BIPOLAR INTUITIONISTIC FUZZY GENERALISED CLOSED SETS VIA BIPOLAR INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

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Abstract : The aim of this paper is to introduce the concept of Bipolar Intuitionistic fuzzy generalised alpha closed sets in Bipolar Intuitionistic fuzzy topological spaces environment and thereby studying its relative properties. Also few other sets in Bipolar Intuitionistic fuzzy topological spaces are introduced and its interrelationships with other sets are investigated.

***IndexTerms* - Bipolar Intuitionistic fuzzy topological spaces, Bipolar Intuitionistic fuzzy generalised alpha closed sets, Bipolar Intuitionistic fuzzy generalised alpha open sets.**

I. INTRODUCTION

The concept of fuzzy set was introduced by Zadeh[14] in the year 1965 and later Atanassov[2] generalised this idea to a new class of intuitionistic fuzzy sets using the notions of fuzzy sets. C.L. Chang[3] described the new concept of Fuzzy Topological Spaces and Dogan Coker[5] gave an introduction to intuitionistic fuzzy topological spaces. Bipolar valued fuzzy sets, which was introduced by Lee[7] in 2000,which is an extension of fuzzy sets whose membership degree range is enlarged from the interval [0,1] to [-1,1]. D. Ezhilmaran & K. Sankar[6] in the year 2015,discussed on the morphism of Bipolar Intuitionistic fuzzy graphs and developed its related properties.Later Bipolar Intuitionistic Fuzzy sets in a soft environment was put forward by Chiranjibe Jana and Madhumangal Pal[4] in the year 2018. In this paper we introduce Bipolar Intuitionistic Fuzzy generalised alpha closed sets and Bipolar Intuitionistic Fuzzy generalised alpha open sets via Bipolar Intuitionistic Fuzzy topological spaces and study some of its properties and relationships.

II.PRELIMINARIES

Definition 2.1[2]:
Let X be the universe. An intuitionistic fuzzy set (or IFS) A in X is an object having the form A={<x,$\mu_A(x),\gamma_A(x)$>/x∈X}where the functions μ_A:X→[0,1] and γ_A:X→[0,1] defines the degree of membership and degree of non-membership respectively of the element x∈X to the set A, and ∀ x∈X 0≤ $\mu_A(x)+\gamma_A(x) \leq 1$.Fuzzy sets can be viewed as an intuitionisitc fuzzy sets but not conversely.

Definition 2.2[7]:
Let X be the universe. A bipolar valued fuzzy set,A on X is defined by the positive membership function μ_A^+ ,i.e., μ_A^+ :X→[0,1], and a negative membership function μ_A^-, i.e., μ_A^- :X→[-1,0].For the sake of simplicity we shall use the symbol,
A={<x, μ_A^+, μ_A^->/x∈ X}.

Definition 2.3[6]:
Let X be a non empty set. A bipolar intuitionistic fuzzy set B = {(x,$\mu^P(x)$,$\mu^N(x)$,$\gamma^P(x)$,$\gamma^N(x)$)| $x \in X$}where
$\mu^P : X \to [0,1]$, $\mu^N : X \to [-1,0]$,$\gamma^P : X \to [0,1]$, $\gamma^N : X \to [-1,0]$ are the mappings such that $0\leq \mu^P(x)+\gamma^P(x) \leq 1$ and
$-1 \leq \mu^N(x)+\gamma^N(x) \leq 0$.
We use the positive membership degree $\mu^P(x)$, which denote how for an element x satisfies the property corresponding to a bipolar intuitionistic fuzzy set B and the negative membership degree $\mu^N(x)$, which denote how for an element x satisfies the implicit counter property corresponding to a bipolar intuitionistic fuzzy set. We use the positive nonmembership degree $\gamma^P(x)$, which is one minus the positive membership degree and the negative nonmembership degree $\gamma^N(x)$, which is one minus the negative membership degree.
If $\mu^P(x) \leq 0$,$\mu^N(x) = 0$ and $\gamma^P(x) = 0$,$\gamma^N(x) = 0$ it is the situation that x regarded as having only the positive membership property of a bipolar intuitionistic fuzzy set. If $\mu^P(x) = 0$,$\mu^N(x) \leq 0$ and $\gamma^P(x) = 0$,$\gamma^N(x) = 0$ it is the situation that x is regarded as having only the negative membership property of a bipolar intuitionistic fuzzy set.
$\mu^P(x) = 0$,$\mu^N(x) = 0$ and $\gamma^P(x) \leq 0$,$\gamma^N(x) = 0$ it is the situation that x regarded as having only the positive non membership property of a bipolar intuitionistic fuzzy set. $\mu^P(x) = 0$,$\mu^N(x) = 0$ and $\gamma^P(x) = 0$, $\gamma^N(x) \leq 0$ it is the situation that x regarded as having only the negative nonmembership property of a bipolar intuitionistic fuzzy set. It is possible for an element x to be such that $\mu^P(x) \leq 0$,$\mu^N(x) \leq 0$ and $\gamma^P(x) \leq 0$, $\gamma^N(x) \leq 0$ when the membership and non membership function of the property overlaps with its counter properties over some portion of X. If $\mu^P(x) = 0$,$\mu^N(x) = 0$ and $\gamma^P(x) = 0$,$\gamma^N(x) = 0$ then the element x is irrelevant to the corresponding property.

