NEUTROSOPHIC GENERALIZED REGULAR SETS IN NEUTROSOPHIC TOPOLOGICAL SPACES

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ABSTRACT-
In this paper,the concept of Neutrosophic generalized regular closed set and Neutrosophic generalized regular closure in Neutrosophic topological spaces and its properties are investigated.

KEYWORDS-
Neutrosophic regular closed set, Neutrosophic regular open set, Neutrosophic generalized regular closed set and Neutrosophic generalized regular open set.

1. INTRODUCTION

C.L.Chang[3] introduced and developed fuzzy topological space by using L.A. Zadeh[12] fuzzy sets. Coker[4] introduced the notion of Intuitionistic fuzzy topological spaces by using Atanassov[2] intuitionistic fuzzy set . Neutrality the degree of indeterminacy, as an independent concept, was introduced by Smarandache[7] He also defined the Neutrosophic set on three component Neutrosophic topological spaces (t,f,i) =(Truth, Falsehood, Indeterminacy) .The Neutrosophic crisp set concept was converted to Neutrosophic topological spaces by Salama[10]. R.Dhavaseelan[5] introduced Neutrosophic generalized closed sets. V.K.Shanthi[11] introduced Neutrosophic Generalized Semi Closed Sets In Neutrosophic Topological Spaces.

2. PRELIMINARIES

Definition 2.1 [5,11]
Let X be a non-empty fixed set. A Neutrosophic set A has the form
$A = \{(x,\mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\}$
where $\mu_A(x), \sigma_A(x), \gamma_A(x)$ are topological spaces and $\mu_A(x)$ is the degree of membership function, $\sigma_A(x)$ is the degree of indeterminacy and $\gamma_A(x)$ is the degree of non-membership function respectively of each $x \in X$ to the set A.

Remark 2.2 [5,11]
A Neutrosophic set $A = \{(x,\mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\}$ can be identified to an ordered triple $(\mu_A, \sigma_A, \gamma_A)$ in $]0^-, 1^+[$on X.

Example 2.3 [5,11]
Since our main purpose is to construct the tools for developing Neutrosophic set and Neutrosophic topology, we must introduce the Neutrosophic set

0_N and 1_N in X as follows:
0_N may be defined as
$(0_1)0_N = \{(x, 0, 0, 1): x \in X\}$
$(0_2)0_N = \{(x, 0, 1, 1): x \in X\}$
$(0_3)0_N = \{(x, 0, 1, 0): x \in X\}$
$(0_4)0_N = \{(x, 0, 0, 0): x \in X\}$
1_N may be defined as

$(1_1) 1_N = \{(x, 1, 0, 0): x \in X\}$
$(1_2) 1_N = \{(x, 1, 0, 1): x \in X\}$
$(1_3) 1_N = \{(x, 1, 1, 0): x \in X\}$
$(1_4) 1_N = \{(x, 1, 1, 1): x \in X\}$.

Definition 2.4 [11]
Let $A = \{(x, \mu_A, \sigma_A, \gamma_A)\}$ be Neutrosophic set on X , then Complement of set A i.e., A^C is defined as $A^C = \{(x, \gamma_A(x), 1-\sigma_A(x), \mu_A(x)) : x \in X\}$.

Definition 2.5 [11]
Let X be a non-empty set and A and B are Neutrosophic sets of the form
$A = \{(x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\}$ and
$B = \{(x, \mu_B(x), \sigma_B(x), \gamma_B(x)) : x \in X\}$
then we consider the definition of subset $(A \subseteq B)$ is defined as
$A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x), \sigma_A(x) \le \sigma_B(x), \gamma_A(x) \ge \gamma_B(x)$, for all $x \in X$.

Proposition 2.6 [11]
For any Neutrosophic set A the following condition holds
(i) $0_N \subseteq A$, $0_N \subseteq 0_N$,
(ii) $A \subseteq 1_N$, $1_N \subseteq 1_N$.

Definition 2.7 [11]
Let X be a non-empty set and
$A = \{x, \mu_A(x), \sigma_A(x), \gamma_A(x)\}$ and
$B = \{x, \mu_B(x), \sigma_B(x), \gamma_B(x)\}$ are Neutrosophic sets then $A \cap B$ is defined as
$A \cap B = \{x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\}$ then $A \cup B$ is defined as
$A \cup B = \{x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x)\}$.

