ORTHOGONALLY BLOCKED RESPONSE SURFACE SPLIT-PLOT DESIGNS

Renu Kaul[1] *Anita Bansal[2] Sanjoy Roy Chowdhury[1] Archana Verma[3]

[1]Department of Statistics, Lady Shri Ram College, University of Delhi, India

[2]Department of Statistics, Ramjas College, University of Delhi, India

[3]Department of Statistics, Mata Sundri College, University of Delhi, India

*Corresponding Author E-mail: anitabansal68@yahoo.in

ABSTRACT

In Response surface experiments, we often encounter situations where levels of some factors are difficult, time consuming and/or costly to change while levels of other factors are comparatively easy to change. This results in a practical restriction on randomization giving rise to the split-plot design structure. Also, in many experiments, the study is too large to allow all the design points to be run under homogeneous conditions. Thus, hard to change factors are needed to be blocked. In this paper we have developed second-order response surface designs under split-plot structure using the designs given by [1]. Further we have given the method for orthogonal blocking of these designs.

Key words: Response surface designs, split-plot structure, orthogonal blocking, qualitative and quantitative factors.

INTRODUCTION

The concept of Response Surface Methodology (RSM) introduced by [2] in the context of experimentation in chemical industry. Later this concept has been extensively used in a wide variety of areas. Excellent reviews of RSM are given by [3-6].

In classical RSM we assume that the levels of all the factors are equally easy to change justifying the use of completely randomized designs. However, in many agricultural experiments there are factors like irrigation methods and ploughing methods which require large areas of land, called whole-plots and there are factors like different fertilizers or different seed varieties which require smaller areas of land, called sub-plots. Likewise, in industrial experiments there are factors whose levels are difficult, time consuming and/or costly to change called hard to change (HTC) factors along with factors whose levels are easy to change called easy to change (ETC) factors. The HTC factors are associated with whole-plots whereas ETC factors are associated with the sub-plots. During experimentation, it is therefore, desirable to change the levels of HTC factors less often. This, leads to the split-plot design structure. In a split-plot design, randomization is done in two stages giving rise to two errors, whole-plot errors and sub-plot errors.

Although the concept of response surface methodology has been extensively used but comparatively less research has been done regarding the construction of SPD in RS experiments. [7] gave second-order response surface designs under split-plot structure. [8] gave second-order split-plot designs in which ordinary least square estimators were equivalent to generalized least square estimators. [9-10] generalized the equivalence condition given by [8] and developed systematic construction procedures, VKM and

MWP for constructing Box-Behnken Designs (BBDs) and Central Composite Designs (CCDs) under split-plot structure. They considered balanced designs where whole-plots are of equal size. [11] & [12] have also discussed response surface designs under split-plot structure.

There are many situations in RSM where the study is too large to allow all runs to be made under homogeneous conditions. This is why blocking is needed. It is done to reduce the experimental error. A block comprises of a group of experimental runs that are run under relatively homogeneous conditions. A split-plot design is similar in structure to a blocked design if we view the WP as blocks. Thus, it is important to consider second-order designs that facilitate blocking and the design points should be assigned to blocks so as to minimize their effect on the estimation of the model coefficients. The concept of orthogonal blocking for second-order response surface designs was introduced by [13]. Orthogonal blocking provides the same estimate of the model parameters as would have been obtained had there been no blocking.

The conditions for orthogonal blocking of five-level CCDs under balanced split-plot structure were obtained by [14]. However, in practice, there are situations in which less number of distinct levels is desired. [15] First gave three level response surface designs. Further, [16] obtained three-level response surface designs that can be orthogonally blocked. [17] Proposed definitive screening designs. These designs are three level designs for estimates of quantitative factors that provide estimates of main effects that are unbiased by any second order effect and required only one more than twice as many run as there are factors. They have provided blocking schemes for both fixed and random blocks. A general method of constructing three-level second-order designs was given by [1] which can be orthogonally blocked. He used BIB designs with two plots per block to construct three level second order designs and showed that these designs have high D-efficiencies. However, the designs given by [1] were not under split-plot structure.