Definition 2.4: A subset A of a topological space (X,τ) is called a

- preclosed set[11] if cl(int (A))⊆A.
- semi-closed set[8] if int(cl(A))⊆A.
- regular closed set[13] if A=cl(int(A)).
- α closed set if[12] cl(int(cl(A)))⊆A.
- generalised semi-closed set[1] (briefly gs-closed set) scl(A)⊆U whever A⊆U and U is open set in X.

- generalised pre-closed set[10](briefly gp-closed set)pcl(A)⊆U whenever A⊆U and U is open set in X.
- generalised alpha closed[9](briefly gα-closed set)αcl(A)⊆U whenever A⊆U and U is α-open in X.

III. BIPOLAR INTUITIONISTIC TOPOLOGICAL SPACES

In this section we define Bipolar Intuitionistic fuzzy topological spaces and define Bipolar Intuitionistic fuzzy interior and Bipolar Intuitionistic fuzzy closure along with the properties.

Definition 3.1[4]:
Let A an B be two bipolar intuitionistic fuzzy sets given by A={<$x, \mu_A^p(x), \mu_A^N(x), \gamma_A^P(x), \gamma_A^N(x)$>;x∈X} and B={< $x, \mu_B^p(x), \mu_B^N(x), \gamma_B^P(x), \gamma_B^N(x)$>;x∈X >} in the universe X .If $\mu_A^p(x) \leq \mu_B^p(x), \mu_A^N(x) \geq \mu_B^N(x), \gamma_A^P(x) \geq \gamma_B^P(x), \gamma_A^N(x) \leq \gamma_B^N(x)$ then A⊆B.

Definition 3.2[6]:
The union of two bipolar intuitionistic fuzzy sets A and B in the universe X written as
C= A∪B which is given by
(A∪B)(x)={<$\mu_A^p(x) \vee \mu_B^p(x)$, $\mu_A^N(x) \wedge \mu_B^N(x)$, $\gamma_A^P(x) \wedge \gamma_B^P(x), \gamma_A^N(x) \vee \gamma_B^N(x)$>}

Definition 3.3[6]:
The intersection of two bipolar intuitionistic fuzzy sets A and B in the universe X written as
C= A∩B which is given by
(A∩B)(x)={< $\mu_A^p(x) \wedge \mu_B^p(x)$, $\mu_A^N(x) \vee \mu_B^N(x)$, $\gamma_A^P(x) \vee \gamma_B^P(x), \gamma_A^N(x) \wedge \gamma_B^N(x)$>}

Definition 3.4[4]:
The complement of bipolar intuitionistic fuzzy set A in the universe X written as A^c,which is given by
A^c={1-$\mu_A^p(x)$,-1-$\mu_A^N(x)$,1-$\gamma_A^P(x)$,-1-$\gamma_A^N(x)$}

Definition 3.5[4]:
Two bipolar intuitionistic fuzzy sets A and B in the universe X is said to be equal and written as A=B if and only if A⊆B and B⊆A.

Definition 3.6:
The bipolar intuitionistic fuzzy empty set may be defined as: $0\sim = \{< x, 0,0,1, -1 >$: x ∈ X}

Definition 3.7:
The bipolar intuitionistic fuzzy absolute set may be defined as : 1~={<x,1,-1,0,0>:x∈ X}

Definition 3.8

A Bipolar intuitionistic fuzzy topology (BIFTS for short) on a nonempty set X is a family τ of bipolar intuitionistic fuzzy subsets in X satisfying the following axioms

Axiom1:0,1 ∈ τ
Axiom 2:∪ $G_i \in \tau$ for any $\{G_i/i \in J\} \subseteq \tau$
Axiom 3:A∩B ∈ $\tau$for any $A, B \in \tau$

In this case the pair (X,τ) is called a bipolar intuitionistic fuzzy topological space(BIFTS for short) and any BIFS in τ is known as a Bipolar intuitionistic fuzzy open set (BIFOS for short) in X.
The complement A^c of a BIFOS is called a bipolar intuitionistic fuzzy closed set.

Definition 3.9

Let (X,τ) be a BIFTS and A={<x,$\mu^P(x), \mu^N(x), \gamma^P(x), \gamma^N(x)$ >} be a BIFTS in X.Then the bipolar intuitionistic fuzzy interior and bipolar intuitionistic fuzzy closure are defined by
BIFint(A)=∪{G/G is anBIFOS in X and G⊆A}
BIFcl(A)=∩{K/K is an BIFCS in X and A⊆K }

Proposition 3.10
For any BIFS set A={<x,$\mu^P(x), \mu^N(x), \gamma^P(x), \gamma^N(x)$ >} the following results holds
(i)0~ ⊆A,0~ ⊆ 0~
(ii)1~ ⊆ 1~ ,A ⊆ 1~.