Proposition 2.8 [11]
For all A and B are two Neutrosophic sets then the following condition are true :
(i) $(A \cap B)^C = (A)^C \cup (B)^C$,
(ii) $(A \cup B)^C = (A)^C \cap (B)^C$, where C refers to Complement.

Definition 2.9 [5,11]
A Neutrosophic topology is a non-empty set X is an family τ_N of Neutrosophic subsets in X satisfying the axioms
(i) 0_N , $1_N \in \tau_N$
(ii) $G_1 \cap G_2 \in \tau_N$ for any $G_1, G_2 \in \tau_N$
(iii) $\cup G_i \in \tau_N$ for every $\{ G_j : j \in J\} \subseteq \tau_N$
the pair (X, τ_N) is called Neutrosophic topological space.
The element in Neutrosophic topological space (X, τ_N) are called Neutrosophic open sets.
A Neutrosophic set F is closed if and only if $(F)^C$ is Neutrosophic open.

Example 2.10 [11]
Let $X = \{x\}$ and
$A_1 = \{(x, 0.6, 0.6, 0.5): x \in X\}$
$A_2 = \{(x, 0.5, 0.7, 0.9): x \in X\}$
$A_3 = \{(x, 0.6, 0.7, 0.5): x \in X\}$
$A_4 = \{(x, 0.5, 0.6, 0.9): x \in X\}$
Then the family $\tau_N = \{0_N, A_1, A_2, A_3, A_4, 1_N\}$ is called a Neutrosophic topological space X.

Definition 2.11 [1,11]
Let (X, τ_N) Neutrosophic topological spaces and
$A = \{(x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\}$ be Neutrosophic set in X. Then the *Neutrosophic closure* and *Neutrosophic interior* are defined as
$Ncl(A) = \cap \{K : K$ is Neutrosophic closed set in X and $A \subseteq K\}$,
$Nint(A) = \cup \{G : G$ is Neutrosophic open set in X and $G \subseteq A\}$.

Definition 2.12 [11]
A is Neutrosophic open set if and only if $A = Nint(A)$,
A is Neutrosophic closed set if and only if $A = Ncl(A)$.

Proposition 2.13 [11]
For Neutrosophic set A in (X, τ_N) we have
$Ncl(A^C) = (Nint(A))^C$
$Nint(A^C) = (Ncl(A))^C$

Proposition 2.14 [11]
Let (X, τ_N) Neutrosophic topological spaces and A, B be two Neutrosophic sets in X. Then the following properties holds:
(i) $Nint(A) \subseteq A$,
(ii) $A \subseteq Ncl(A)$,
(iii) $A \subseteq B \Rightarrow Nint(A) \subseteq Nint(B)$,
(iv) $A \subseteq B \Rightarrow Ncl(A) \subseteq Ncl(B)$,
(v) $Nint(Nint(A))=Nint(A)$,
(vi) $Ncl(Ncl(A))=Ncl(A)$,
(vii) $Nint(A \cap B) = Nint(A) \cap Nint(B)$,
(viii) $Ncl(A \cup B) = Ncl(A) \cup Ncl(B)$,
(ix) $Nint(0_N)=0_N$,
(x) $Nint(1_N)=1_N$,
(xi) $Ncl\ (0_N)=0_N$,
(xii) $Ncl\ (1_N)=1_N$,
(xiii) $A \subseteq B \Rightarrow (B)^C \subseteq (A)^C$,
(xiv) $Ncl(A \cap B) \subseteq Ncl(A) \cap Ncl(B)$,
(xv) $Nint(A \cup B) \supseteq Nint(A) \cup Nint(B)$.

Definition 2.15 [1]
A subset A of Neutrosophic space (X, τ_N) is called Neutrosophic regular open (in short *NR open*) if $A = Nint(Ncl(A))$ .The Complement of *NR open* set is called *NR closed*.

Definition 2.16 [11]
A subset A of Neutrosophic space (X, τ_N) is called Neutrosophic generalized closed (in short *NG closed*) if $Ncl(A) \subseteq U$, whenever $A \subseteq U$ and U is Neutrosophic open.
The Complement of a *NG closed* set is called *NG open* set.