In this paper, in Section 2, we have used [1] designs to construct the second-order response surface designs under split-plot structure. Method for orthogonal blocking of these split-plot designs is given in Section 3. We have restricted ourselves to a maximum of two HTC factors in constructing these designs having up to 6 factors.

METHODOLOGY

Method for Constructing Response Surface Designs under Split-Plot Structure

A new class of three-level designs was given by [1] which was not under the split-plot structure. These designs satisfy the following moment conditions:

$$\sum_{u=1}^{n}\prod_{i=1}^{m} x_{iu}^{\alpha_i} = 0 \; if\ any\ \alpha_i\ is\ odd, for\ \alpha_i = 0,1,2\ or\ 3\ and\ \sum_{i=1}^{m}\alpha_i \le 4; \qquad (1)$$

$$\sum_{u=1}^{n} x_{iu}^2 = constant, \sum_{u=1}^{n} x_{iu}^4 = constant, \sum_{u=1}^{n} x_{iu}^2 x_{ju}^2 = constant,\ 1 \le i,j \le m, i \ne j \qquad (2)$$

where, for $1 \le i \le m, 1 \le u \le n, x_{iu}$ denotes the level of the i^{th} factor in the u^{th} run, m is the number of factors, n is the total number of runs in the second order design.

Let X be the design matrix having m factors. Let k_1 be the number of HTC factors (here, k_1 = 1, 2) and k_2 be the number of ETC factors such that $k_1+k_2=m$. Let z_i, i=1,2…., k_1 denote the HTC factors and x_j, j = 1,2, .., k_2 the ETC factors. The method for constructing the designs is as follows:

(i) Consider the designs given by [1] for m (=k_1+k_2) number of factors.

(ii) Allocate HTC factors to the first k_1 columns in the design.

(iii) Allocate k_2 ETC factors to the remaining columns.

(iv) The design is then sorted on the basis of HTC factors, in order to give it the split-plot structure where HTC factors form the whole-plots and ETC factors form the sub-plots.

(v) Some of the runs are repeated to make all whole-plots of equal size.

(vi) Add a whole-plot with overall center runs.

We illustrate the above method by constructing two second-order response surface designs under split-plot structure.

RESULTS AND DISCUSSION

Example 1: The orthogonal design developed by [1] for m = 4 factors is

Table 1: Orthogonal Design for m=4

±1	±1	**0**	**0**
±1	**0**	±1	**0**
±1	**0**	**0**	±1
0	±1	±1	**0**
0	±1	**0**	±1
0	**0**	±1	±1

The **0** represents a 4x1 vector of zeros and (±1, ±1) represent the four points of a 2^2 factorial with levels -1 and +1. This gives 24 non-center runs. The four columns of the design represent the four factors x_1, x_2, x_3 and x_4. Let us consider first the example of one HTC factor and three ETC factors. Allocate HTC factor z_1 to the first column and ETC factors to the rest of the three columns. The design is sorted on the basis of the HTC factor. After sorting the design some of the runs are repeated to make all the whole-plots of equal size. This generates 8 whole-plots of size four each. One whole-plot consisting of four center runs is added to the design. The split-plot design so generated is given below where WP column gives the whole-plot number.

Table 2: Split-Plot Design for m=4 & one HTC factor and three ETC factors

WP	z_1	x_2	x_3	x_4	WP	z_1	x_2	x_3	x_4	WP	z_1	x_2	x_3	x_4
1	-1	1	0	0	4	0	0	1	1	7	1	0	1	0
	-1	-1	0	0		0	0	1	-1		1	0	-1	0
	-1	1	0	0		0	0	1	1		1	0	0	1
	-1	-1	0	0		0	0	1	-1		1	0	0	-1
2	-1	0	1	0	5	0	-1	1	0	8	1	1	0	0
	-1	0	-1	0		0	-1	-1	0		1	-1	0	0
	-1	0	0	1		0	-1	0	1		1	1	0	0
	-1	0	0	-1		0	-1	0	-1		1	-1	0	0
3	0	0	-1	1	6	0	1	1	0	9	0	0	0	0
	0	0	-1	-1		0	1	-1	0		0	0	0	0
	0	0	-1	1		0	1	0	1		0	0	0	0
	0	0	-1	-1		0	1	0	-1		0	0	0	0