Properties 3.11
Let (X,τ) be a BIFTS and A and B are any two BIF subsets of X.Then

(i) A⊂BIFcl (A)
(ii) A⊂B⇒ BIFcl (A) ⊂BIFcl (B)
(iii) A is a BIFCS⇔ A=BIFcl(A)
(iv) BIFcl (BIFcl (A)) = BIFcl (A)
(v) BIFcl (0~) =0~,BIFcl (1~) = 1~

(ix) BIFint(A)⊂A.
(x) A⊂B⟹ BIFint(A)⊂BIFint(B)
(xi)A is BIFOS ⇔ BIFint(A)=A
(xii)BIFint(BIFint(A)) = BIFint(A)
(xiii) BIFint (0~) =0~ ,BIFint (1~) = 1~

(vi) BIFcl (A∪B) = BIFcl (A) ∪ BIFcl (B)

(vii) $(BIFcl\ (A))^c$ = BIF int (A^c)

(viii) $(\text{BIF cl(A)})^c \in \tau$

(xiv)BIFint (A∩B) = BIFint (A) ∩BIFint (B)

(xv)$(BIFint\ (A))^c$ = BIFcl (A^c)

(xvi)BIFint(A)∈ τ

Proof: obvious

Definition 3.12: A subset A of a Bipolar intuitionistic fuzzy topological space (X,τ) is said to be

- Bipolar intuitionistic fuzzy pre closed set if *BIF cl*(*BIF int(A)*)⊆A.
- Bipolar intuitionistic fuzzy semi closed set if *BIF int*(*BIF cl(A)*)⊆A.
- Bipolar intuitionistic fuzzy regular closed set if A=BIF *cl(*BIF *Int A).*
- Bipolar intuitionistic fuzzy alpha closed set if BIF *cl(BIF int(BIF cl(A)))*⊆A.
- Bipolar intuitionistic fuzzy generalised closed set if BIF *cl(A)*⊆U whenever A⊆U and U is BIF open in X.
- Bipolar intuitionistic fuzzy generalised semi closed set if BIF scl(A)⊆U whenever A⊆U and U is BIF open in X.
- Bipolar intuitionistic fuzzy generalised pre closed set if BIF pcl(A)⊆U whenever A⊆U and U is BIF open in X.

IV. BIPOLAR INTUITIONISTIC FUZZY GENERALISED ALPHA CLOSED SETS

Definition 4.1:A subset A of a Bipolar intuitionistic fuzzy topological space (X,τ) is said to be Bipolar intuitionistic fuzzy generalised alpha closed set if BIF αcl(A)⊆U whenever A⊆U and U is BIF α open in X.

Example 4.2: Let X={a,b} and let τ={0~,1~,G} be a BIFT on X where G={<a,0.5,-0.2,0.3,-0.6>,<b,0.2,-0.5,0.3,-0.2 >}.Let A={<a,0.2,-0.3,0.5,-0.4>,<b,0.3,-0.2,0.4,-0.3>} be any BIFS in X.Then A is BIFGαCS in X.

Theorem 4.3: Every BIFCS is BIFGαCS in X but not conversely.
Proof: Let A be a BIFCS and let A⊆U and U be BIFαOS in (X,τ).Since A is BIFCS, BIFCl(A) =A.But,every BIFCS is BIFαCS.Therefore $BIF\alpha cl(A)$ ⊆BIFcl(A)=A⊆ U.Therefore BIF αcl(A)⊆U whenever A⊆U and U is BIFαOS in X.Therefore A is BIFGαCS in X.

Example 4.4: Let X={a,b} and let τ={0~,1~,G} be a BIFT on X where G={<a,0.4,-0.1,0.1,-0.1>,<b,0.4,-0.9,0.1,-0.1>}.Let A={<a,0.4,-0.8,0.1,-0.2>,<b,0.4,-0.6,0.1,-0.4>} be any BIFS in X.Here BIF αcl(A)⊆G whenever A⊆G and G is BIF αOS in X, but A is not BIFCS since BIFcl(A)=1≠A.

Theorem 4.5: Every BIFαCS is BIFGαCS in X but not conversely.
Proof: Let A be a BIFαCS and let A⊆U and U be BIFαOS in (X,τ).Since A is BIFαCS, BIFαCl(A) =A.Therefore $BIF\alpha cl(A)$=A⊆ U.Therefore BIF αcl(A)⊆U whenever A⊆U and U is BIF αOS in X.Therefore A is BIF αCS in X.

Example 4.6: Let X={a,b} and let τ={0~,1~,G} be a BIFT on X where G={<a,0.3,-0.1,0.4,-0.3>,<b,0.2,-0.5,0.1,-0.2>}.Let A={<a,0.1,-0.2,0.4,-0.5>,<b,0.2,-0.1,0.3,-0.4>} be any BIFS in X.Here BIF αcl(A)⊆G whenever A⊆G and G is BIF αOS in X, but A is not BIFαCS since BIFcl(BIFint(BIFcl(A)))=G^c ⊈A.