3. NEUTROSOPHIC GENERALIZED REGULAR CLOSED SET IN NEUTROSOPHIC TOPOLOGICAL SPACES

In this section we introduce the concept Neutrosophic generalized regular closed set in Neutrosophic topological spaces.

Definition 3.1
Let A be a subset of Neutrosophic space (X, τ_N) is called Neutrosophic generalized regular closed *(NGR closed)* if *Neutrosophic Regular* $cl(A) \subseteq U$ (in short $NRcl(A) \subseteq U$), whenever $A \subseteq U$ and U is Neutrosophic open.
The Complement of a *NGR closed* set is called *NGR open* set.

Example 3.2
Let $X = \{a, b\}$ and where $A_1 = \{(0.4, 0.6, 0.5), (0.7, 0.3, 0.6)\}$,
$A_2 = \{(0.3, 0.7, 0.8), (0.6, 0.4, 0.2)\}$, $A_3 = \{(0.4, 0.7, 0.5), (0.7, 0.4, 0.2)\}$,
$A_4 = \{(0.3, 0.6, 0.8), (0.6, 0.3, 0.6)\}$ and $\tau_N = \{0_N, A_1, A_2, A_3, A_4, 1_N\}$ is Neutrosophic topological space. Then $A_5 = \{(0.5, 0.7, 0.5), (0.9, 0.4, 0.5)\}$ is *NGR closed* set.

Definition 3.3

Let (X, τ_N) be a Neutrosophic topological space. Then for a Neutrosophic subset A of X the Neutrosophic generalized regular interior of A is the union of all Neutrosophic generalized regular open sets of X contained in A. i.e., $NGR\ int(A) = \cup$ {G:G is a *NGR open* sets in X and G $\subseteq$A}.

Proposition 3.4

Neutrosophic subsets A and B of Neutrosophic topological space X . We have

(i)$NGR\ int(A) \subseteq A$

(ii)A is *NGR-open* set in $X \Leftrightarrow NGR\ int(A) = A$

(iii)$NGR\ int(NGR\ int(A)) = NGR\ int(A)$

(iv)If $A \subseteq B$ then $NGR\ int(A) \subseteq NGR\ int(B)$

Proof :

(i)It follows from definition 3.3, We get $NGR\ int(A) \subseteq A$.

(ii) Let A be Neutrosophic subset of X and *NGR-open* sets in X.

Then $A \subseteq NGR\ int(A)$

By using (i) we get, $NGR\ int(A) \subseteq A$

Then we can say, $A = NGR\ int(A)$

Conversely, Assume $A = NGR\ int(A)$

By using definition 3.3, A is *NGR-open* set in X.

(iii) Put $A = NGR\ int(A)$ in (ii),We get $NGR\ int(NGR\ int(A)) = NGR\ int(A)$.

(iv) Given $A \subseteq B$, Using (i) we get, $NGR\ int(A) \subseteq A \subseteq B$

i.e., $NGR\ int(A) \subseteq B$

Using (iii) we get, $NGR\ int(NGR\ int(A)) \subseteq NGR\ int(B)$

(Since By using proposition 2.14(v))

We get, $NGR\ int(A) \subseteq NGR\ int(B)$

Hence proved.

Proposition 3.5

Let (X, τ_N) be Neutrosophic topological space.Then for any Neutrosophic subsets A and B of Neutrosophic topological spaces. We get

(i)$NGR\ int(A \cap B) = NGR\ int(A) \cap NGR\ int(B)$

(ii)$NGR\ int(A \cup B) \supseteq NGR\ int(A) \cup NGR\ int(B)$.

Proof:

(i) Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$

By using proposition 3.4(iv) We get, $NGR\ int(A \cap B) \subseteq NGR\ int(A)$ and

$NGR\ int(A \cap B) \subseteq NGR\ int(B)$

Therefore $NGR\ int(A \cap B) \subseteq NGR\ int(A) \cap NGR\ int(B)$-------------------------------*(1)*

Using proposition 3.4(i),we get, $NGR\ int(A) \subseteq A$ and $NGR\ int(B) \subseteq B$