Example 2: Again, consider the above design given by [1] for m = 4 factors to develop a design for two HTC factors and two ETC factors. The two HTC factors z_1 and z_2 are allocated to the first two columns of the design. The design is then sorted first on the basis of first HTC factor z_1 and then on the basis of second HTC factor z_2 within the sorted first HTC factor z_1. Then some runs are repeated to make all whole-plots of equal size. This generates 9 whole-plots of size 4 each. A whole-plot with overall center runs is added to the design. The split-plot design so generated is given as follows:

Table 3: Split-Plot Design for m=4 & two HTC factors and two ETC factors

WP	z_1	z_2	x_3	x_4	WP	z_1	z_2	x_3	x_4	WP	z_1	z_2	x_3	x_4
1	-1	1	0	0	5	0	0	1	1	9	1	-1	0	0
	-1	1	0	0		0	0	1	-1		1	-1	0	0
	-1	1	0	0		0	0	-1	1		1	-1	0	0
	-1	1	0	0		0	0	-1	-1		1	-1	0	0
2	-1	0	1	0	6	0	1	1	0	10	0	0	0	0
	-1	0	-1	0		0	1	-1	0		0	0	0	0
	-1	0	0	1		0	1	0	1		0	0	0	0
	-1	0	0	-1		0	1	0	-1		0	0	0	0
3	-1	-1	0	0	7	1	1	0	0					
	-1	-1	0	0		1	1	0	0					
	-1	-1	0	0		1	1	0	0					
	-1	-1	0	0		1	1	0	0					
4	0	-1	1	0	8	1	0	1	0					
	0	-1	-1	0		1	0	-1	0					
	0	-1	0	1		1	0	0	1					
	0	-1	0	-1		1	0	0	-1					

Method for orthogonal blocking of the second order response surface designs under split-plot structure for k_1 HTC and k_2 ETC factors

The following conditions for orthogonal blocking in second-order designs were given by [13]. Let X = [**1**, x_1, x_2, ……, x_m] be the columns of the design matrix having m factors. Let the design be divided into b blocks. Then, the conditions for the design to be orthogonal are

$$(i)\ \sum_{u}^{n_b} x_{iu}x_{ju}= 0, for\ i \neq j, 0 \leq i,j \leq m, and\ for\ all\ b\ ; \qquad (3)$$

$$(ii)\ \frac{\sum_{u}^{n_b} x_{iu}^2}{\sum_{u=1}^{n} x_{iu}^2} = \frac{n_s}{n}, for\ 1 \leq i \leq m\ , and\ for\ all\ b \qquad (4)$$

Where, for $1 \leq i \leq m, 1 \leq u \leq n$, x_{iu} denotes the level of the ith factor in the uth run, m is the number of factors, n is the total number of runs in the second order design divided into b blocks and n_b is the number of runs in the bth block.

The above conditions for orthogonally blocked second-order response surface designs under split-plot structure were extended by [14]. Let the design with n runs be divided into b blocks. Let s_i be the size of ith block with n_i runs. Let k_1 be the number of HTC factors denoted by z_i , i=1,2….,k_1 and k_2 be the number of ETC factors denoted by x_j, j= 1,2,.., k_2 where k_1+k_2=m. The conditions for second order response surface designs under split-plot structure to be orthogonally blocked are:

$$i)\ \sum_{u=1}^{n_1}\frac{z_{ui}}{n_1} = \cdots = \sum_{u=1}^{n_b}\frac{z_{ui}}{n_b} = \sum_{u=1}^{n}\frac{z_{ui}}{n}\ \ and\ \sum_{u=1}^{n_1}\frac{x_{uj}}{n_1} = \cdots = \sum_{u=1}^{n_b}\frac{x_{uj}}{n_b} = \sum_{u=1}^{n}\frac{x_{uj}}{n} \qquad (5)$$

$$ii)\ \sum_{u=1}^{n_1}\frac{z_{ui}x_{uj}}{n_1} = \cdots = \sum_{u=1}^{n_b}\frac{z_{ui}x_{uj}}{n_b} = \sum_{u=1}^{n}\frac{z_{ui}x_{uj}}{n} \qquad (6)$$

$$iii)\ \sum_{u=1}^{n_1}\frac{z_{ui}^2}{n_1} = \cdots = \sum_{u=1}^{n_b}\frac{z_{ui}^2}{n_b} = \sum_{u=1}^{n}\frac{z_{ui}^2}{n}\ and\ \sum_{u=1}^{n_1}\frac{x_{ui}^2}{n_1} = \cdots = \sum_{u=1}^{n_b}\frac{x_{ui}^2}{n_b} = \sum_{u=1}^{n}\frac{x_{ui}^2}{n} \quad (7)$$