Theorem 4.7: Every BIFGαCS is BIFGPCS in X but not conversely.
Proof: Let A⊆U and U be BIFOS in (X,τ).Every BIFOS is BIFαOS.Therefore BIF αcl(A)⊆U whenever A⊆U and U is BIF α open in X.But every BIFαCS is BIFPCS.Hence BIF pcl(A)⊆BIFαcl(A)⊆U. Therefore BIF $p$cl(A)⊆U whenever A⊆U and U is BIF αOS in X. Therefore A is BIFGPCS in X.

Example 4.8: Let X={a,b} and let τ={0~,1~,G} be a BIFT on X where G={<a,0.1,-0.1,0.9,-0.9>,<b,0.2,-0.1,0.1,-0.1>}.Let A={<a,0.1,-0.1,0.9,-0.9>,<b,0.1,-0.1,0.1,-0.1>} be any BIFS in X..Here A is BIFGPCS but A is not BIFGαCS in X since BIFαcl(A)=1⊈{1,G}.

Theorem 4.9: Every BIFGαCS is BIFGSCS in X but not conversely.
Proof: Let A⊆U and U be BIFOS in (X,τ).Every BIFOS is BIFαOS.Therefore BIF αcl(A)⊆U whenever A⊆U and U is BIF α open in X.But every BIFαCS is BIFSCS. Hence BIFscl(A)⊆BIFαcl(A)⊆U Therefore BIF $s$cl(A)⊆U whenever A⊆U and U is BIF αOS in X. Therefore A is BIFGSCS in X.

Example 4.10: Let X={a,b} and let τ={0~,1~,G} be a BIFT on X where G={<a,0.1,-0.4,0.6,-0.6>,<b,0.2,-0.3,0.6,-0.6>}.Let A={<a,0.1,-0.4,0.6,-0.6>,<b,0.2,-0.3,0.6,-0.6>} be any BIFS in X.Here A is BIFGSCS but A is not BIFGαCS in X since NHαcl(A)=G^c ⊈{1,G}.

Theorem 4.11: Every BIFRCS is BIFGαCS in X but not conversely.
Proof: Let A be a BIFRCS.Then A=BIFcl(BIFInt(A)).This implies BIFcl(A)=BIFcl(BIFInt(A)).Therefore BIFcl(A)=A.Hence A is BIFCS.But by theorem 4.3,every BIFCS is BIFGαCS.Therefore A is BIFGαCS in X.

Example 4.12: Let X={a,b} and let τ={0~,1~,G} be a BIFT on X where G={<a,0.4,-0.1,0.1,-0.2>,<b,0.4,-0.2,0.2,-0.3>}.Let A={<a,0.4,-0.3,0.3,-0.4>,<b,0.4,-0.4,0.4,-0.5>} be any BIFS in X.Here A is BIFGαCS but A is not BIFRCS in X since BIFcl(BIF int(A))=0≠A.

Theorem 4.13: Every BIFCS is BIFαCS in X but not conversely.

Proof: Let A be a BIFCS.Since BIFint (A)⊆A and BIF cl(A)=A,we have BIFint(BIF cl(A))⊆BIFcl(A)⟹ BIF cl(BIFint (BIF cl(A))) ⊆A.Therefore A is BIFαCS in X.

Example 4.14: Let X={a,b} and let τ={0∼,1∼,G} be a BIFT on X where G={<a,0.4,-0.4,0.5,-0.4>,<b,0.4,-0.4,0.6,-0.4>}.Let A={<a,0.3,-0.1,0.6,-0.7>,<b,0.3,-0.2,0.5,-0.7>} be any BIFS in X.Here A is BIFαCS but A is not BIFCS in X since BIFcl(A)=G^c ≠A.

Theorem 4.15: Every BIFCS is BIFPCS in X but not conversely.
Proof: Let A be a BIFCS in X. Since BIF int(A)⊆A,BIFcl (BIF int(A))⊆ BIF cl(A)=A⟹BIFcl (BIFint(A))⊆A.Therefore A is BIFPCS in X.

Example 4.16: Let X={a,b} and let τ={0∼,1∼,G} be a BIFT on X where G={<a,0.1,-0.1,0.5,-0.5>,<b,0.1,-0.1,0.6,-0.6>}.Let A={<a,0.1,-0.1,0.7,-0.7>,<b,0.1,-0.1,0.8,-0.8>} be any BIFS in X.Here A is BIFPCS but A is not BIFCS in X since BIFcl(A)=G^c ≠A.

Theorem 4.17: Every BIFαCS is BIFPCS in X but not conversely.
Proof: Let A be a BIFαCS in X.Then BIFcl(BIFint(BIFcl(A)))⊆A.Since A⊆BIF cl(A) ⟹BIF cl (BIF int(A))⊆A.Therefore A is BIFPCS in X.

