ON NEUTROSOPHIC ORBIT CO-KERNAL SPACES

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Abstract : In this paper, the concepts of neutrosophic orbit co-kernal functions and neutrosophic hardly open orbit functions are introduced and studied. In this connection, some interesting properties and characterizations are established.

***IndexTerms* - neutrosophic orbit co-kernal spaces, neutrosophic O_δ -sets, neutrosophic orbit meager* sets, neutrosophic orbit comeager* sets, neutrosophic orbit meager* spaces, neutrosophic orbit quasi regular spaces and neutrosophic orbit strongly complete spaces**

I. INTRODUCTION

The notion of fuzzy set was introduced by Zadeh[19] in 1965 where each element had a certain degree of membership. Following this concept K.Atanassov[1,2,3] in 1983 introduced the idea of intuitionistic fuzzy set on a universe X as a generalization of the fuzzy set. Here besides the degree of membership a degree of non-membership for each element is also defined. Smarandache[15,16] originally gave the definition of a neutrosophic set and neutrosophic logic. The neutrosophic logic is a formal frame trying to measure the truth, indeterminacy and falsehood. Chaos and fractal are among the greatest discoveries of the 20th century, which have been widely investigated with significant progress and achievements. It has become an exciting emerging interdisciplinary area in which a broad spectrum of technologies and methodologies have emerged to deal with large-scale, complex and dynamical systems and problems. In 1989, R.L. Deveney[6] defined chaotic function in general metric space. T. Thrivikraman and P.B. Vinod Kumar[19] defined Chaos and fractals in general topological spaces. M. Kousalyaparasakthi, E. Roja, M.K. Uma[10] introduced the concept of intuitionistic chaotic continuous functions. M. Kousalyaparasakthi and E. Roja [11] have contributed to the study on Intuitionistic Orbit Co-kernal Spaces and Intuitionistic Uniform* AB-Border Spaces.

The concept of Baire spaces was introduced by R.C Haworth and R.C. McCoy [9]. The concept of orbit of a point plays a vital role in chaos and fractals. These concepts are applied in animation, computer graphics and medical field. Also the concepts may be applied in game theory to find the solution of the game. Motivated by the varied applications of neutrosophic and chaotic spaces. T.Madhumathi and F.Nirmala Irudayam[12] introduced the concept of neutrosophic chaotic set and hence continuous functions. T.Madhumathi and F.Nirmala Irudayam[13] introduced the concept of application of neutrosophic chaotic continuous functions. In this paper, the concepts of neutrosophic orbit co-kernal spaces, neutrosophic O_δ -sets, neutrosophic orbit meager* sets, neutrosophic orbit comeager* sets, neutrosophic orbit meager* spaces, neutrosophic orbit quasi regular spaces and neutrosophic orbit strongly complete spaces are introduced and studied.

II. PRELIMINARIES

Definition 2.1([16]). Let X be a non empty set. A neutrosophic set (NS for short) A is an object having the form A = $<x,A^1,A^2,A^3>$ where A^1,A^2, A^3 represent the degree of membership , the degree of indeterminacy and the degree of non-membership respectively of each element x∈X to the set A.

Definition 2.2 ([16]). Let X be a non empty set, A = $<x,A^1,A^2,A^3>$ and B = $<x,B^1,B^2,B^3>$ be neutrosophic sets on X, and let $\{A_i : i \in J\}$ be an arbitrary family of neutrosophic sets in X, where $A^i = <x,A^1, A^2, A^3>$

(i) $A \subseteq B$ if and only if $A^1 \subseteq B^1$, $A^2 \supseteq B^2$ and $A^3 \supseteq B^3$

(ii) A = B if and only if $A \subseteq B$ and $B \subseteq A$.

(iii) $\overline{A} = <x,A^3,A^2,A^1>$

(iv)$A\cap B=<x,A^1\cap B^1,A^2\cup B^2,A^3\cup A^3>$

(v) $A\cup B=<x,A^1\cup B^1,A^2\cap B^2,A^3\cap B^3>$

(vi) $\cup A_i = <x,\cup A_i^1,\cap A_i^2,\cap A_i^3>$

(vii) $\cap A_i = <x,\cap A_i^1,\cup A_i^2,\cup A_i^3>$

(viii)$A - B = A \cap \overline{B}$.

(iX) $\varphi_N = <x,\varphi,X ,X>$; $X_N = <x,X, \varphi, \varphi >$.

Definition 2.3 ([18]). A neutrosophic topology (NT for short) on a nonempty set X is a family τ of neutrosophic set in X satisfying the following axioms:

(i) $\varphi_N, X_N \in \tau$.

(ii) $G_1\cap G_2 \in \tau$ for any $G_1,G_2 \in \tau$.

(iii) $\cup G_i \in \tau$ for any arbitrary family $\{G_i : i\in J\} \subseteq \tau$.

Definition 2.4 ([7]). (a) If B = $<y,B^1,B^2,B^3>$ is a neutrosophic set in Y , then the preimage of B under f, denoted by $f^{-1}(B)$, is the neutrosophic set in X defined by $f^{-1}(B) = <x,f^{-1}(B^1),f^{-1}(B^2),f^{-1}(B^3)>$.

