

Some new operations and its properties on intuitionistic fuzzy matrices

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ABSTRACT

In this paper, some new binary and unary operators are extended from intuitionistic fuzzy sets to intuitionistic fuzzy matrices. Some basic properties like commutative, associative etc., are studied. Also we discuss the distributive property of the above said operators with other predefined operators on intuitionistic fuzzy matrices. Several inequalities are obtained which relate modal operators with them. Finally we generalize these operations with some result. Some properties of two operations - conjunction and disjunction from Lukasiewicz type – over Intuitionistic Fuzzy Matrices are studied.

Keywords and Phrases: Intuitionistic Fuzzy Set (IFS), Intuitionistic Fuzzy Matrix (IFM).

1. INTRODUCTION:

There have been theories evolved over the years to deal with the various types of uncertainties. These evolved theories are put into practice and when found to be wanting are improved upon, paving the way for new theories to handle the tricky uncertainties. The Probability theory is one such important theory concerned with the analysis of random phenomena. Zadeh [13] came out with the concept of Fuzzy Set which is indeed an extension of the classical notion of set. Fuzzy Set has been found to be an effective tool to deal with fuzziness. However, it often falls short of the expected standard when describing the neutral state. As a result, a new concept namely Intuitionistic Fuzzy Set (IFS) was worked out and the same was introduced in 1983 by Atanassov [1][2]. Using the concept of IFS, Im et al. [6][7] studied Intuitionistic Fuzzy Matrix (IFM). IFM generalizes the Fuzzy Matrix introduced by Thomson [11] and has been useful in dealing with areas such as decision making, relational equations, clustering analysis etc., Z.S.Xu [12] and Zhang [14] studied Intuitionistic Fuzzy Value and also IFMs. He defined intuitionistic fuzzy similarity relation and also utilize it in clustering analysis. A lot of research activities have been carried out over the years on IFMs in Pal et al. [9] and Pradhan [10].

Intuitionistic fuzzy matrices (IFMs) have been proposed to represent intuitionistic fuzzy relations on finite universes where relationships between elements are more or less vague. Let X and Y be two universes. It is well known that an intuitionistic fuzzy relation $X \times Y$ can be presented by an IFM (say R). Linear systems of equations with uncertainty on the parameters play a major role in several applications in the areas mentioned above. In many applications, the parameters of the system (or at least some of them) should be represented by intuitionistic fuzzy rather than crisp or fuzzy numbers. Hence, it is important to develop the mathematical procedures that would appropriately treat intuitionistic fuzzy linear systems to solve them. This motivates us to extend all the above mentioned operators to IFM by studying many properties of them, and highlight some applications when we use the above said operators in IFMs. In this way, we extend some operations which were introduced by Atanassov in [3][4] on IFSs.

2. PRELIMINARIES:

Definition 2.1 . [1][2]

Let a set $X = \{x_1, x_2, \dots, x_n\}$ be fixed, then an intuitionistic fuzzy set (IFS) can be defined as $A = \{(x_i, \mu_A(x_i), \nu_A(x_i)) / x_i \in X\}$ which assigns to each element x_i a membership degree $\mu_A(x_i)$ and a non membership degree $\nu_A(x_i)$ with the condition $0 \leq \mu_A(x_i) + \nu_A(x_i) \leq 1$ for all $x_i \in X$.

For our convenience let us consider the element of an IFS as (x, x')

Definition 2.2.[1][2]

For any two $(x, x'), (y, y') \in \text{IFS}$, define

$$(i) (x, x') \vee (y, y') = (\max\{x, y\}, \min\{x', y'\})$$

$$(ii) (x, x') \wedge (y, y') = (\min\{x, y\}, \max\{x', y'\})$$

Definition 2.3[12][14]

The two tuple $(\mu_A(x_i), \nu_A(x_i)) = (x, x')$ called an Intuitionistic fuzzy value that such that $0 \leq x + x' \leq 1$ and $x, x' \in [0, 1]$

Definition 2.4[12]

Let $A = (a_{ij})$ be a matrix of order $m \times n$, if all $a_{ij} (i = 1, 2, \dots, m, j = 1, 2, \dots, n)$ are IFVs, then A is called an intuitionistic fuzzy matrix (IFM). Hereafter F_{mn} denotes the set of all IFMs of order $m \times n$.