Method for orthogonally blocking the response surface designs under split-plot structure is as follows:

Take the split plot design developed in Section 2 for k_1 HTC factors. Allocate the whole-plots in two blocks in such a way so that all the above three conditions for the orthogonal blocking of second order response surface designs under split-plot structure get satisfied. If all the conditions are still not met then repeat some of the whole-plots. A whole-plot with overall center runs is added to both the blocks.

We illustrate the above method by blocking the response surface design under split-plot structure into two orthogonal blocks in the following two examples.

Example 3: Consider example 1 for four factors with one HTC factor z_1. As explained above, the split-plot design is divided into two blocks. Whole-plots (WP) with numbers 1, 3, 4, 8 and 9 will form block 1 and whole-plots with number 2, 5, 6, 7 and 9 will form block 2. The blocking satisfies all three orthogonality conditions mentioned in [14]. Each block contains five whole-plots of size 4 each. Within each block, the above design is first order orthogonal.

Example 4: Now take example 2 with four factors with two HTC and two ETC factors. The design is blocked into the following two orthogonal blocks:

Whole-plots with numbers 1, 2, 4, 5, 7, 9 and 10 form block 1 and whole-plots with numbers 1, 3, 5, 6, 8, 9 and 10 form block 2. These two blocks satisfy the orthogonality conditions given above. Each block consists of 7 whole-plots of size 4 each. Within each block, the above design is first order orthogonal.

In the Annexure, we have given the orthogonally blocked split-plot designs for 5 and 6 factors, involving one HTC and two HTC factors.

Conclusion

Although there has been a significant growth in the field of RSM but no extensive research has been undertaken on response surface split-plot designs specially with blocking. In this paper we have given a method of construction as well as blocking of II order RSSPD paving the way for obtaining more efficient designs with fewer runs.

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Annexture A

Table A.1:Response surface design under split-plot structure for m = 5 factors with one HTC factor.

WP	z1	x2	x3	x4	x5	WP	z1	x2	x3	x4	x5	WP	z1	x2	x3	x4	x5
1	1	1	0	0	0	5	0	1	1	0	0	9	0	1	0	1	0
	1	-1	0	0	0		0	1	-1	0	0		0	1	0	-1	0
	1	0	1	0	0		0	-1	1	0	0		0	-1	0	1	0
	1	0	-1	0	0		0	-1	-1	0	0		0	-1	0	-1	0
2	-1	0	0	1	0	6	0	0	1	0	1	10	-1	1	0	0	0
	-1	0	0	-1	0		0	0	1	0	-1		-1	-1	0	0	0
	-1	0	0	0	1		0	0	-1	0	1		-1	0	1	0	0
	-1	0	0	0	-1		0	0	-1	0	-1		-1	0	-1	0	0
3	0	0	0	-1	1	7	0	0	1	1	0	11	1	0	0	1	0
	0	0	0	-1	-1		0	0	1	-1	0		1	0	0	-1	0
	0	0	0	1	1		0	0	-1	1	0		1	0	0	0	1
	0	0	0	1	-1		0	0	-1	-1	0		1	0	0	0	-1
4	0	-1	0	0	1	8	0	0	0	0	0						
	0	-1	0	0	-1		0	0	0	0	0						
	0	1	0	0	1		0	0	0	0	0						
	0	1	0	0	-1		0	0	0	0	0						

This split-plot design is divided into two orthogonal blocks with whole-plots 1, 2, 3, 4, 5, 7, 9 and 11 forming Block 1 and whole-plots 2, 3, 5, 6, 8, 9, 10 and 12 forming Block 2.