$\Rightarrow NGR\ int(A) \cap NGR\ int(B) \subseteq A \cap B$

By using proposition 3.4(iv) we get,

$NGR\ int(NGR\ int(A) \cap NGR\ int(B)) \subseteq NGR\ int(A \cap B)$

$\Rightarrow NGR\ int(NGR\ int(A)) \cap NGR\ int(NGR\ int(B)) \subseteq NGR\ int(A \cap B)$

(Since By using proposition 3.4(iii))

We get, $NGR\ int(A) \cap NGR\ int(B) \subseteq NGR\ int(A \cap B)$----------------------------------*(2)*

From *(1)* and *(2)*, $NGR\ int(A \cap B) = NGR\ int(A) \cap NGR\ int(B)$

(ii) Since $A \subseteq A \cup B$ and $B \subseteq A \cup B$

By using proposition 3.4(iv) $NGR\ int(A) \subseteq NGR\ int(A \cup B)$ and

$NGR\ int(B) \subseteq NGR\ int(A \cup B)$

$\Rightarrow NGR\ int(A) \cup NGR\ int(B) \subseteq NGR\ int(A \cup B)$.

Therefore $NGR\ int(A \cup B) \supseteq NGR\ int(A) \cup NGR\ int(B)$

Hence proved.

The following example shows that the equality need not be hold in proposition 3.5(ii)

Example 3.6

Let $X = \{a, b, c\}$ and $\tau_N = \{0_N, A_1, A_2, A_3, A_4, 1_N\}$
where $A_1 = \{(0.41, 0.71, 0.11), (0.51, 0.61, 0.21), (0.91, 0.71, 0.31)\}$,
$A_2 = \{(0.41, 0.61, 0.11), (0.71, 0.71, 0.21), (0.91, 0.51, 0.11)\}$,
$A_3 = \{(0.41, 0.71, 0.11), (0.71, 0.71, 0.21), (0.91, 0.71, 0.11)\}$,
$A_4 = \{(0.41, 0.61, 0.11), (0.51, 0.61, 0.21), (0.91, 0.51, 0.31)\}$
and (X, τ_N) be Neutrosophic topological spaces.
Consider two Neutrosophic sets,
$E = \{(0.71, 0.61, 0.11), (0.71, 0.61, 0.11), (0.91, 0.51, 0.01)\}$,
$F = \{(0.41, 0.61, 0.11), (0.51, 0.71, 0.21), (0.97, 0.71, 0.11)\}$.
Then *NGR int(E)=D* and *NGR int(F)=D* . This implies that *NGR int*$(E \cup F)$ = D
Now $E \cup F = \{(0.71, 0.61, 0.11), (0.71, 0.71, 0.11), (0.97, 0.71, 0.01)\}$and
NGR int$(E \cup F)$ = B. Then *NGR int*$(E \cup F) \nsubseteq$ *NGR int*$(E) \cup$ *NGR int*(F).

4. NEUTROSOPHIC GENERALIZED REGULAR CLOSURE IN NEUTROSOPHIC TOPOLOGICAL SPACES

In this section we introduce the concept of Neutrosophic generalized regular closure in Neutrosophic topological spaces.

Definiton 4.1

Let (X, τ_N) be Neutrosophic topological space. Then for a Neutrosophic subset A of X the Neutrosophic generalized regular closure of A is the intersection of all Neutrosophic generalized regular closed sets of X contained in A.
i.e., *NGR cl(A)* = ∩ {K:K is a *NGR closed* sets in X and K ⊇ A}.

Proposition 4.2

Let (X, τ_N) be Neutrosophic topological space. Then for any Neutrosophic subset A of X
(i) $(NGR\ int(A))^C = NGR\ cl\ (A)^C$
(ii) $(NGR\ cl\ (A))^C = NGR\ int(A)^C$.
Proof :
(i)By using definition 3.3 i.e.,
NGR int(A) = ∪ {G:G is a *NGR open* sets in X and G ⊆ A}
Taking Complement on Both sides , we get
$(NGR\ int(A))^C = \cap (\{G:G$ is a *NGR open* sets in X and $G \subseteq A\})^C$
$\Rightarrow \cap\{(G)^C : (G)^C$ is a *NGR closed* sets in X and $(G)^C \supseteq (A)^C\}$
Replace $(G)^C$ by K , we get
$NGR\ cl\ (A)^C = \cap\ \{K:K$ is a *NGR closed* sets in X and $K \supseteq (A)^C\}$
By using definition 4.1, we get $(NGR\ int(A))^C = NGR\ cl\ (A^C)$.