(b) If A = $<x,A^1,A^2,A^3>$ is a neutrosophic set in X, then the image of A under f, denoted by f(A), is the neutrosophic set in Y defined by $f(A) = <y,f(A^1),f(A^2),Y\text{-}f(X\text{-}A^3)>$ where

$$f(A^1)=\begin{cases}\sup_{x\in f^{-1}(y)} A^1 & if\ f^{-1}(y)\neq\phi\\ 0 & otherwise\end{cases}$$

$$f(A^2)=\begin{cases}\sup_{x\in f^{-1}(y)} A^2 & if\ f^{-1}(y)\neq\phi\\ 0 & otherwise\end{cases}$$

$$Y\text{-}f(X\text{-}A^3)=\begin{cases}\inf_{x\in f^{-1}(y)} A^3 & if\ f^{-1}(y)\neq\phi\\ 1 & otherwise\end{cases}$$

Definition 2.5 ([17]). Let (X, τ) and (Y,σ) be any two neutrosophic topological spaces and let f : X → Y be a function. Then f is said to be continuous if and only if the preimage of each neutrosophic set in σ is a neutrosophic set in τ.

Definition 2.6 ([17]). Let (X, τ) and (Y,σ) be two neutrosophic topological spaces and let f : (X, τ) → (Y,σ) be a function. Then f is said to be open iff the image of each neutrosophic set in τ is a neutrosophic set in σ.

Definition 2.7 ([6]). Orbit of a point x in X under the mapping f is $O_f(x)=\{x, f(x), f^2(x),...\}$

Definition 2.8 ([6]). x in X is called a periodic point of f if $f^n(x) = x$, for some $n \in Z^+$. Smallest of these n is called period of x.

Definition 2.9 ([6]). f is sensitive if for each $\delta>0$ there exist (a) $\varepsilon > 0$ (b) $y\in X$ and (c) $n\in Z_+$ such that $d(x,y)< \delta$ and $d(f^n(x),f^n(y))> \varepsilon$.

Definition 2.10 ([6]). f is chaotic on (X,d) if (i) Periodic points of f are dense in X (ii) Orbit of x is dense in X for some x in X and (iii) f is sensitive.

Definition 2.11 ([19]). Let (X, τ) be a topological space and f : (X, τ) → (X, τ) be continuous map. Then f is sensitive at x ∈ X if given any open set U containing x there exists (i) y ∈ U (ii) $n \in Z^+$ and (iii) an open set V such that $f^n(x) \in V$, $f^n(y) \notin cl(V)$. We say that f is sensitive on a F if f|F is sensitive at every point of F.

Definition 2.12.([19]) Let (X, τ) be a topological space and F ∈ K(X). Let f : F → F be a continuous. Then f is chaotic on F if

(i) $cl(O_f(x)) = F$ for some x ∈ F.

(ii) periodic points of f are dense in F.

(iii) f ∈ S(F).

Notation 2.13.([19]) (i) C(F) = {f : F → F | f is chaotic on F} and (ii) CH(X) = {F ∈ NK(X) | C(F) ≠ φ}.

Definition 2.14.([19]) A topological space (X, τ) is called a chaos space if CH(X) ≠ φ. The members of CH(X) are called chaotic sets.

Definition 2.15.([12]) Let (X, τ) be a neutrosophic topological space. A neutrosophic orbit set in X under the function f : (X, τ) → (X, τ) is denoted and defined as $NO_f(x) = \langle x,O_{fT}(x),O_{fI}(x),O_{fF}(x)\rangle$ for x ∈ X.

Definition 2.16.([12]) Let (X, τ) be a neutrosophic topological space and f : (X, τ) → (X, τ) be a neutrosophic continuous function. Then f is said to be neutrosophic sensitive at x ∈ X if given any neutrosophic open set $U = \langle x,U^1,U^2,U^3\rangle$ containing x there exists a neutrosophic open set $V = \langle x,V^1,V^2,V^3\rangle$ such that $f^n(x) \in V$, $f^n(y) \notin Ncl(V)$ and y ∈ U, $n \in Z^+$. We say that f is neutrosophic sensitive on a neutrosophic compact set $F = \langle x,F^1,F^2.F^3\rangle$ if f|F is neutrosophic sensitive at every point of F.

Definition 2.17.([12]) Let (X, τ) be a two neutrosophic topological space. Let f : (X, τ) → (X, τ) be a function. A neutrosophic periodic set is denoted and defined as $NP_f(x) = \langle x,\{x \in X \mid f^n{}_T(x) = x\},\{x \in X \mid f^n{}_I(x) = x\},\{x \in X \mid f^n{}_F(x) = x\}\rangle$

Definition 2.18.([12]) Let (X, τ) be a neutrosophic topological space and $F = \langle x,F^1,F^2,F^3\rangle \in NK(X)$. Let f : F → F be a neutrosophic continuous function. Then f is said to be neutrosophic chaotic on F if

(i) $Ncl(NO_f(x)) = F$ for some x ∈ F.

(ii) neutrosophic periodic points of f are neutrosophic dense in F. That is, $Ncl(NP_f(x)) = F$.

(iii) f ∈ S(F).

Notation 2.19.([12]) Let (X, τ) be a neutrosophic topological space then $C(F) = \langle x,C(F)^1,C(F)^2,C(F)^3\rangle$ where $C(F)^1$ = {f : F → F | f is neutrosophic chaotic on F}, $C(F)^2$ = {f : F → F | f is indeterminacy neutrosophic chaotic on F},and $C(F)^3$ = {f : F → F | f is not neutrosophic chaotic on F}.