Definition 2.4[6][7]:

Let $A = [(a_{ij}, a'_{ij})]$ and $B = [(b_{ij}, b'_{ij})]$ be two IFMs of order $m \times n$. Then the ij^{th} element of all the operations are given below.

- i. $A \vee B = [(a_{ij}, a'_{ij}) \vee (b_{ij}, b'_{ij})]$
- ii. $A \wedge B = [(a_{ij}, a'_{ij}) \wedge (b_{ij}, b'_{ij})]$
- iii. $A \leq B \Rightarrow a_{ij} \leq b_{ij}$ and $a'_{ij} \geq b'_{ij}$
- iv. An IFM $U = [(1, 0)]$ for all entries is known as Universal Matrix and $O = [(0, 1)]$ for all entries is known as Zero matrix.
- v. An IFM $I = \langle 1, 0 \rangle$ for all $i = j$ and $\langle 0, 1 \rangle$ for all $i \neq j$ known as Identity Matrix
- vi. $A^c = [(a'_{ij}, a_{ij})]$. for all i, j
- vii. If A is reflexive the $A \geq I_n$ where I_n is the identity IFM contains $\langle 1, 0 \rangle$ when $i = j$ otherwise $\langle 0, 1 \rangle$.
- viii. If A is irreflexive then $\langle a_{ij}, a'_{ij} \rangle = \langle 1, 0 \rangle$ when $i = j$.
- ix. $\square A = [(a_{ij}, 1 - a_{ij})]$
- x. $\diamond A = [(1 - a'_{ij}, a_{ij})]$
- xi. $A + B = [a_{ij} + b_{ij} - a_{ij}b_{ij}, a'_{ij}b'_{ij}]$
- xii. $A \cdot B = [a_{ij}b_{ij}, a'_{ij} + b'_{ij} - a'_{ij}b'_{ij}]$

Definition 2.4[3][4]

For any two $(x, x'), (y, y') \in \text{IFS}$, define

$$(i) \oplus A = \left\{ \left(\frac{x}{2}, \frac{x'+1}{2} \right) / x \in E \right\}$$

$$(ii) \ominus A = \left\{ \left(\frac{x+1}{2}, \frac{x'}{2} \right) / x \in E \right\}$$

$$(iii) (x, x') @ (y, y') = \left\{ \left(\frac{x+y}{2}, \frac{x'+y'}{2} \right) \right\}$$

$$(iv) (x, x') \textcircled{R} (y, y') = \{(\sqrt{x \cdot y}, \sqrt{x' \cdot y'})\}$$

$$(v) (x, x') \textcircled{C} (y, y') = \left\{ \left(\frac{2xy}{x+y}, \frac{2x'y'}{x'+y'} \right) \right\}$$

$$(vi) (x, x') * (y, y') = \left\{ \left(\frac{x+y}{2(xy+1)}, \frac{x'+y'}{2(x'y'+1)} \right) \right\}$$

3. Properties of new operations on IFM

Throughout this section matrices means intuitionistic fuzzy matrices. In this section four new binary and two unary operators are extended to IFM and several properties are studied.

Definition 3.1.

Let $A = [\langle a_{ij}, a'_{ij} \rangle]$ and $B = [\langle b_{ij}, b'_{ij} \rangle]$ be two IFMs of order $m \times n$ then for all i, j define

$$(i) A @ B = \left[\left(\frac{a_{ij} + b_{ij}}{2}, \frac{a'_{ij} + b'_{ij}}{2} \right) \right]$$

$$(ii) A \textcircled{R} B = [\langle \sqrt{a_{ij} \cdot b_{ij}}, \sqrt{a'_{ij} \cdot b'_{ij}} \rangle]$$

$$(iii) A \textcircled{C} B = \left[\left(\frac{2a_{ij}b_{ij}}{a_{ij} + b_{ij}}, \frac{2a'_{ij}b'_{ij}}{a'_{ij} + b'_{ij}} \right) \right]$$

$$(iv) A * B = \left[\left(\frac{a_{ij} + b_{ij}}{2(a_{ij}b_{ij} + 1)}, \frac{a'_{ij} + b'_{ij}}{2(a'_{ij}b'_{ij} + 1)} \right) \right]$$

$$(v) \oplus A = \left[\left(\frac{a_{ij}}{2}, \frac{a'_{ij} + 1}{2} \right) \right]$$

$$(vi) \ominus A = \left[\left(\frac{a_{ij} + 1}{2}, \frac{a'_{ij}}{2} \right) \right]$$

Theorem 3.1:

For any two IFMs $A = [\langle a_{ij}, a'_{ij} \rangle]$ and $B = [\langle b_{ij}, b'_{ij} \rangle]$ of order $m \times n$, we have the following

(i) $A @ B = B @ A$ means the operator '@' is commutative.

