# OBSERVATIONS ON TERNARY QUADRATIC DIOPHANTINE EQUATION $9(x^2+y^2)-17xy+5x+5y+25=51z^2$

[1]G.Janaki, [2]P.Dhivyabharathi

Assistant Professor, Department of Mathematics, Cauvery College for Women, Tiruchirappalli, Tamil Nadu, India[1]

PG Student, Department of Mathematics, Cauvery College for Women,Tiruchirappalli, Tamil Nadu, India[2]

**ABSTRACT**: The Ternary Quadratic Diophantine Equation speaking to non-homogeneous cone given by $9(x^2+y^2)-17xy+5x+5y+25=51z^2$ is investigated for its non-zero particular number focuses on it. Six distinct examples of essential arrangements fulfilling the cone under thought are acquired. A couple of intriguing relations between the arrangements and some extraordinary number examples spoke to.

**KEYWORDS:** Ternary non homogenous Quadratic, Integral solutions.

## I. INTRODUCTION

Ternary quadratic conditions are wealthy in assortment. For a broad audit of different one may allude $[1-7]$. In , $[8]$ the ternary quadratic Diophantine condition of the structure $kxy+m(x+y)=z^2$ has been considered for non-unimportant fundamental arrangements. In , $[9,10]$ an exceptional Pythagorean triangle issue have been examined for its fundamental arrangement. In , $[11,12]$ ternary quadratic Diophantine conditions have been examined for its fundamental arrangements.

This correspondence worries with one more fascinating ternary quadratic condition $9(x^2+y^2)-17xy+5x+5y+25=51z^2$ speaking to a cone for deciding its unendingly numerous non-zero essential focuses. Likewise fascinating connection among the arrangements are introduced.

## NOTATIONS

$\Pr_r$ = Pronic number of rank 'n'.

$T_{m,n}$ = Polygonal number of rank 'n' with sides 'm'.

$4DF_n$ = Four Dimensional Figurate number of rank 'n'.

$CS_n$ = Centered Square number of rank 'n'.

$SO_n$ = Stella Octangula number of rank 'n'.

$O_n$ = Octahedral number of rank 'n' with sides 'm'.

$P^5_n$ = Pyramidal number of rank 'n' with sides 'm'.

$Gno_n$ = Gnomonic number of rank 'n'.

$Star_n$ = Star number of rank 'n'.

**METHOD OF ANALYSIS**

The ternary quadratic Diophantine equation to be solved for its non-zero integral solution is

$$9\left(x^2+y^2\right)-17xy+5x+5y+25=51z^2 \qquad (1)$$

The substitution of linear transformation $x=u+v$ and $y=u-v$ $\qquad (2)$

$(2)$ in $(1)$ leads to,

$$(u+5)^2+35v^2=51z^2 \qquad (3)$$

We illustrate below six different patterns of non-zero distinct integer solutions to $(1)$

**PATTERN : 1**

Assume $\qquad z=z(a,b)=a^2+35b^2 \qquad (4)$

where, a and b are non-zero integers,

and we write $\qquad 51=\left(4+i\sqrt{35}\right)\left(4-i\sqrt{35}\right) \qquad (5)$

Using $(4)$ and $(5)$ in equation $(3)$ and using factorization method,

$$\left((u+5)+i\sqrt{35}\right)\left((u+v)-i\sqrt{35}\right)$$

$$=\left(4+i\sqrt{35}\right)\left(4-i\sqrt{35}\right)\left[\left(a+i\sqrt{35}b\right)^2\left(a-i\sqrt{35}b\right)^2\right] \qquad (6)$$

Equating the like terms and comparing real and imaginary parts we get,

$$u=u(a,b)=4a^2-140b^2-70ab$$

$$v=v(a,b)=a^2-35b^2+8ab$$

Substituting the above integer of u & v in equation (2) the corresponding integer solutions of (1) are given by

$$x=x(a,b)=5a^2-175b^2-62ab-5$$

$$y=y(a,b)=3a^2-105b^2-78ab-5$$

$$z=z(a,b)=a^2+35b^2$$

**OBSERVATIONS:**

(1). $x(a,1)+y(a,1)+z(a,1)-3T_{8,a}+67Gno_a\equiv 0(\mathrm{mod}\,322)$

(2). $[y(a,a)-x(a,a)+4z(a,a)]$ is a perfect square.