Notation 2.20.([12]) Let (X, τ) be a neutrosophic topological space then $CH(X) = \{F = \langle x,F^1,F^2,F^3\rangle \in NK(X) \mid C(F) \neq \varphi\}$.

Definition 2.21.([12]) A neutrosophic topological space (X, τ) is called a neutrosophic chaos space if CH(X) ≠ φ. The members of CH(X) are called neutrosophic chaotic sets.

Definition 2.22.([4]) Let (X, τ) be a neutrosophic topological space. Let A be a neutrosophic subset os X. Then the co-kernal and kernel of A are denoted and defined by

(i) co-ker(A) = ∪{ G :G is a closed set and G ⊆ A }

(ii) ker(A) = ∩{ K : K is a open set and A ⊆ K }

Definition 2.23.([9]) A topological space (X, τ) is a Baire space if the intersection of each countable family of dense open sets is dense.

Definition 2.24.([14]) A topological space is quasi regular if for every non empty open set U there is a non empty open set V such that $\overline{V} \subseteq U$.

Definition 2.25.([5]) Let A be an open covering of a space X. Then a subset S of X is said to be A-small if S is contained in a member of A.

Definition 2.26.([8]) A space X is said to be strongly countably complete if there is a sequence $(A_i, i=1,2,3,...n)$ of open covering of X such that a sequence (F_i) of non empty closed sets of X has a non empty intersection provided that $F_{i+1} \subseteq F_i$ for all i and each F_i is A_i-small.

III. CHARACTERIZATIONS AND PROPERTIES OF NEUTROSOPHIC ORBIT CO-KERNAL SPACES

Definition 3.1. Let (X, τ) be a neutrosophic topological space. Let $NO_f(x)$ be a neutrosophic orbit set of X. Then the neutrosophic co-kernal and neutrosophic kernal of $NO_f(x)$ are denoted and defined by

Nco-ker($NO_f(x)$) = $\cup\{ G = (x, G^1, G^2, G^3)$:G is a neutrosophic closed set and $G \subseteq NO_f(x)$ }

Nker($NO_f(x)$) = $\cap\{ K = (x, K^1, K^2, K^3)$: K is a neutrosophic open set and $NO_f(x) \subseteq K$ }

Remark 3.2. Let (X, τ) be a neutrosophic topological space. Let $NO_f(x)$ be a neutrosophic orbit set of X .

(i) $\overline{\text{Nker}(NO_f(x))}$ = Nco-ker($\overline{NO_f(x)}$).

(ii) $\overline{\text{Nco-ker}(NO_f(x))}$ = ker($\overline{NO_f(x)}$).

(iii) Nker(φ_N) = φ_N and Nco-ker(φ_N) = φ_N.

(iv) Nker(X_N) = X_N and Nco-ker(X_N) = X_N.

(v) If A is a neutrosophic open orbit set then Nker($NO_f(x)$) = $NO_f(x)$.

(vi) If A is a neutrosophic closed orbit set then Nco-ker($NO_f(x)$) = $NO_f(x)$.

(vii) Nco-ker($NO_f(x_1) \cup NO_f(x_2)$) = Nco-ker($NO_f(x_1)$) $\cup$ Nco-ker($NO_f(x_2)$).

Proof The proof is simple.

Definition 3.3. A neutrosophic topological space (X, τ) is said to be a neutrosophic orbit co-kernal space if the following condition holds:

Given any finite collection { $NO_f(x_i)$, $x_i \in X$, i = 1, 2, 3, ..., n } of neutrosophic open orbit sets of (X, τ) with Nco-ker($NO_f(x_i)$) = φ_N, i = 1, 2, 3, ..., n and Nco-ker($\bigcap_{i=1}^{n} NO_f(x_i)$) = φ_N.

Example 3.4. Let X = { a, b, c, d }. Then the neutrosophic sets A, B, C and D of X are defined by A = <x, { a, b } , {a, c} { a, d }> , B = < x, {b, c}{ b, d } , { a, b}> , C = < x, { a, b, c } , φ, { a }> and D = < x, {b}, X, { a, b, d}> . Let τ = { X_N, φ_N, A, B, C, D } be a neutrosophic topology on X. Clearly, (X, τ) is a neutrosophic topological space. Let f : (X, τ) → (X, τ) be a function defined by f (a) = <b, c, d>, f (b) = <c, d, a>, f (c) = <d, a, b> and f (d) = <a, b, c>. Then A and B are neutrosophic open orbit sets. Now, Nco-ker (A) = φ_N, Nco-ker(B) = φ_N and Nco-ker (A ∩ B) = φ_N. Hence the neutrosophic topological space (X, τ) is called a neutrosophic orbit co-kernal space.

Definition 3.5. Let (X, τ) be a neutrosophic topological space. A neutrosophic orbit set $NO_f(x)$, $x \in X$ is said to be a neutrosophic O_δ -set if it equals the intersection of a finite collection of neutrosophic open orbit sets of (X, τ). The complement of a neutrosophic O_δ -set is a neutrosophic C_σ -set.

Theorem 3.6. Let (X, τ) be a neutrosophic topological space. Then the following statements are equivalent:

(i) (X, τ) is a neutrosophic orbit co-kernal space.