(ii) $A \textcircled{R} B = B \textcircled{R} A$ means the operator '\textcircled{R}' is commutative.

(iii) $A \textcircled{C} B = B \textcircled{C} A$ means the operator '\textcircled{C}' is commutative.

(iv) $A * B = B * A$ means the operator '*' is commutative.

(v) $[A^c @ B^c]^c = A @ B$

(vi) $[A^c \textcircled{R} B^c]^c = A \textcircled{R} B$

(vii) $[A^c \textcircled{C} B^c]^c = A \textcircled{C} B$

(viii) $[A^c * B^c]^c = A * B$

Proof:

From the definition 3.1 results (i) to (iv) are obvious.

(v) Consider an IFM $A = [\langle a_{ij}, a'_{ij} \rangle]$ then $A^c = [\langle a'_{ij}, a_{ij} \rangle]$ for all i, j

From definition 3.1, we have $A @ B = \left[\left(\frac{a_{ij} + b_{ij}}{2}, \frac{a'_{ij} + b'_{ij}}{2} \right) \right]$ ------(1)

$$A^c @ B^c = \left[\left(\frac{a'_{ij} + b'_{ij}}{2}, \frac{a_{ij} + b_{ij}}{2} \right) \right]$$

$$\text{And } [A^c @ B^c]^c = \left[\left(\frac{a_{ij} + b_{ij}}{2}, \frac{a'_{ij} + b'_{ij}}{2} \right) \right]$$
------(2)

From (1) and (2) we have $[A^c @ B^c]^c = A @ B$

(vi) Similar to (v)

$$(vii) \quad [A^c @ B^c] = \left[\left(\frac{2a'_{ij}b'_{ij}}{a'_{ij}+b'_{ij}}, \frac{2a_{ij}b_{ij}}{a_{ij}+b_{ij}} \right) \right] \text{ and } [A^c @ B^c]^c = \left[\left(\frac{2a_{ij}b_{ij}}{a_{ij}+b_{ij}}, \frac{2a'_{ij}b'_{ij}}{a'_{ij}+b'_{ij}} \right) \right] = A @ B$$

(viii) Similar to (vii)

Theorem 3.2.

For any IFM $A = [(a_{ij}, a'_{ij})]$, we have the following results

- (i) $\oplus A \leq A$ and $\odot A \geq A$
- (ii) $\oplus O = O$ and $\odot U = U$
- (iii) $\oplus \odot A \leq \odot \oplus A$
- (iv) $\oplus A^c = (\odot A)^c$
- (v) $\odot A^c = (\oplus A)^c$
- (vi) If A is irreflexive then $\oplus A$ is irreflexive and if A is reflexive then $\odot A$ reflexive

Proof:

(i) Consider any (i,j)th element of $\oplus A = \left(\frac{a_{ij}}{2}, \frac{a'_{ij}+1}{2} \right)$, obviously $\frac{a_{ij}}{2} \leq a_{ij}$ and $\frac{a'_{ij}+1}{2} \geq a'_{ij}$ therefore from definition 2.3 $\oplus A \leq A$. Similarly we can prove $\odot A \geq A$

(ii) $\oplus O = \left(\frac{0}{2}, \frac{1+1}{2} \right) = (0,1)$ for all i,j, $\oplus O = O$. Similarly $\odot U = U$.

(iii) The (i,j)th element $\oplus \odot A = \left(\frac{a_{ij}+1}{4}, \frac{a'_{ij}+2}{4} \right)$ and also $\odot \oplus A = \left(\frac{a_{ij}+2}{4}, \frac{a'_{ij}+1}{4} \right)$

Now it is clear that $\frac{a_{ij}+1}{4} \leq \frac{a_{ij}+2}{4}$ and $\frac{a'_{ij}+2}{4} \geq \frac{a'_{ij}+1}{4}$

Thus $\oplus \odot A \leq \odot \oplus A$

(iv) $\oplus A^c = \left(\frac{a'_{ij}}{2}, \frac{a_{ij}+1}{2} \right) = (\odot A)^c$

(v) Similar to (iv).