Proposition 4.3

Let (X, τ_N) be Neutrosophic topological space. Then for any Neutrosophic subsets A and B of Neutrosophic topological space X. We have,
(i)$A \subseteq NGR\ cl(A)$
(ii)A is *NGR closed set in X* ⇔ $NGR\ cl(A) = A$
(iii)$NGR\ cl(NGR\ cl(A)) = NGR\ cl(A)$
(iv)If $A \subseteq B$ then $NGR\ cl(A) \subseteq NGR\ cl(B)$.
Proof :
(i)By using definition 4.1, We can say $A \subseteq NGR\ cl(A)$.

(ii)Let A be *NGR closed set in X* Then$(A)^C$ be *NGR open set in X*
By using proposition 4.2(ii), $NGR\ int(A)^C = (A)^C$

$\Leftrightarrow (NGR\ cl\ (A))^C = (A)^C$
Taking Complement on both sides, we get
$\Leftrightarrow NGR\ cl(A) = A$.

(iii)Put $A = NGR\ cl(A)$ in (ii), we get $NGR\ cl(NGR\ cl(A)) = NGR\ cl(A)$.

(iv)Since $A \subseteq B$ then $(B)^C \subseteq (A)^C$
By using proposition 3.4(iv),
$NGR\ int(B)^C \subseteq NGR\ int(A)^C$
Taking Complement on both sides,
$(NGR\ int(B)^C)^C \supseteq (NGR\ int(A)^C)^C$
By using proposition 4.2 (ii), $((NGR\ cl\ (B))^C)^C \supseteq ((NGR\ cl\ (A))^C)^C$
$\Rightarrow NGR\ cl(A) \subseteq NGR\ cl(B)$.
Hence proved.

Proposition 4.4
Let A be a Neutrosophic set in a Neutrosophic topological space X. Then
$Nint(A) \subseteq NGR\ int(A) \subseteq A \subseteq NGR\ cl(A) \subseteq Ncl(A)$
The result is obvious.

Proposition 4.5
Let (X, τ_N) be Neutrosophic topological space. Then for a Neutrosophic subsets and B of Neutrosophic topological space X. We have,
(i)$NGR\ cl(A \cup B) = NGR\ cl(A) \cup NGR\ cl(B)$
(ii)$NGR\ cl(A \cap B) \subseteq NGR\ cl(A) \cap NGR\ cl(B)$.

Proof :
(i)Since $NGR\ cl(A \cup B) = NGR\ cl\ ((A \cup B)^C)^C$
By using proposition 4.2(i), $NGR\ cl(A \cup B) = (NGR\ int(A \cup B)^C)^C$
$NGR\ cl(A \cup B) = (NGR\ int(A^C) \cap (B^C))^C$
By using proposition 3.5(i), we get
$NGR\ cl(A \cup B) = (NGR\ int(A^C) \cap (NGR\ int(B^C))^C$
$NGR\ cl(A \cup B) = (NGR\ int(A^C))^C \cup (NGR\ int(B^C))^C$
By using proposition 4.2(i) ,the above equation becomes
$NGR\ cl(A \cup B) = NGR\ cl\ (A^C)^C \cup NGR\ cl\ (B^C)^C$
$NGR\ cl(A \cup B) = NGR\ cl(A) \cup NGR\ cl(B)$
Hence we obtain the result.

(ii)Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$
By using proposition 4.3 (iv),we get
$NGR\ cl(A \cap B) \subseteq NGR\ cl(A)$ and $NGR\ cl(A \cap B) \subseteq NGR\ cl(B)$
Hence $NGR\ cl(A \cap B) \subseteq NGR\ cl(A) \cap NGR\ cl(B)$.
Hence proved.