(ii) Nco-ker($NO_f(x_i)$) = φ_N, $x_i \in X$, i = 1, 2, 3, ..., n and Nco-ker($NO_f(z)$) = φ_N, for every neutrosophic O_δ -set $NO_f(z)$ of (X, τ).

(iii) Nker($\overline{NO_f(x_i)}$) = X_N, $x_i \in X$, i = 1, 2, 3, ..., n and Nker($\overline{NO_f(z)}$) = X_N, for every neutrosophic C_σ -set $\overline{NO_f(z)}$ of (X, τ).

Proof

(i) ⇒ (ii) Let $NO_f(z)$ be a neutrosophic O_δ -set. By Definition 3.5., $NO_f(z) = \bigcap_{i=1}^{n} NO_f(x_i)$, $x_i \in X$. Nco-ker($NO_f(z)$) = Nco-ker($\bigcap_{i=1}^{n} NO_f(x_i)$) Since (X, τ) is a neutrosophic orbit co-kernal space, Nco-ker($NO_f(x_i)$) = φ_N and Nco-ker($\bigcap_{i=1}^{n} NO_f(x_i)$) = φ_N .

This implies that, Nco-ker($NO_f(z)$) = φ_N . Hence, Nco-ker($NO_f(x_i)$) = φ_N and Nco-ker($NO_f(z)$) = φ_N.

(ii) ⇒ (iii) The proof is simple.

(iii) ⇒ (i) Let Nker($\overline{NO_f(x_i)}$) = X_N, $x_i \in X$, i = 1, 2, 3, ..., n this implies that,

$$\text{Nco-ker}(NO_f(x_i)) = \varphi_N. \quad (1)$$

Let $NO_f(z) = \bigcap_{i=1}^{n} NO_f(x_i)$, be a neutrosophic O_δ -set. Then, $\overline{NO_f(z)}$ is a neutrosophic C_σ -set. By assumption, Nker($\overline{NO_f(z)}$) = X_N, that is $\overline{\text{Nker}(\overline{NO_f(z)})} = \overline{X_N}$ This implies that, Nco-ker($NO_f(z)$) = φ_N.

That is, Nco-ker($\bigcap_{i=1}^{n} NO_f(x_i)$) = φ_N . (2)

From (1) and (2), we have, (X, τ) is a neutrosophic orbit co-kernal space.

Theorem 3.7. Let (X, τ) be a neutrosophic orbit co-kernal space. If $NO_f(z)$, $z \in X$ is a neutrosophic C_σ -set then $NO_f(z) \cap NO_f(y) = \varphi_N$, for every neutrosophic open orbit set $NO_f(y)$, $y \in X$ of (X, τ).

Proof Let NO_f (z) be a neutrosophic C_σ -set. By Definition 3.5., $NO_f(z) = \bigcup_{i=1}^{n} NO_f(x_i)$

$$\overline{NO_f(z)} = \overline{\bigcup_{i=1}^{n} NO_f(x_i)} = \bigcap_{i=1}^{n} \overline{NO_f(x_i)}$$

Since (X, τ) is a neutrosophic orbit co-kernal space, Nco-ker($\bigcap_{i=1}^{n} \overline{NO_f(x_i)}$) = φ_N.

This implies that, Nco-ker($\overline{NO_f(z)}$) = φ_N (3)

Suppose that, $NO_f(z) \cap \overline{NO_f(y)} = \varphi_N$

$\overline{NO_f(z) \cap \overline{NO_f(y)}} = \varphi_N$

$\overline{NO_f(z)} \cup NO_f(y) = X_N$.

Now, Nco-ker($\overline{NO_f(z)} \cup NO_f(y)$) = Nco-ker($X_N$)

By Remark 3.2., Nco-ker($\overline{NO_f(z)}$)∪ Nco-ker(NO_f (y)) = X_N

By (3), φ_N ∪ Nco-ker(NO_f (y)) = X_N. That is, Nco-ker(NO_f (y)) = X_N.

Which is impossible. Hence, $NO_f(z) \cap NO_f(y) = \varphi_N$.

Definition 3.8. Let (X, τ) be a neutrosophic topological space. A neutrosophic orbit set NO_f (x), x∈X is said to be a neutrosophic orbit meager* set if $NO_f(x) = \bigcup_{i=1}^{n}$ Nker(Nco-ker($NO_f(x_i)$)), x_i∈X with Nker(Nco-ker($NO_f(x_i)$)) = X_N. The complement of a neutrosophic orbit meager* set is a neutrosophic orbit comeager* set.

Theorem 3.9. Let (X, τ) be a neutrosophic orbit co-kernal space. Then the neutrosophic kernal of every neutrosophic orbit meager* set is X_N.

Proof Let NO_f (x), x ∈X be an neutrosophic orbit meager* set. By Definition 3.8., $NO_f(x) = \bigcup_{i=1}^{n}$ Nker(Nco-ker($NO_f(x_i)$)), x_i ∈X with Nker(Nco-ker($NO_f(x_i)$)) = X_N. Now, Nker(N O_f (x)) = Nker($\bigcup_{i=1}^{n}$ Nker(Nco-ker($NO_f(x_i)$))) = Nker($\bigcup_{i=1}^{n} X_N$)= Nker(X_N)=X_N. Hence the neutrosophic kernal of every neutrosophic orbit meager* set NO_f (x) is X_N.

Remark 3.10. Let (X, τ) be a neutrosophic orbit co-kernal space. Then the neutrosophic co-kernal of every neutrosophic orbit comeager* set is φ_N.