Example 4.18. Let X={a,b} and let τ={0∼,1∼,G} be a BIFT on X where G={<a,0.4,-0.1,0.1,-0.5>,<b,0.4,-0.1,0.1,-0.6>}.Let A={<a,0.4,-0.1,0.1,-0.7>,<b,0.4,-0.1,0.1,-0.8>} be any BIFS in X.Here A is BIFPCS but A is not BIFαCS in X since BIFcl(BIF int(BIF cl(A)))=1 ⊈A.

Theorem 4.19: Every BIFRCS is BIFCS in X but not conversely.
Proof: Let A be BIFRCS in X.Then BIFcl(BIFint(A))=A⟹BIFcl(A)= BIF cl(BIF int (A)).Therefore BIFcl(A)=A.Therefore A is BIFCS in X.

Example 4.20. Let X={a,b} and let τ={0∼,1∼,G} be a BIFT on X where G={<a,0.7,-0.9,0.3,-0.1>,<b,0.6,-0.8,0.4,-0.2>}.Let A={<a,0.3,-0.1,0.7,-0.9>,<b,0.4,-0.2,0.6,-0.8>} be any BIFS in X.Here A is BIFCS but A is not BIFRCS in X since BIFcl(BIF int(A))=0 ≠A.

Theorem 4.21: Every BIFαCS is BIFSCS in X but not conversely.
Proof: Let A be a BIFαCS in X.Then BIF cl(BIF int(BIF cl(A)))⊆A and since BIFcl(A)⊆A,BIFint (BIFcl(A))⊆A.Therefore A is BIFSCS in X.

Example 4.22: Let X={a,b} and let τ={0∼,1∼,G} be a BIFT on X where G={<a,0.2,-0.3,0.5,-0.5>,<b,0.3,-0.4,0.6,-0.5>}.Let A={<a,0.2,-0.3,0.5,-0.5>,<b,0.3,-0.4,0.6,-0.5>} be any BIFS in X.Here A is BIFSCS but A is not BIFαCS in X since BIFcl(BIF int (BIF cl(A)))=G^c ⊈A.

Remark 4.23:BIFPCS and BIFGαCS are independent to each other.

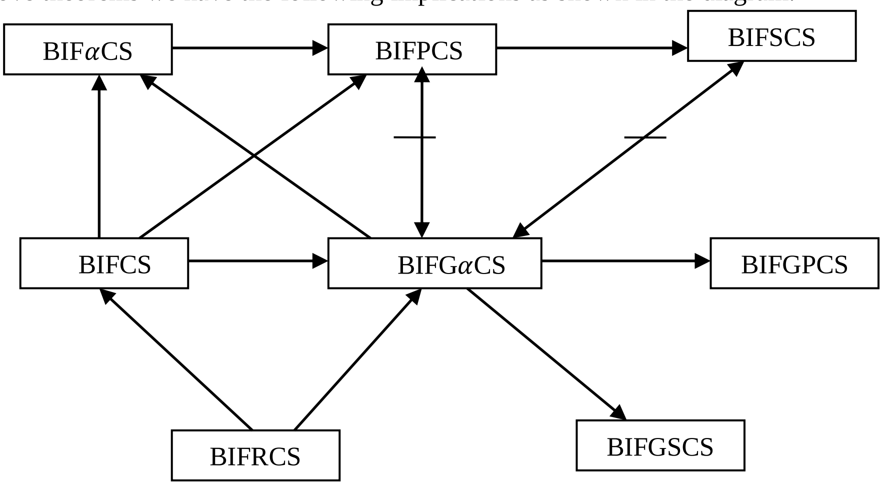
Example 4.24: Let X={a,b} and let τ={0∼,1∼,G} be a BIFT on X where G={<a,0.1,-0.1,0.1,-0.1>,<b,0.1,-0.1,0.2,-0.2>}.Let A={<a,0.1,-0.1,0.3,-0.3>,<b,0.1,-0.1,0.4,-0.4>} be any BIFS in X.Here A is BIFPCS but A is not BIFGαCS in X.

Example 4.25: Let X={a,b} and let τ={0∼,1∼,G} be a BIFT on X where G={<a,0.1,-0.1,0.9,-0.9>,<b,0.2,-0.1,0.1,-0.1>}.Let A={<a,0.5,-0.4,0.1,-0.1>,<b,0.6,-0.5,0.1,-0.1>} be any BIFS in X. Here A is BIFGαCS but A is not BIFPCS in X.
Remark 4.26:BIFSCS and BIFGαCS are independent of each other.

Example 4.27: Let X={a,b} and let τ={0∼,1∼,G} be a BIFT on X where G={<a,0.1,-0.2,0.7,-0.6>,<b,0.1,-0.2,0.5,-0.5>}.Let A={<a,0.1,-0.2,0.7,-0.6>,<b,0.1,-0.2,0.5,-0.5>} be any BIFS in X. Here A is BIFSCS but A is not BIFGαCS in X.