(vi) If A is irreflexive then $(a_{ii}, a'_{ii}) = (0,1)$, from (ii) the $(i,i)^{th}$ element of $\oplus A = (0,1)$ thus $\oplus A$ irreflexive. Similarly we can prove $\odot A$ is reflexive.

Theorem 3.3.

For any three arbitrary IFMs $A, B, C \in F_{mn}$, we have the following

- (i) $[A \wedge B] @ C = [A @ C] \wedge [B @ C]$
- (ii) $[A \vee B] @ C = [A @ C] \vee [B @ C]$
- (iii) $[A \wedge B] @ C = [A @ C] \wedge [B @ C]$
- (iv) $[A \vee B] @ C = [A @ C] \vee [B @ C]$

Proof:

(i) $A \wedge B = [(a_{ij} \wedge b_{ij}, a'_{ij} \vee b'_{ij})]$ for all i, j

If $A \leq B$ then $[A \wedge B] @ C = \left[\left(\frac{a_{ij}+c_{ij}}{2}, \frac{a'_{ij}+c'_{ij}}{2} \right) \right]$ for all i, j

$A @ C = \left[\left(\frac{a_{ij}+c_{ij}}{2}, \frac{a'_{ij}+c'_{ij}}{2} \right) \right]$ and $B @ C = \left[\left(\frac{b_{ij}+c_{ij}}{2}, \frac{b'_{ij}+c'_{ij}}{2} \right) \right]$ for all i, j

$[A @ C] \wedge [B @ C] = \left[\left(\frac{a_{ij}+c_{ij}}{2}, \frac{a'_{ij}+c'_{ij}}{2} \right) \right]$, Thus $[A \wedge B] @ C = [A @ C] \wedge [B @ C]$

The proof of (ii),(iii) and (iv) are similar to (i)

Theorem 3.4.

For any three arbitrary IFMs A, B, C $\in F_{mn}$, we have the following

- (i) $[A@B]+C=[A+C]@[B+C]$
- (ii) $[A@B].C=[A.C]@[B.C]$
- (iii) $[A+B]@C \leq [A+C]@[B+C]$
- (iv) $[A.B]@C \geq [A@C].[B@C]$

That means '+' and '.' are right distributive over '@' but '@' is not right distributive over '+' and '.'.

Proof:

$$[A+B]@C = \left[\left(\frac{a_{ij} + b_{ij} - a_{ij}b_{ij} + c_{ij}}{2}, \frac{a'_{ij}b'_{ij} + c'_{ij}}{2} \right) \right]$$

$$[A@C] + [B@C] = \left[\left(\frac{a_{ij} + c_{ij}}{2} + \frac{b_{ij} + c_{ij}}{2} - \frac{(a_{ij} + c_{ij})(b_{ij} + c_{ij})}{4}, \frac{a'_{ij} + b'_{ij} - a'_{ij}b'_{ij} + c'_{ij}}{2} \right) \right]$$

Consider the membership value of the previous two equations

$$\left(\frac{a_{ij} + c_{ij}}{2} + \frac{b_{ij} + c_{ij}}{2} - \frac{(a_{ij} + c_{ij})(b_{ij} + c_{ij})}{4} \right) - \left(\frac{a_{ij} + b_{ij} - a_{ij}b_{ij} + c_{ij}}{2} \right) =$$

$$\frac{2c_{ij} + a_{ij}b_{ij} - a_{ij}c_{ij} - b_{ij}c_{ij} - c_{ij}^2}{4} = \frac{c_{ij}(2 - a_{ij} - b_{ij} - c_{ij}) + a_{ij}b_{ij}}{4}$$

$$= \frac{c_{ij}(2 - a_{ij} - b_{ij}) - c_{ij}^2 + a_{ij}b_{ij}}{4} \geq a_{ij}b_{ij} - c_{ij}^2 \geq 0 \text{ when } a_{ij}b_{ij} \geq c_{ij}^2$$

If $a_{ij}b_{ij} \geq c_{ij}^2$ then $c_{ij}(2 - a_{ij} - b_{ij} - c_{ij}) + a_{ij}b_{ij} \geq \sqrt{a_{ij}b_{ij}}(1 - a_{ij} - b_{ij} + \sqrt{a_{ij}b_{ij}})$