Proof The proof is simple.

Theorem 3.11. Let (X, τ) be a neutrosophic topological space. Then the following statements are equivalent:

(i) (X, τ) is a neutrosophic orbit co-kernal space.

(ii) Nker($\bigcup_{i=1}^{n} NO_f(x_i)$) = X_N , for every neutrosophic closed orbit set $NO_f(x_i)$, x_i∈X , i = 1, 2, 3, ..., n with Nker($NO_f(x_i)$) = X_N.

Proof (i) ⇒ (ii)

Let $NO_f(x_i)$, x_i ∈X , i = 1, 2, 3, ..., n be neutrosophic closed orbit sets with Nker($NO_f(x_i)$) = X_N. Then $\overline{NO_f(x_i)}$ is neutrosophic open orbit sets with Nco-ker($\overline{NO_f(x_i)}$) = φ_N. Since (X, τ) is a neutrosophic orbit co-kernal space, Nco-ker($\bigcap_{i=1}^{n} \overline{NO_f(x_i)}$)= φ_N. $\overline{\text{Nco-ker}(\bigcap_{i=1}^{n} \overline{NO_f(x_i)})} = \overline{\varphi_N}$. Nker($\bigcup_{i=1}^{n} NO_f(x_i)$)=$X_N$.

(ii) ⇒ (i) The proof is simple

Definition 3.12. Let $NO_f(x_i)$, x_i ∈X be neutrosophic orbit sets. A neutrosophic topological space (X, τ) is called a neutrosophic orbit meager* space if $\bigcup_{i=1}^{n}$ Nker(Nco-ker($NO_f(x_i)$)) = φ_N.

Theorem 3.13. If (X, τ) is a neutrosophic orbit meager* space then (X, τ) is not an neutrosophic orbit co-kernal space.

Proof Since (X, τ) is a neutrosophic orbit meager* space, $\bigcup_{i=1}^{n}$ Nker(Nco-ker($NO_f(x_i)$))= φ_N, $x_i \in$ X . This implies that, Nco-ker($NO_f(x_i)$) = φ_N, $x_i \in$ X . Hence,

$$\bigcup_{i=1}^{n} \text{Nco-ker}(NO_f(x_i)) = \varphi_N \qquad (4)$$

Let $NO_f(y_i)$ = $\overline{\text{Nco-ker}(NO_f(x_i))}$. Which implies that each $NO_f(y_i)$, $y_i \in$ X is a neutrosophic open orbit set. Now, $\bigcap_{i=1}^{n} NO_f(y_i) = \bigcap_{i=1}^{n} (\overline{\text{Nco-ker}(NO_f(x_i))})$.

$$\bigcap_{i=1}^{n} NO_f(y_i) = \overline{\bigcup_{i=1}^{n}(\text{Nco-ker}(NO_f(x_i)))}$$

By (4), $\bigcap_{i=1}^{n} NO_f(y_i) = \overline{\varphi_N}$.

$$\bigcap_{i=1}^{n} NO_f(y_i) = X_N$$

$$\text{Nco-ker}(\bigcap_{i=1}^{n} NO_f(y_i)) = \text{Nco-ker}(X_N)$$

$$\text{Nco-ker}(\bigcap_{i=1}^{n} NO_f(y_i)) = X_N$$

$$\text{Nco-ker}(\bigcap_{i=1}^{n} NO_f(y_i)) = \varphi_N.$$

Hence, (X, τ) is not a neutrosophic orbit co-kernal space.

Theorem 3.14. Let (X, τ) be a neutrosophic topological space and $NO_f(x)$, x $\in$ X be a neutrosophic orbit set of X . Then Nco-ker($NO_f(x)$) = φ_N if and only if $NO_f(y) \cup NO_f(x) = X_N$, for every neutrosophic open orbit set $NO_f(y)$, $NO_f(y) = X_N$, y $\in$ X .

Proof Assume that Nco-ker($NO_f(x)$) = φ_N. Suppose that $NO_f(y) \cup NO_f(x) = X_N$, for Every neutrosophic open orbit set $NO_f(y)$, $NO_f(y) = X_N$. Now, $NO_f(y) \cup NO_f(x) = X_N$

$$\overline{NO_f(y) \cup NO_f(x)} = \overline{X_N}$$

$$\overline{NO_f(y)} \cap \overline{NO_f(x)} = \varphi_N.$$

This implies that, $\overline{NO_f(y)} \subseteq \overline{\overline{NO_f(x)}}$

$$\overline{NO_f(y)} \subseteq NO_f(x)$$

$$\text{Nco-ker}(\overline{NO_f(y)}) \subseteq \text{Nco-ker}(NO_f(x))$$

By assumption, Nco-ker($\overline{NO_f(y)}$) = φ_N. Since $\overline{NO_f(y)}$ is a neutrosophic closed orbit set, Nco-ker($\overline{NO_f(y)}$) = $\overline{NO_f(y)}$. Thus, $\overline{NO_f(y)}$ = φ_N. This implies that, $NO_f(y) = X_N$. Which is a contradiction. Hence, $NO_f(y) \cup NO_f(x) = X_N$.