Table A2: Response surface design under split-plot structure for m = 5 factors with two HTC factors.

WP	z1	z2	x1	x2	x3	WP	z1	z2	x1	x2	x3	WP	z1	z2	x1	x2	x3
1	1	-1	0	0	0	5	1	1	0	0	0	9	-1	1	0	0	0
	1	-1	0	0	0		1	1	0	0	0		-1	1	0	0	0
	1	-1	0	0	0		1	1	0	0	0		-1	1	0	0	0
	1	-1	0	0	0		1	1	0	0	0		-1	1	0	0	0
	1	-1	0	0	0		1	1	0	0	0		-1	1	0	0	0
	1	-1	0	0	0		1	1	0	0	0		-1	1	0	0	0
2	1	0	1	0	0	6	0	-1	1	0	0	10	-1	-1	0	0	0
	1	0	-1	0	0		0	-1	-1	0	0		-1	-1	0	0	0
	1	0	0	1	0		0	-1	0	1	0		-1	-1	0	0	0
	1	0	0	-1	0		0	-1	0	-1	0		-1	-1	0	0	0
	1	0	0	0	1		0	-1	0	0	1		-1	-1	0	0	0
	1	0	0	0	-1		0	-1	0	0	-1		-1	-1	0	0	0
3	0	0	1	1	0	7	-1	0	1	0	0	11	0	0	0	0	0
	0	0	1	-1	0		-1	0	-1	0	0		0	0	0	0	0
	0	0	-1	1	0		-1	0	0	1	0		0	0	0	0	0
	0	0	-1	-1	0		-1	0	0	-1	0		0	0	0	0	0
	0	0	0	1	1		-1	0	0	0	1		0	0	0	0	0
	0	0	0	-1	-1		-1	0	0	0	-1		0	0	0	0	0
4	0	0	-1	0	1	8	0	1	1	0	0						
	0	0	-1	0	-1		0	1	-1	0	0						
	0	0	1	0	1		0	1	0	1	0						
	0	0	1	0	-1		0	1	0	-1	0						
	0	0	0	1	-1		0	1	0	0	1						
	0	0	0	-1	1		0	1	0	0	-1						

This split-plot design is divided into two orthogonal blocks with whole-plots 1, 2, 3, 4, 5, 7, 9 and 11 forming Block 1 and whole-plots 2, 3, 5, 6, 8, 9, 10 and 12 forming Block 2.

Table A.3: Response surface design under split-plot structure for m = 6 factors with one HTC factor.

WP	z1	x2	x3	x4	x5	x6	WP	z1	x2	x3	x4	x5	x6	WP	z1	x2	x3	x4	x5	x6
1	1	1	0	0	0	0	7	1	0	0	1	0	0	13	0	1	0	1	0	0
	1	-1	0	0	0	0		1	0	0	-1	0	0		0	1	0	-1	0	0
	1	0	1	0	0	0		1	0	0	0	1	0		0	-1	0	1	0	0
	1	0	-1	0	0	0		1	0	0	0	-1	0		0	-1	0	-1	0	0
2	1	0	0	0	0	1	8	-1	1	0	0	0	0	14	0	0	1	1	0	0
	1	0	0	0	0	-1		-1	-1	0	0	0	0		0	0	1	-1	0	0
	1	0	0	0	0	1		-1	0	1	0	0	0		0	0	-1	1	0	0
	1	0	0	0	0	-1		-1	0	-1	0	0	0		0	0	-1	-1	0	0
3	-1	0	0	1	0	0	9	-1	0	0	0	0	1	15	0	0	1	0	1	0
	-1	0	0	-1	0	0		-1	0	0	0	0	-1		0	0	1	0	-1	0
	-1	0	0	0	1	0		-1	0	0	0	0	1		0	0	-1	0	1	0
	-1	0	0	0	-1	0		-1	0	0	0	0	-1		0	0	-1	0	-1	0
4	0	1	1	0	0	0	10	0	1	0	0	0	1	16	0	0	0	0	1	1
	0	1	-1	0	0	0		0	1	0	0	0	-1		0	0	0	0	1	-1
	0	-1	0	1	0	0		0	-1	0	0	0	1		0	0	0	0	-1	1
	0	-1	0	-1	0	0		0	-1	0	0	0	-1		0	0	0	0	-1	-1
5	0	1	0	0	1	0	11	0	0	1	0	0	1	17	0	0	0	0	0	0
	0	1	0	0	-1	0		0	0	1	0	0	-1		0	0	0	0	0	0
	0	-1	0	0	1	0		0	0	-1	0	0	1		0	0	0	0	0	0
	0	-1	0	0	-1	0		0	0	-1	0	0	-1		0	0	0	0	0	0
6	0	0	0	1	1	0	12	0	0	0	1	0	1							
	0	0	0	1	-1	0		0	0	0	1	0	-1							
	0	0	0	-1	1	0		0	0	0	-1	0	1							
	0	0	0	-1	-1	0		0	0	0	-1	0	-1							

