctly for all of the questions related to any of the factors 1-8, it was observed that factor 2, direction concept describing velocity and acceleration as a vector, was better understood since hardly any of these student answered wrongly to both the Factor 2 questions. The other related Factor 5, concept of velocity and acceleration as rate and vector also had similar responses that is very less students answering correctly to any of the factors, answering factor 5 wrong.

There seems to be wide misunderstanding about factors 1 and 4 among students who answered other questions all correct. This shows lack of qualitative understanding of the concept of area under the curve and qualitative description of motion given acceleration (concept of double integration).

The percentages of those answering all wrong among those answering all correct for any factor are comparatively low (mostly below 40%). This shows the relative clarity of concepts as also the inter-relationship for these concepts. This shows that understanding and scores could improve significantly, if basics are stronger.

Similarly, when we analyse responses for students answering wrongly for any of the eight concepts, we can identify concepts which had been misunderstood (Table 4 below). The base gives counts for each factor answered wrongly while the counts below provide how many out of the students answering wrongly for a factor, answered wrongly for another factor.

Table 4: Distribution of 8 Factors Solution All answered Wrongly

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6	Factor 7	Factor 8
Total # Questions	6	2	2	2	3	2	2	2
Base (unwtd)	71	32	45	90	37	76	73	82
Base (wtd)	71	32	45	90	37	76	73	82
Factor 1 (all wrong)	71	13	11	34	18	23	30	30
Percent	100	40.63	24.44	37.78	48.65	30.26	41.1	36.59
Factor 2	13	32	9	20	8	17	17	18
Percent	18.31	100	20	22.22	21.62	22.37	23.29	21.95
Factor 3	11	9	45	22	10	13	15	14
Percent	15.49	28.13	100	24.44	27.03	17.11	20.55	17.07
Factor 4	34	20	22	90	19	37	39	41
Percent	47.89	62.5	48.89	100	51.35	48.68	53.42	50
Factor 5	18	8	10	19	37	17	16	19
Percent	25.35	25	22.22	21.11	100	22.37	21.92	23.17
Factor 6	23	17	13	37	17	76	33	58
Percent	32.39	53.13	28.89	41.11	45.95	100	45.21	70.73
Factor 7	30	17	15	39	16	33	73	32
Percent	42.25	53.13	33.33	43.33	43.24	43.42	100	39.02
Factor 8	30	18	14	41	19	58	32	82
Percent	42.25	56.25	31.11	45.56	51.35	76.32	43.84	100
Remaining	0	0	0	0	0	0	0	0

Results in table 4 show that Factors 2, 3 and 5 (velocity and acceleration and rate and vector) seem to be better understood even among those who had answered all wrong for any of these factors.

Factor 4, the qualitative description of motion relating speed, velocity and acceleration, is the lowest scoring among these students pointing out basic lack of understanding of these concepts. This confusion can be seen from comparable low sores for factor 8 - concept of integration involving verbal description.

This shows that the concept of first derivative as velocity or acceleration given speed / velocity information is clearer than that of the reverse even among students who responded 100% wrong to any of the factors.

Similarly, when we analyse responses for students answering correctly for any of the eight concepts, we can identify other concepts which had also been correctly (Table 5 below). The base gives counts for each factor answered correctly while the counts below provide how many out of the students answering correctly for a factor, answered correctly for another factor.

Table 5: Distribution of 8 Factors Solution answered correctly

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6	Factor 7	Factor 8
Total # Questions	6	2	2	2	3	2	2	2
Base (unwtd)	12	58	7	27	38	37	43	35
Base (wtd)	12	58	7	27	38	37	43	35
Factor 1 (Correct)	12	5	1	4	5	6	4	4
Percent	100	8.62	14.29	14.81	13.16	16.22	9.3	11.43
Factor 2	5	58	5	9	14	18	18	13
Percent	41.67	100	71.43	33.33	36.84	48.65	41.86	37.14
Factor 3	1	5	7	1	1	1	3	1
Percent	8.33	8.62	100	3.7	2.63	2.7	6.98	2.86
Factor 4	4	9	1	27	8	7	8	9
Percent	33.33	15.52	14.29	100	21.05	18.92	18.6	25.71
Factor 5	5	14	1	8	38	9	16	12
Percent	41.67	24.14	14.29	29.63	100	24.32	37.21	34.29
Factor 6	6	18	1	7	9	37	12	12
Percent	50	31.03	14.29	25.93	23.68	100	27.91	34.29
Factor 7	4	18	3	8	16	12	43	12
Percent	33.33	31.03	42.86	29.63	42.11	32.43	100	34.29
Factor 8	4	13	1	9	12	12	12	35
	33.33	22.41	14.29	33.33	31.58	32.43	27.91	100

From Table 5 it was observed that Factor 2, direction concept describing velocity and acceleration as a vector, is answered correctly by most of those who answered 100% correctly to any of the factors.

The surprising outlier was the low scores for factor 3, concept of velocity and acceleration as rate. Corresponding factors 2 and 5 had high scores among those who answered any factors all correct.

Conclusion:

In order to evaluate students' understanding of individual factors it is desirable to have at least three questions instead of two in a multiple choice test instrument so that any result of variation due to accidentally correct response is reduced. The results also show that the students' knowledge and ability to apply mathematical concepts to physically observed phenomenon needs to improve. We feel that the conceptual understanding would be much improved if the students are taught visualization of abstract mathematical concepts.

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