Example 4.28: Let X={a,b} and let τ={0∼,1∼,G} be a BIFT on X where G={<a,0.5,-0.2,0.4,-0.3>,<b,0.3,-0.5,0.2,-0.3>}.Let A={<a,0.2,-0.1,0.7,-0.8>,<b,0.1,-0.1,0.9,-0.8>} be any BIFS in X Here A is BIFGαCS but A is not BIFSCS in X.

From the above theorems we have the following implications as shown in the diagram:



In this diagram A→B means A implies B but not conversely.A↮B means A and B are independent of each other.None of them are reversible.

Remark 4.29: The intersection of any two BIFGαCS is not an BIFGαCS in general as shown in the example below.

Example 4.30: Let X={a,b} and let τ={0~,1~,G} be a BIFT on X where G={<a,0.7,-0.6,0.1,-0.1>,<b,0.8,-0.7,0.1,-0.1>}.Let A={<a,0.1,-0.9,0.1,-0.1>,<b,0.1,-0.2,0.1,-0.1>} and B={<a, 0.8,-0.3,0.1,-0.7>,<b,0.7,-0.3,0.1,-0.6>} be any two BIFS in X.Here A and B are BIFαCS but A∩B is not BIFGαCS in X.

Theorem 4.31:If A and B are BIFGαCS then their union A∪B is a BIFGαCS.
Proof: Assume that A and B are BIFGαCS in (X,τ).Then BIFGαcl(A)=A and BIFGαcl(B)=B.Let A∪B⊆U whenever U is BIFαOS in (X,τ).Then BIFcl(BIF int(BIF cl(A∪B)))= BIF cl(BIFint(A∪B))⊆ BIFcl(A∪B)= A∪B⊆U.Hence BIFαcl(A∪B)⊆U.Therefore A∪B is a BIFGαCS.

V.BIPOLAR INTUITIONISTIC FUZZY GENERALISED ALPHA OPEN SETS:

Definition 5.1:A BIFS is said to be Bipolar intuitionistic fuzzy generalised alpha open set (BIFGαOS in short) in (X,τ) if its complement A^c is BIFGαCS in X.The family of all BIFGαOS of a BIFTS (X,τ) is denoted by BIFGαO(X).

Example 5.2 : Let X={a,b} and let τ={0~,1~,G} be a BIFT on X where G={<a,0.3,-0.2,0.2,-0.1>,<b,0.3,-0.2,0.2,-0.2>}.Let A={<a,0.3,-0.2,0.2,-0.3>,<b,0.3,-0.2,0.2,-0.4>} be any BIFS in X.Then A is BIFGαOS in X.

Theorem 5.3 :For any BIFTS(X,τ),we have the following results:

(i) Every BIFOS is BIFGαOS in X but not conversely.
(ii) Every BIFαOS is BIFGαOS in X but not conversely.
(iii) Every BIFROS is BIFGαOS in X but not conversely.
(iv) Every BIFGαOS is BIFGPOS in X but not conversely.
(v) Every BIFGαOS is BIFGSOS in X but not conversely.

Proof:Obvious.

The converse of the above theorem need not be true as shown in the examples below.

Example 5.4: Let X={a,b} and let τ={0~,1~,G} be a BIFT on X where G={<a,0.1,-0.2,0.2,-0.2>,<b,0.1,-0.2,0.3,-0.3>}.Let A={<a,0.1,-0.1,0.4,-0.3>,<b,0.1,-0.2,0.5,-0.4>} be any BIFS in X. Here A is BIFGαOS but A is not BIFOS in X.

Example 5.5: Let X={a,b} and let τ={0~,1~,G} be a BIFT on X where G={<a,0.4,-0.5,0.5,-0.1>,<b,0.4,-0.1,0.4,-0.2>}.Let A={<a,0.4,-0.1,0.1,-0.3>,<b,0.4,-0.1,0.1,-0.4>} be any BIFS in X.Here A is BIFGαOS but A is not BIFαOS in X.

Example 5.6: Let X={a,b} and let τ={0~,1~,G} be a BIFT on X where G={<a,0.3,-0.2,0.1,-0.1>,<b,0.4,-0.3,0.1,-0.2>}.Let A={<a,0.1,-0.1,0.9,-0.9>,<b,0.2,-0.1,0.1,-0.1>} be any BIFS in X.Here A is BIFGαOS but A is not BIFROS in X.

Example 5.7: Let X={a,b} and let τ={0~,1~,G} be a BIFT on X where G={<a,0.5,-0.5,0.1,-0.5>,<b,0.5,-0.5,0.2,-0.5>}.Let A={<a,0.7,-0.6,0.3,-0.4>,<b,0.6,-0.6,0.4,-0.4>} be any BIFS in X.Here A is BIFGPOS but A is not BIFGαOS in X.

Example 5.8: Let X={a,b} and let τ={0~,1~,G} be a BIFT on X where G={<a,0.4,-0.2,0.6,-0.8>,<b,0.2,-0.3,0.8,-0.7>}.Let A={<a,0.6,-0.8,0.4,-0.2>,<b,0.8,-0.7,0.2,-0.3>} be any BIFS in X.Here A is BIFGαOS but A is not BIFOS in X.