(3). $x(a,1)+y(a,1)-2z(a,1)-2T_{6,a}+69Gno_a\equiv 0(\mathrm{mod}\,289)$

(4). $4x(a,1)-y(a,1)-z(a,1)-8T_{6,a}+81Gno_a\equiv 0(\mathrm{mod}\,726)$

(5). $6[y(a,a)-x(a,a)+4z(a,a)]$ represents a Nasty number.

**PATTERN: 2**

Instead of (5), we write 51 as

$$51=\frac{(13+i\sqrt{35})(13-i\sqrt{35})}{4} \tag{7}$$

Following the procedure presented above, the corresponding non-zero distinct integer solutions to (1) are obtained as

$$x=x(A,B)=14A^2-490B^2-44AB-5$$

$$y=y(A,B)=12A^2-420B^2-96AB-5$$

$$z=z(A,B)=2A^2+70B^2$$

**OBSERVATIONS:**

(1). $7(y(A,A)-x(A,A))+2z(A,A)$ is a perfect square.

(2). $2y(A,1)+z(A,1)-10T_{8,A}+34Gno_A\equiv 0(\mathrm{mod}\,954)$

(3). $2x(A,1)+y(A,1)-12T_{12,A}+68Gno_A\equiv 0(\mathrm{mod}\,1413)$

(4). $x(A,1)+y(A,1)+7z(A,1)-4T_{22,A}+52Gno_A\equiv 0(\mathrm{mod}\,1462)$

(5). $6[7(y(A,A)-x(A,A))+2z(A,A)]$ represents a Nasty number.

**REMARKABLE NOTE:**

In addition one may write 51 as,

$$(1)\ .\ 51 = \frac{\left(8 + i2\sqrt{35}\right)\left(8 - i2\sqrt{35}\right)}{4}$$

$$(2)\ .51 = \frac{\left(12 + i3\sqrt{35}\right)\left(12 - i3\sqrt{35}\right)}{9}$$

$$(3)\ .\ 51 = \frac{\left(12 + i5\sqrt{35}\right)\left(12 - i5\sqrt{35}\right)}{25}$$

For the choices, one may obtain different patterns of solution of (1).

**PATTERN: 3**

Equation (3) can be written as

$$(u+5)^2 = 51z^2 - 35v^2 \qquad (8)$$

Introducing the linear transformation

$$z = X + 35T^2 \quad \text{and} \quad v = X + 51T \qquad (9)$$

In (8), we get

$$X^2 = 1785T^2 + \left(\frac{u+5}{4}\right)^2 \qquad (10)$$

Which is satisfied by

$$T(r,s) = 2rs$$

$$u(r,s) = 7140r^2 - 4s^2 - 5$$

$$X(r,s) = 1785r^2 + s^2$$

Employing the above values in (9) and using (2), the corresponding integer solutions of (1) are given by

$$x = x(r,s) = 8925r^2 - 3s^2 + 102rs - 5$$

$$y = y(r,s) = 5355r^2 - 5s^2 - 102rs - 5$$

$$z = z(r,s) = 1785r^2 + s^2 + 70rs$$

**OBSERVATONS:**

(1). $x(r,r)-y(r,r)-2z(r,r)$ is a perfect square.

(2). $6[x(r,r)-y(r,r)-2z(r,r)]$ represents a Nasty number.

(3). $3z(r,1)-y(r,1)-156Gno_r \equiv 0(\bmod 169)$

(4). $x(r,1)-y(r,1)-2z(r,1) \equiv 0(\bmod 8^2)$

(5). $x(r,1)-y(r,1)-1785CS_r-1887Gno_r \equiv 0(\bmod 3674)$

**PATTERN: 4**

Consider (3) as

$$(u+5)^2-16v^2=51(z^2-v^2) \qquad (11)$$

Write (12) in the form of ratio as

$$\frac{(u+5)+4v}{z+v}=\frac{51(z-v)}{(u+5)-4v}=\frac{A}{B}, B \neq 0$$

Which is equivalent of the following two equation

$$B(u+5)+v(4B-A)-Az=0 \qquad (i)$$

$$A(u+5)+v(51B-4A)-51Bz=0 \qquad (ii)$$

Applying the method of cross multiplication in $(i)$ and $(ii)$ we get

$$u=u(A,B)=4A^2-102AB+204B^2-5$$

$$v=v(A,B)=A^2-51B^2$$

$$z=z(A,B)=A^2-8AB+51B^2$$

Applying the above values of u and v in equation (2) the corresponding integer solutions of (1) are given by

$$x=x(A,B)=5A^2-102AB+153B^2-5$$

$$y=y(A,B)=3A^2-102AB+255B^2-5$$

$$z=z(A,B)=A^2-8AB+51B^2$$

**OBSERVATIONS:**

(1). $y(A,A)-x(A,A)+z(A,A)$ is a perfect square.