Conversely, let $NO_f(y) \cup NO_f(x) = X_N$. This implies that, $\overline{NO_f(y)} \cap \overline{NO_f(x)} = \varphi_N$. Suppose that, x $\in X_N$ and x $\notin$ Nker($\overline{NO_f(x)}$). So, x $\in$ ($\overline{N\ker(\overline{NO_f(x)})}$), that is, x $\in$ Nco-ker($NO_f(x)$). Therefore, Nco-ker($NO_f(x)$) $\cap$ $\overline{NO_f(x)}$ = φ_N. This is a contradiction. Hence, x $\in$ Nker($\overline{NO_f(x)}$). Thus, $X_N \subseteq$ Nker($NO_f(x)$). That is, X_N = Nker($\overline{NO_f(x)}$).

Hence, Nco-ker($NO_f(x)$) = φ_N.

Definition 3.15. A neutrosophic topological space (X, τ) is said to be neutrosophic orbit quasi regular if for every neutrosophic open orbit set $NO_f(x)$, x$\in$ X with $NO_f(x) = \varphi_N$, there is a neutrosophic open orbit set $NO_f(y)$, y $\in$ X such that Nco-ker($NO_f(y)$) $\supseteq NO_f(x)$ with Nker($NO_f(x)$) = φ_N.

Definition 3.16. Let (X, τ) be a neutrosophic topological space. Let {$NO_f(x_i)$, $x_i \in$ X , i = 1, 2, 3, ..., n } be a neutrosophic open orbit covering of (X, τ). Then the neutrosophic orbit set $NO_f(y)$, y$\in$ X is said to be { $NO_f(x_i)$ } -small if $NO_f(y)$ is contained in a member of {$NO_f(x_i)$, $x_i \in$ X , i = 1, 2, 3, ..., n } .

Definition 3.17. A neutrosophic topological space (X, τ) is said to be neutrosophic orbit strongly complete if there is a sequence { $NO_f(x_i)$, $x_i \in$ X , i = 1, 2, 3, ..., n } of neutrosophic open orbit covering of (X, τ) such that a sequence { $NO_f(y_i)$, $y_i \in$ X , i = 1, 2, 3, ..., n } of neutrosophic closed orbit sets with $NO_f(y_i) = X_N$ of (X, τ) with $\bigcup_{i=1}^{n} NO_f(y_i) = X_N$ provided that $NO_f(y_{i+1}) \supseteq NO_f(y_i)$ for all i and each $NO_f(y_i)$ is { $NO_f(x_i)$ } -small.

Theorem 3.18. Every neutrosophic orbit quasi regular space and neutrosophic orbit strongly complete space is a neutrosophic orbit co-kernal space.

Proof Since (X, τ) is a neutrosophic orbit strongly complete space, { $NO_f(x_i)$, $x_i \in X$, i = 1, 2, 3, ..., k } is a sequence of neutrosophic open orbit cover. Let { $NO_f(y_i)$, $y_i \in X$, i = 1, 2, 3, ..., k } be a sequence of neutrosophic open orbit sets with Nco-ker($NO_f(y_i)$) = φ_N.

Let $NO_f(z)$, z∈ X be a neutrosophic open orbit set of (X, T). Since (X, T) is a neutrosophic orbit quasi regular space, there is a neutrosophic open orbit { $NO_f(x_2)$ } -small set $NO_f(z_1)$, $z_1 \in X$ with $NO_f(z_1) = X_N$ such that Nco- ker($NO_f(z_1)$) $\supseteq$ $NO_f(z) \cup NO_f(y_1)$.

Continue inductively to construct a sequence { $NO_f(z_j)$, $z_j \in X$, j = 1, 2, 3, ..., n } of neutrosophic open orbit sets with $NO_f(z_j) = X_N$ such that Nco-ker($NO_f(z_j)$) $\supseteq$ $NO_f(z_{j-1}) \cup NO_f(y_{j-1})$ and $NO_f(z_j)$, $z_j \in X$, j = 1, 2, 3, ..., n is { $NO_f(x_{j+1})$ } -small.

Now, { Nco-ker($NO_f(x_j)$), $x_j \in X$, j = 1, 2, 3, ..., n } is a increasing sequence of neutrosophic closed orbit sets and Nco-ker($NO_f(z_j)$) is { $NO_f(x_j)$ } -small. Since (X, τ) is a neutrosophic orbit strongly complete space,

$$\bigcup_{j=1}^{n} \text{Nco-ker}(NO_f(x_j)) = X_N \quad (5)$$

Now, $\bigcup_{j=1}^{n}$ Nco-ker($NO_f(z_j)$) $\supseteq NO_f(z) \cup (\bigcap_{j=1}^{n} NO_f(y_j))$.

By (5), $NO_f(z) \cup (\bigcap_{j=1}^{n}(NO_f(y_j))) = X_N$. By Theorem 3.14., Nco-ker($\bigcap_{j=1}^{n} NO_f(y_j)$) = φ_N . Hence, (X, τ) is a neutrosophic orbit co-kernal space.

Theorem 3.19. Let (X, τ) be a neutrosophic orbit co-kernal space. Let { $NO_f(x_i)$, $x_i \in X$, i = 1, 2, 3, ..., n } be a sequence of neutrosophic closed orbit sets and $NO_f(y)$, y∈ X be a neutrosophic open orbit set with $NO_f(y) = \varphi_N$ such that $\overline{NO_f(y)} \cap (\bigcup_{i=1}^{n} \overline{NO_f(x_i)}) = \varphi_N$. Then there is an integer j such that $NO_f(y) \cap$ Nker($NO_f(x_j)$) = φ_N.