Theorem 5.9: Let (X,τ) be a BIFTS.If A is an BIFS of X then the following properties are equivalent:

(i) A∈ BIFGαO(X)
(ii) V⊆BIFint(BIF cl (BIF int(A))) whenever V⊆A and V is BIFαCS in X.
(iii) There exist BIFOS G_1,such that $G_1 \subseteq$ V⊆ BIF int(BIF cl(G)) where G= BIF int(A),V⊆A and V is a BIFαCS in X.

Proof:
(i)⟹(ii)Let A ∈ BIFGαO(X).Then A^c is BIFGαCS in X.Therefore BIFαcl(A^c)⊆U,whenever A^c ⊆U and U is BIFαOS in X.That is BIFcl(BIF int(BIF cl(A^c))) ⊆U.This implies that U^c ⊆BIF int(BIF cl(BIF int(A))) whenever U^c ⊆A and U^c is BIFαCS in X.Replacing U^c by V,we get V⊆BIF int(BIF cl (BIF int(A))) whenever V⊆A and V is BIFαCS in X.
(ii)⟹(iii) Let V⊆BIF int(BIF cl (BIF int(A))) whenever V⊆A and V is BIFαCS in X.Hence BIFint(V)⊆ V ⊆ BIF int(BIFcl(BIF int(A))) .Then there exist BIFOS G_1 $in\ X, such\ that\ G_1 \subseteq$ V⊆ BIF int(BIF cl(G)) where G= BIF int(A) and $G_1 =$ int(V).
(iii)⇒(i) Suppose that there exist BIFOS G_1,such that $G_1 \subseteq$ V⊆ BIF int(BIF cl(G)) where G= BIF int(A),V⊆A and V is a BIFαCS in X.It is obvious that $BIF\ int(BIF\ cl(G))^{\ c} \subseteq V^c$,that is $BIF\ int(BIF\ cl(BIFint(A)))^{\ c} \subseteq V^c$.Therefore BIFcl(BIF int(BIF cl(A^c))) ⊆ V^c,$A^c \subseteq V^c$ and V^c is BIFαOS in X.Hence BIF αcl(A^c) ⊆ V^c,that is A^c is BIFGαCS in X.Therefore A ∈ BIFGαO(X)

Theorem 5.10: Let (X,τ) be a BIFTS.Then for every A $\in$ BIFGαO(X) and for every B $\in$ BIFS(X),BIFα int(A)$\subseteq$B$\subseteq$A implies B $\in$ BIFGαO(X).

Proof: By hypothesis, BIF α int(A)$\subseteq$B$\subseteq$A.Taking complements on both sides, we get $A^c \subseteq B^c \subseteq (BIF\ \alpha\ int(A))^c$.Let $B^c \subseteq$ U and U is BIFαO S in X.Since $A^c \subseteq B^c$,$A^c \subseteq$U and since A^c is BIFGαCS,BIFαcl(A)$\subseteq$U.Also $B^c \subseteq (BIF\ \alpha\ int(A))^c$=$BIF\alpha cl(A^c)$.Therefore BIF$\alpha$ $cl(B^c) \subseteq$ BIF α $cl(A^c)$ $\subseteq$U.Hence B^c is a BIFGαCS in X.Therefore B is an BIFGαOS in X.That is B $\in BIFG\alpha O$(X).

Theorem 5.11: A BIFS A of a BIFTS (X,τ) is a BIFGαOS if and only if G$\subseteq$BIFαcl(A),whenever G is a BIFαCS(X) and G$\subseteq$A.

Proof: Necesssity: Let A be a BIFGαOS in X and G a BIFαCS and G$\subseteq$A.Then G^c is a BIFαOS in X such that $A^c \subseteq G^c$. Since A^c is a BIFGαCS , BIFαcl(A^c)$\subseteq G^c$.But BIFαcl(A^c)=$(BIF\alpha\ int\ (A))^c$.Hence $(BIF\alpha\ int\ (A))^c \subseteq G^c$.Hence $G \subseteq BIF\alpha int(A)$.
Sufficiency: Let G $\subseteq$BIFαcl(A),whenever G is a BIFαCS(X) and G$\subseteq$A.Then BIFαint(A^c) $\subseteq G^c$,whenever G^c *is a BIF*αOS and $A^c \subseteq G^c$. *Hence* A^c is a BIFGαCS which implies A is BIFGαOS.

VI.APPLICATIONS OF BIPOLAR INTUITIONISTIC FUZZY GENERALISED ALPHA CLOSED SETS:

Definition 6.1:A bipolar intuitionistic fuzzy topological space (X,τ) is said to be

- $T_{g\alpha}$ space if every BIFGαCS is BIFCS in X. The collection of all BIFGαOS in (X,τ) is denoted by BIFGαO(τ).
- T_r space if every BIFGαCS is BIFRCS in X. The collection of all BIFROS in (X,τ) is denoted by BIFRO(τ).
- T_α space if every BIFGαCS is BIFαCS in X. The collection of all BIFαOS in (X,τ) is denoted by BIFαO(τ).