The following example shows that the equality need not be hold in proposition 4.5(ii)
Example 4.6
Let $X = \{a, b, c\}$ and $\tau_N = \{0_N , A_1, A_2, A_3, A_4, 1_N \}$ and
$(\tau_N)^C = \{1_N , A_5, A_6, A_7, A_8, 0_N \}$
where $A_1 = \{(0.5, 0.6, 0.1), (0.6, 0.7, 0.1), (0.9, 0.5, 0.2)\}$
$A_2 = \{(0.4, 0.5, 0.2), (0.8, 0.6, 0.3), (0.9, 0.7, 0.3)\}$
$A_3 = \{(0.4, 0.5, 0.2), (0.6, 0.6, 0.3), (0.9, 0.5, 0.3)\}$
$A_4 = \{(0.5, 0.6, 0.1), (0.8, 0.7, 0.1), (0.9, 0.7, 0.2)\}$
$A_5 = \{(0.1, 0.4, 0.5), (0.1, 0.3, 0.6), (0.2, 0.5, 0.9)\}$
$A_6 = \{(0.2, 0.5, 0.4), (0.3, 0.4, 0.8), (0.3, 0.3, 0.9)\}$
$A_7 = \{(0.2, 0.5, 0.4), (0.3, 0.4, 0.6), (0.3, 0.5, 0.9)\}$
$A_8 = \{(0.1, 0.4, 0.5), (0.1, 0.3, 0.8), (0.2, 0.3, 0.9)\}$
And (X, τ_N) be Neutrosophic topological spaces.

Consider the Neutrosophic sets,
A_9 = {(0.1, 0.2, 0.5), (0.2, 0.3, 0.7), (0.3, 0.3, 1)} and
A_{10} = {(0.2, 0.4, 0.8), (0.1, 0.2, 0.8), (0.2, 0.5, 0.9)}
then $NGR\ cl\ (A_9) = A_7$ and $NGR\ cl\ (A_{10}) = A_7$ This implies that
$NGR\ cl\ (A_9) \cap NGR\ cl\ (A_{10}) = A_7$
Now $A_9 \cap A_{10}$ ={(0.1, 0.2, 0.8), (0.1, 0.2, 0.8), (0.2, 0.3, 1)}. It follows that
$NGR\ cl(A_9) \cap NGR\ cl(A_{10}) = A_8$
Then we get $NGR\ cl(A_9) \cap NGR\ cl(A_{10}) \nsubseteq NGR\ cl(A_9 \cap A_{10})$.

Proposition 4.7

Let (X, τ_N) be Neutrosophic topological space. Then for a Neutrosophic subsets A and B of Neutrosophic topological space X. We have

(i)$NGR\ cl(A) \supseteq A \cup NGR\ cl(NGR\ int(A))$
(ii)$NGR\ int(A) \subseteq A \cap NGR\ int(NGR\ cl(A))$
(iii)$Nint(NGR\ cl(A)) \subseteq Nint(Ncl(A))$
(iv)$Nint(NGR\ cl(A)) \supseteq Nint(NGR\ cl(NGR\ int(A)))$.

Proof :
(i)By using proposition 4.3(i), we get $A \subseteq NGR\ cl(A)$
By again proposition 3.4(i) , we get $NGR\ int(A) \subseteq A$
Taking *NGR cl* on both sides above,
We get $NGR\ cl(NGR\ int(A)) \subseteq NGR\ cl(A)$.

(ii)By proposition 3.4(i) , we get $NGR\ int(A) \subseteq A$ -----------------------------*(1)*
By using proposition 4.3(i), we get $A \subseteq NGR\ cl(A)$
Taking *NGR int* on both sides , we get
$NGR\ int(A) \subseteq NGR\ int(NGR\ cl(A))$--*(2)*
From *(1)* and *(2)*,$NGR\ int(A) \subseteq A \cap NGR\ int(NGR\ cl(A))$.

(iii)By using proposition 4.4, we get $NGR\ cl(A) \subseteq Ncl(A)$
Taking *Nint* on both sides we get, $Nint(NGR\ cl(A)) \subseteq Nint(Ncl(A))$.

(iv) By (i) we know that , $NGR\ cl(A) \supseteq A \cup NGR\ cl(NGR\ int(A))$
Taking *Nint* on both sides we get ,
$Nint(NGR\ cl(A)) \supseteq Nint(A \cup NGR\ cl(NGR\ int(A)))$
By using proposition 3.5(ii), we get,
$Nint(NGR\ cl(A)) \supseteq Nint\ (A) \cup Nint(NGR\ cl(NGR\ int(A)))$
$\supseteq Nint(NGR\ cl(NGR\ int(A)))$
Hence the Proof.

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