Proof Suppose that, $NO_f(y) \cap$ Nker($NO_f(x_j)$) = φ_N for all j. Let $NO_f(x_i)$ − Nker($NO_f(x_i)$) = NOf $(x_i) \cap$ Nco-ker($\overline{NO_f(x_i)}$) be a neutrosophic closed orbit set. Let $NO_f(b_i)$ = $\overline{NO_f(x_i) - Nker(NO_f(x_i))}$ be a neutrosophic open orbit set with $\overline{NO_f(x_i) - Nker(NO_f(x_i))} = \varphi_N$. By assumption, $\overline{NO_f(y)} \cap (\bigcup_{i=1}^{n} \overline{NO_f(x_i) - Nker(NO_f(x_i))}) = \varphi_N$. $\overline{NO_f(y)} \cap (\bigcup_{i=1}^{n} NO_f(b_i)) = \varphi_N$. $\overline{NO_f(y) \cup (\bigcap_{i=1}^{n}(\overline{NO_f(b_i)}))} = \overline{X_N}$. $NO_f(y) \cup (\bigcap_{i=1}^{n}(\overline{NO_f(b_i)})) = X_N$.

By Theorem 3.14., Nco-ker($\bigcap_{i=1}^{n} NO_f(b_i)$) = φ_N (6)

Then there is an integer j such that Nco-ker($NO_f(b_j)$) = φ_N. (7)

From (6) and (7), we have (X, τ) is not a neutrosophic orbit co-kernal space.

Which is a contradiction. Hence, $NO_f(y) \cap$ Nker($NO_f(x_j)$) = φ_N.

IV. ON NEUTROSOPHIC ORBIT CO-KERNAL CONTINUOUS FUNCTIONS

Definition 4.1. Let (X, τ) and (X, σ) be any two neutrosophic topological spaces. Let f : (X, τ) → (X, σ) be a function. Then f is said to be a neutrosophic orbit co-kernal function if Nco-ker($\bigcap_{i=1}^{n} f^{-1}(NO_f(y_i))$) = φ_N and Nco-ker($f^{-1}(NO_f(y))$) = φ_N in (X, τ) for every neutrosophic open orbit set $NO_f(y)$, y ∈ X in (X, σ).

Theorem 4.2. Let (X, τ) be a neutrosophic topological space. If f : (X, τ) → (X, τ) is a neutrosophic orbit co-kernal function then (X, τ) is a neutrosophic orbit co-kernal space.

Proof The proof is simple.

Definition 4.3. Let (X, τ) and (X, σ) be any two neutrosophic topological spaces. Let f : (X, τ) → (X, σ) be a function. Then f is said to be a neutrosophic orbit continuous function if the inverse image of each neutrosophic open orbit set in (X, σ) is a neutrosophic open orbit set in (X, τ).

Theorem 4.4. Let (X, τ), (X, σ) and (X, η) be any three neutrosophic topological spaces. If f : (X, τ) → (X, σ) is a neutrosophic orbit co-kernal function and g : (X, σ) → (X, η) is a neutrosophic orbit continuous function then g ∘ f : (X, τ) → (X, η) is a neutrosophic orbit co-kernal function.

Proof Let $NO_f(x)$, x ∈ X be a neutrosophic open orbit set in (X, η). Since g is a neutrosophic orbit continuous function, $g^{-1}(NO_f(x))$ is a neutrosophic open orbit set in (X, S). Since f is a neutrosophic orbit co-kernal function, Nco-ker($\bigcap_{i=1}^{n} f^{-1}(g^{-1}(NO_f$

$(x_i))) = \varphi_N$ and Nco-ker$(f^{-1}(g^{-1}(NO_f(x)))) = \varphi_N$. This implies that Nco-ker$(\bigcap_{i=1}^{n}(g \circ f)^{-1}(NO_f(x_i)))) = \varphi_N$ and Nco-ker$(g \circ f)^{-1}(NO_f(x))) = \varphi_N$. Hence, $g \circ f$ is a neutrosophic orbit co-kernal function.

Theorem 4.5. Let (X, τ), (X, σ) and (X, η) be any three neutrosophic topological spaces. Let $g : (X, \sigma) \to (X, \eta)$ be a neutrosophic open orbit function. If $g \circ f : (X, \tau) \to (X, \eta)$ is a neutrosophic orbit co-kernal function then $f : (X, \tau) \to (X, \sigma)$ is a neutrosophic orbit co-kernal function .

Proof Let $NO_f(x)$, $x \in X$ be a neutrosophic open orbit set in (X, σ). Since g is a neutrosophic open orbit function, $g(NO_f(x))$ is a neutrosophic open orbit set in (X, η). Since $g \circ f$ is a neutrosophic orbit co-kernal function, Nco-ker$(\bigcap_{i=1}^{n}(g \circ f)^{-1}(g(NO_f(x_n)))) = \varphi_N$ and Nco-ker$((g \circ f)^{-1}(g(NO_f(x)))) = \varphi_N$.

This implies that, Nco-ker$(\bigcap_{i=1}^{n} f^{-1}(g^{-1}(g(NO_f(x_n))))) = \varphi_N$ and Nco-ker$(f^{-1}(g^{-1}(g(NO_f(x))))) = \varphi_N$. Therefore, Nco-ker$(\bigcap_{i=1}^{n} f^{-1}(NO_f(x_n))) = \varphi_N$ and Nco-ker$(f^{-1}(NO_f(x)))) = \varphi_N$. Hence, f is a neutrosophic orbit co-kernal function.