The above expression greater than or equal to zero when $a_{ij} \geq b_{ij}$ or $a_{ij} \leq b_{ij}$

$$\text{Thus } \left(\frac{a_{ij} + c_{ij}}{2} + \frac{b_{ij} + c_{ij}}{2} - \frac{(a_{ij} + c_{ij})(b_{ij} + c_{ij})}{4} \right) \geq \left(\frac{a_{ij} + b_{ij} - a_{ij}b_{ij} + c_{ij}}{2} \right)$$

Similarly we can prove the following inequality for non membership value

$$\frac{a'_{ij} + b'_{ij} - a'_{ij}b'_{ij} + c'_{ij}}{2} \leq \frac{a'_{ij}b'_{ij} + c'_{ij}}{2} \text{ for all } i, j.$$

$$\text{Thus } [A+B]@C \leq [A@C] + [B@C]$$

Theorem 3.5.

For any three arbitrary IFMs A, B $\in F_{mn}$, we have the following

- (i) $\square[A@B] = \square A @ \square B$
- (ii) $\square[A\otimes B] \leq \square A \otimes \square B$
- (iii) $\square[A\odot B] \leq \square A \odot \square B$
- (iv) $\diamond[A@B] = \diamond A @ \diamond B$
- (v) $\diamond[A\otimes B] \geq \diamond A \otimes \diamond B$
- (vi) $\diamond[A\odot B] \geq \diamond A \odot \diamond B$

Proof:

$$(i) \square A = [(a_{ij}, 1 - a_{ij})] \text{ and } \square B = [(b_{ij}, 1 - b_{ij})]$$

$$\square A @ \square B = \left[\left(\frac{a_{ij} + b_{ij}}{2}, \frac{2 - a_{ij} - b_{ij}}{2} \right) \right]$$

$$A @ B = \left[\left(\frac{a_{ij} + b_{ij}}{2}, \frac{a'_{ij} + b'_{ij}}{2} \right) \right] \text{ and } \square[A@B] = \left[\left(\frac{a_{ij} + b_{ij}}{2}, 1 - \frac{a_{ij} + b_{ij}}{2} \right) \right] = \left[\left(\frac{a_{ij} + b_{ij}}{2}, \frac{2 - a_{ij} - b_{ij}}{2} \right) \right]$$

$$\text{Thus } \square[A@B] = \square A @ \square B$$

$$(ii) \square[A\otimes B] = [(\sqrt{a_{ij}b_{ij}}, 1 - \sqrt{a_{ij}b_{ij}})] \text{ and}$$

$$\square A \otimes \square B = [(\sqrt{a_{ij}b_{ij}}, \sqrt{(1 - a_{ij})(1 - b_{ij})})]$$

Since $1 - \sqrt{a_{ij}b_{ij}} \geq \sqrt{(1-a_{ij})(1-b_{ij})}$, we have $\square[A \circledast B] \leq \square A \circledast \square B$

$$(iii) \square[A \circledast B] = \left[\frac{2a_{ij}b_{ij}}{a_{ij}+b_{ij}}, 1 - \frac{2a_{ij}b_{ij}}{a_{ij}+b_{ij}} \right] \text{ and also}$$

$$\square A \circledast \square B = \left[\frac{2a_{ij}b_{ij}}{a_{ij}+b_{ij}}, \frac{2(1-a_{ij})(1-b_{ij})}{(1-a_{ij})+(1-b_{ij})} \right], \text{ since } 1 - \frac{2a_{ij}b_{ij}}{a_{ij}+b_{ij}} \geq \frac{2(1-a_{ij})(1-b_{ij})}{(1-a_{ij})+(1-b_{ij})}$$

We have $\square[A \circledast B] \leq \square A \circledast \square B$

Proofs of (iv), (v) and (vi) are similar to the above.

Theorem 3.6.

For any three arbitrary IFMs $A, B, C \in F_{m \times n}$, we have the following

- (i) $[A \circledast B] \circledast C \neq A \circledast [B \circledast C]$
- (ii) $[A \circledast B] \circledast C \neq A \circledast [B \circledast C]$
- (iii) $[A \circledast B] \circledast C \neq A \circledast [B \circledast C]$
- (iv) $[A * B] * C \neq A * [B * C]$

The operators '@', '⊗', '⊙' and '*' are not associative.