The collection of all BIFOS is denoted by BIFO(τ).

Theorem 6.1: Every $T_{g\alpha}$ space is T_α space but not conversely.
Proof: Let (X,τ) be a $T_{g\alpha}$ space and let A $\in T_{g\alpha}$ space.Then A is BIFCS.But every BIFCS is BIF αCS.Therefore,every BIFGαCS is BIFαCS and hence A $\in T_\alpha$ space.

Example 6.2: Let X={a,b} and let τ={0~,1~,G} be a BIFT on X where G={<a,0.3,-0.4,0.4,-0.5>,<b,0.4,-0.4,0.5,-0.5>}.Let A={<a,0.2,-0.2,0.6,-0.6>,<b,0.3,-0.3,0.5,-0.6>} be any BIFS in X. Here A is BIF$G\alpha$CS but A is not BIFCS in X since BIFcl(A)=G^c $\neq$A.

Theorem 6.3: Every T_r space is $T_{g\alpha}$ space but not conversely.
Proof: Let (X,τ) be a T_r space and let A $\in T_r$ space.Then A is BIFRCS.But every BIFRCS is BIFCS.Therefore,every BIFGαCS is BIFCS and hence A $\in T_{g\alpha}$ space.

Example 6.4: Let X={a,b} and let τ={0~,1~,G} be a BIFT on X where G={<a,0.8,-0.8,0.2,-0.2>,<b,0.6,-0.8,0.4,-0.2>}.Let A={<a,0.2,-0.2,0.8,-0.8>,<b,0.4,-0.2,0.6,-0.8>} be any BIFS in X. Here A is BIFGαCS but is not BIFRCS in X since BIFcl(BIF int(A))=0 $\neq$A.

Theorem 6.5:Let (X,τ) be a BIFTS,then

(i) BIFO(τ) $\subset$ BIFGαO(τ)

(ii) A space (X,τ) is $T_{g\alpha}$space iff BIFO(τ)= BIFGαO(τ)

Proof:
(i)Let A be a BIFOS in X.Then X-A is BIFCS and hence it is BIFGαCS.Then A is BIFGαOS in X.Therefore BIFO(τ) $\subset$ BIFGαO(τ).
(ii)Let A be a $T_{g\alpha}$space.Let A be BIFGαOS in X.Then X-A is BIFGαCS.By hypothesis X-A is BIFCS and hence A is BIFOS in X.Therefore BIFO(τ) $\supset$ BIFGαO(τ).By(i), BIFO(τ) $\subset$ BIFGαO(τ),hence BIFO(τ)= BIFGαO(τ).
Conversely let BIFO(τ)= BIFGαO(τ).Let A be a BIFGαCS in X.Then X-A is BIFGαOS and hence X-A is BIFOS.Then A is BIFCS.Therefore (X,τ) is $T_{g\alpha}$space.

Theorem 6.6: Let (X,τ) be a BIFTS,then

(i) BIFRO(τ) $\subset$ BIFGαO(τ)

(ii) A space (X,τ) is T_r space iff BIFRO(τ)= BIFGαO(τ)

Proof:
(i)Let A be a BIFROS in X.Then X-A is BIFRCS and hence it is BIFGαCS.Then A is BIFGαOS in X.Therefore BIFRO(τ) $\subset$ BIFGαO(τ).
(ii)Let A be a T_r space.Let A be BIFGαOS in X.Then X-A is BIFGαCS.By hypothesis X-A is BIFRCS and hence A is BIFRCS in X.Therefore BIFRO(τ) $\supset$ BIFGαO(τ).By(i), BIFRO(τ) $\subset$ BIFGαO(τ),hence BIFRO(τ)= BIFGαO(τ).
Conversely let BIFRO(τ)= BIFGαO(τ).Let A be a BIFGαCS in X.Then X-A is BIFGαOS and hence X-A is BIFROS.Then A is BIFRCS.Therefore (X,τ) is $T_r$space.

Theorem 6.7: Let (X,τ) be a BIFTS,then

(i) BIF αO(τ) $\subset$ BIFGαO(τ)

(ii) A space (X,τ) is T_α space iff BIFα O(τ)= BIFGαO(τ)

Proof:
(i)Let A be a BIFαOS in X.Then X-A is BIF α CS and hence it is BIFGαCS.Then A is BIFGαOS in X.Therefore

BIFαO(τ) ⊂ BIFGαO(τ).

(ii)Let A be a $T_\alpha$space.Let A be BIFGαOS in X.Then X-A is BIFGαCS.By hypothesis X-A is BIFαCS and hence A is BIFαOS in X.Therefore BIFαO(τ) ⊃ BIFGαO(τ).By(i), BIFαO(τ) ⊂ BIFGαO(τ),hence BIFOα(τ)= BIFGαO(τ).

Conversely let BIF αO(τ)= BIFGαO(τ).Let A be a BIFGαCS in X.Then X-A is BIFGαOS and hence X-A is BIFαOS.Then A is BIFαCS.Therefore (X,τ) is T_α space.

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