Definition 4.6. Let (X, τ) and (X, σ) be any two neutrosophic orbit co- kernal spaces. Let $f : (X, \tau) \to (X, \sigma)$ be a function. Then f is said to be a neutrosophic hardly open orbit function if for each neutrosophic orbit set $NO_f(x)$, $x \in X$ of (X, σ) with Nco-ker$(NO_f(x)) = \varphi_N$ such that $NO_f(x) \subset NO_f(y) \subset X_N$ for some neutrosophic open orbit set $NO_f(y)$, $y \in X$ of (X, σ), Nco-ker$(f^{-1}(NO_f(x))) = \varphi_N$ in (X, τ).

Theorem 4.5. Let (X, τ) and (X, σ) be any two neutrosophic orbit co-kernal spaces. If $f : (X, \tau) \to (X, \sigma)$ is onto function then the following statements are equivalent:

(i) f is an neutrosophic hardly open orbit function.

(ii) Nker$(f(NO_f(x))) = X_N$, for all neutrosophic orbit set $NO_f(x)$ of (X, τ) with Nker$(NO_f(x)) = X_N$ and there exists a neutrosophic closed orbit set $NO_f(y) = X_N$ of (X, σ) such that $NO_f(y) \subseteq f(NO_f(x))$.

(iii) Nker$(f(NO_f(x))) = X_N$, for all neutrosophic orbit set $NO_f(x)$ of (X, τ) with Nker$(N O_f(x)) = X_N$ and there exists a neutrosophic closed orbit set $NO_f(y) = X$ of (X, σ) such that $f^{-1}(NO_f(y)) \subseteq NO_f(x)$.

Proof (i) $\Rightarrow$ (ii)

Assume that (i) is true. Let $NO_f(x)$ be a neutrosophic orbit set of (X, τ) with Nker$(NO_f(x)) = X_N$ and $NO_f(y) = X_N$ be a neutrosophic closed orbit set of (X, σ) such that $NO_f(y) \subseteq f(NO_f(x))$. Suppose that Nker$(NO_f(x)) = X_N$. This implies that Nco-ker$(\overline{f(NO_f(x))}) = \varphi_N$ in (X, σ) and $\overline{f(NO_f(x))} \subseteq \overline{NO_f(y)}$. By assumption, Nco-ker$(f^{-1}(\overline{f(NO_f(x))})) = \varphi_N$ in (X, τ). Now, Nker$(NO_f(x))$ = Nker$(f^{-1}(f(NO_f(x))))$ = $\overline{\text{Nco-ker}(\overline{f^{-1}(f(NO_f(x)))})} = \overline{\phi_N} = X_N$. This is a contradiction. Hence, (i) $\Rightarrow$ (ii).

(ii) $\Rightarrow$ (iii)

Assume that (ii) is true. Since f is onto function and by assumption, $NO_f(y) \subseteq f(NO_f(x))$. This implies that $f^{-1}(NO_f(y)) \subseteq f^{-1}(f(NO_f(x)))$. That is, $f^{-1}(NO_f(y)) \subseteq NO_f(x)$. Hence, (ii) $\Rightarrow$ (iii).

(iii) $\Rightarrow$ (i) Let $NO_f(u) \subseteq \overline{NO_f(v)}$, $u, v \in X$ with Nco-ker$(NO_f(u)) = \varphi_N$ and $NO_f(v)$ is a neutrosophic open orbit set of (X, σ). Let $NO_f(x) = f^{-1}(\overline{NO_f(u)})$ and $NO_f(y) = \overline{NO_f(v)}$. Now, $f^{-1}(NO_f(y)) = f^{-1}(\overline{NO_f(v)}) \subseteq f^{-1}(\overline{NO_f(u)}) = NO_f(x)$. Consider, Nker$(f(NO_f(x))$ = Nker$(f(f^{-1}(\overline{NO_f(u)})))$ = Nker$(\overline{NO_f(u)})$ = $\overline{\text{Nco-ker}(NO_f(u))} = \overline{\phi_N} = X_N$.Hence, Nker$(NO_f(x)) = X_N$ implies that, Nker$(f^{-1}(\overline{NO_f(u)}))$ = Nker$(\overline{f^{-1}(NO_f(u))}) = X_N$. Therefore, $\overline{\text{Nco-ker}(f^{-1}(NO_f(u)))} = X_N$. This implies that Nco-ker$(f^{-1}(NO_f(u))) = \varphi_N$. Therefore, N co-ker$(f^{-1}(NO_f(u))) = \varphi_N$ in (X, τ). This implies that f is a neutrosophic hardly open orbit function. Hence, (iii)$\Rightarrow$(i).

CONCLUSION

In this paper, the concepts of neutrosophic orbit co-kernal spaces, neutrosophic O_δ -sets, neutrosophic orbit meager* sets, neutrosophic orbit comeager* sets, neutrosophic orbit meager* spaces, neutrosophic orbit quasi regular spaces, and neutrosophic orbit strongly complete spaces are studied. Also the concepts of neutrosophic orbit co-kernal functions and neutrosophic hardly open orbit functions are discussed.

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