(2). $6[y(A,A)-x(A,A)+z(A,A)]$ represents a Nasty number.

(3) $y(A,1)-3z(A,1)+39Gno_A \equiv 0(\mathrm{mod}\,58)$

(4). $y(A,1)+x(A,1)-4T_{6,A}+100Gno_A \equiv 0(\mathrm{mod}\,292)$

(5). $x(A,1)-y(A,1)+3z(A,1)-T_{12,A}+10Gno_A \equiv 0(\mathrm{mod}\,41)$

**REMARKABLE NOTE:**

Equation (12) can also be expressed in the form of ratio in two different ways as follows.

$$(i).\ \frac{(u+5)+4v}{3(z+v)}=\frac{17(z-v)}{(u+5)-4v}=\frac{A}{B}, B\neq 0$$

$$(ii).\ \frac{(u+5)+4v}{17(z+v)}=\frac{3(z-v)}{(u+5)-4v}=\frac{A}{B}, B\neq 0$$

Repeating the analysis as above the corresponding integer solutions along with properties are presented below.

**Solution of $(i)$:**

$$x = x(A,B) = 15A^2 - 102AB + 51B^2 - 5$$

$$y = y(A,B) = 9A^2 - 102AB + 85B^2 - 5$$

$$z = z(A,B) = 3A^2 - 8AB + 17B^2$$

**OBSERVATIONS:**

(1). $y(A,A)-x(A,A)+3z(A,A)$ is a perfect square.

(2). $6[y(A,A)-x(A,A)+3z(A,A)]$ represents a Nasty number.

(3). $y(A,1)-x(A,1)-z(A,1)-T_{20,A} \equiv 0(\mathrm{mod}\,17)$

**Solution of** $(ii)$**:**

$$x = x(A,B) = 85A^2 - 102AB + 9B^2 - 5$$

$$y = y(A,B) = 51A^2 - 102AB + 15B^2 - 5$$

$$z = z(A,B) = 17A^2 - 8AB + 3B^2$$

**OBSERVATIONS:**

$(1).\ x(A,1) - y(A,1) - z(A,1) + 17T_{8,A} + 13Gno_A \equiv 0 \pmod{30}$

**IV. CONCLUSION**

To conclude one may search for other patterns of solutions for such types of equation.

V. **REFERRENCES**


[1] Batta.B and Singh.A.N, History of Hindhu Mathematics, Asia Publishing House 1938.

[2] Carmichael, R.D., “The Theory of Numbers and Diophantine Analysis”, Dover Publications, New York, 1959.

[3] Dickson,.L.E., “History of the theory of Numbers”, Chelsia PublishingCo., Vol.II, New York, 1952.

[4] Mollin.R.A., “All solutions of the Diophantine equations $x^2 - Dy^2 = n$,”, For East J.Math. Sci., Social Volume,1998, Part III.

[5] Mordell.L.J., “Diophantine Equations”, Academic Press, London 1969.

[6] Telang.s.G., “Number Theory”, Tata McGraw-Hill Publishing Company, New Delhi 1996.

[7] Gobalan.M.A., Vidhyalakshmi.S and Umarani.J., “On Ternary QuadraticDiophantine equation $6(x^2 + y^2) - 8xy = 21z^2$”, Sch.J. Eng. Tech. 2(2A);108- 112,2014.

[8] Gobalan.M.A., Vidhyalakshmi.S. and Shanthi.J “On Ternary QuadraticDiophantine equation $3(x^2 + y^2) - 5xy + x + y + 1 = 15z^2$”,International Journal of Innovative Science, Engineering & Technology, Vol l , Issue 6,2014.

[9] G.Janaki,C.Saranya, “Observations on Ternary Quadratic Diophantine Equation $6(x^2 + y^2) - 11xy + 3x + 3y + 9 = 72z^2$, International Journalof Innovative Research in Science, Engineering and Technology, Vol. 5,Issue 2, February 2016.

[10] G.Janaki., C.Saranya, “Observations on Ternary Quadratic DiophantineEquation $5x^2 + 7y^2 = 972z^2$”, Jamal Academic Research Journal :An Interdisciplinary, Special Issue, February 2016, pages 305-308.