Proof:

The proof of the above theorem is trivial since division and root operation are not associative

Theorem 3.7.

For any two arbitrary IFMs $A, B \in F_{m \times n}$, we have the following inequalities

- (i) $A \cdot B \leq A \wedge B \leq A \circledast B \leq A \vee B \leq A + B$
- (ii) $A \cdot B \leq A \wedge B \leq A \circledast B \leq A \vee B \leq A + B$
- (iii) $A \cdot B \leq A \wedge B \leq A \circledast B \leq A \vee B \leq A + B$
- (iv) $A \cdot B \leq A \wedge B \leq A * B \leq A \vee B \leq A + B$

Proof: The proof is trivial from the definition 3.1 and 2.7.

Definition 3.2.

For any 'n' IFMs A_1, A_2, \dots, A_n with same order, define

$$A_1 \circledast A_2 \circledast \dots \circledast A_n = \circledast_{k=1}^n A_k = \left[\sum_{k=1}^n \frac{(a_{ij})_k}{n}, \sum_{k=1}^n \frac{(a'_{ij})_k}{n} \right] \text{ for all } i, j$$

Using the definition, we have the following theorem which is the generalization of earlier theorems

Theorem 3.8.

For any n IFMs with same order, we have the following results

- (i) $(\circledast_{k=1}^n A_k^c)^c = \circledast_{k=1}^n A_k$
- (ii) $(\circledast_{k=1}^n A_k) + B = \circledast_{k=1}^n (A_k + B)$
- (iii) $(\circledast_{k=1}^n A_k) \cdot B = \circledast_{k=1}^n (A_k \cdot B)$
- (iv) $\square(\circledast_{k=1}^n A_k) = \circledast_{k=1}^n \square A_k$
- (v) $\diamond(\circledast_{k=1}^n A_k) = \circledast_{k=1}^n \diamond A_k$

4. Conclusion.

Some new kinds of operators are extended from intuitionistic fuzzy sets to intuitionistic fuzzy matrices. Several algebraic properties are discussed

5. References:

1. K.Atanassov, Intuitionistic Fuzzy Sets, VII ITKR's Section, Sofia(Deposed in Central Sci.-Techn. Library of Bulg.Acad. of Sci.,1697/84) June 1983.
2. K.Atanassov, Intuitionistic Fuzzy Sets , Fuzzy Sets and System 20(1) (1986) 87-96.
3. Atanassov. K, New operations defined over the intuitionistic fuzzy sets, Fuzzy Sets and System, 61 (1994) 137--142.
4. K . Atanassov, Intuitionistic fuzzy sets, Springer, Heidelberg, 1999.
5. H.Bustince and P.Burillo, Structures on intuitionistic fuzzy relations, Fuzzy Sets and System, 78 (3)(1996) 293-303.
6. Y.B.IM, E.P. Lee and S.W. Park, The adjoint of square intuitionistic fuzzy matrices, Journal of applied math and computing(series A), 11(2003),No 1-2, 401-412.
7. Y.B. Im, E.P. Lee and S.W. Park, The Determinant of square Intuitionistic Fuzzy Matrices, Far East Journal of Mathematical Sciences, 3(5)(2001) 789-796.
8. K.H. Kim and R.W. Roush, Generalized fuzzy matrices, Fuzzy Sets and System, 4 (1980) 293-315.
9. M. Pal, Khan S.K and Amiya K Shyamala, Intuitionistic Fuzzy Matrices, Notes on Intuitionistic Fuzzy Sets, 8(2) (2002) 51-62.
10. R. Pradhan and M. Pal, Some results on Generalised Inverse of Intuitionistic fuzzy matrices, Fuzzy Information and Engg, 6(2) (2014) 133-145.
11. M.G. Thomson, Convergence of powers of a fuzzy matrix, Journal of mathematical analysis and applications, 57 (1977) 476-480.
12. Z.S. Xu., Intuitionistic Fuzzy Aggregation and Clustering, Studies in Fuzziness and soft computing , 275(2012) 159-190.
13. L.A .Zadeh , Fuzzy Sets, Journal of Information and Control, 8 (1965) 338-353.
14. X. Zhang, A new method for ranking Intuitionistic fuzzy values and its applications in multi attribute decision making Fuzzy Optimization Decision Making, 11(2) (2012) 135-146.