This split-plot design is divided into two orthogonal blocks with whole-plots 1, 2, 3, 4, 5, 7, 9 and 11 forming Block 1 and whole-plots 2, 3, 5, 6, 8, 9, 10 and 12 forming Block 2.

Table A.4: Response surface design under split-plot structure form = 6with two HTC factors.

WP	z1	x2	x3	x4	x5	x6	WP	z1	x2	x3	x4	x5	x6	WP	z1	x2	x3	x4	x5	x6
1	1	0	-1	0	0	0	7	1	0	0	0	1	0	13	0	-1	0	0	1	0
	1	0	1	0	0	0		1	0	0	0	-1	0		0	-1	0	0	-1	0
	1	0	0	1	0	0		1	0	0	0	0	1		0	-1	0	0	0	1
	1	0	0	-1	0	0		1	0	0	0	0	-1		0	-1	0	0	0	-1
2	-1	0	0	0	1	0	8	-1	0	1	0	0	0	14	0	0	1	0	0	1
	-1	0	0	0	-1	0		-1	0	-1	0	0	0		0	0	1	0	0	-1
	-1	0	0	0	0	1		-1	0	0	1	0	0		0	0	-1	0	0	1
	-1	0	0	0	0	-1		-1	0	0	-1	0	0		0	0	-1	0	0	-1
3	0	1	1	0	0	0	9	0	-1	1	0	0	0	15	0	0	0	1	1	0
	0	1	-1	0	0	0		0	-1	-1	0	0	0		0	0	0	1	-1	0
	0	1	0	1	0	0		0	-1	0	1	0	0		0	0	0	-1	1	0
	0	1	0	-1	0	0		0	-1	0	-1	0	0		0	0	0	-1	-1	0
4	0	1	0	0	1	0	10	0	0	1	0	1	0	16	0	0	-1	0	1	0
	0	1	0	0	-1	0		0	0	1	0	-1	0		0	0	-1	0	-1	0
	0	1	0	0	0	1		0	0	0	1	0	1		0	0	0	-1	0	1
	0	1	0	0	0	-1		0	0	0	1	0	-1		0	0	0	-1	0	-1
5	0	0	1	1	0	0	11	1	-1	0	0	0	0	17	-1	1	0	0	0	0
	0	0	1	-1	0	0		1	-1	0	0	0	0		-1	1	0	0	0	0
	0	0	-1	1	0	0		1	-1	0	0	0	0		-1	1	0	0	0	0
	0	0	-1	-1	0	0		1	-1	0	0	0	0		-1	1	0	0	0	0
6	0	0	0	0	1	1	12	-1	-1	0	0	0	0	18	1	1	0	0	0	0
	0	0	0	0	1	-1		-1	-1	0	0	0	0		1	1	0	0	0	0
	0	0	0	0	-1	1		-1	-1	0	0	0	0		1	1	0	0	0	0
	0	0	0	0	-1	-1		-1	-1	0	0	0	0		1	1	0	0	0	0
														19	0	0	0	0	0	0
															0	0	0	0	0	0
															0	0	0	0	0	0
															0	0	0	0	0	0

This split-plot design is divided into two orthogonal blocks with whole-plots 1, 2, 3, 4, 5, 7, 9 and 11 forming Block 1 and whole-plots 2, 3, 5, 6, 8, 9, 10 and 12 forming Block 2.