Padavon Graceful Labeling For Some Complete Graphs And Cycle Related Graphs

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Abstract

A (p,q) connected graph is padavon graceful graph if there exists an injective map f: E(G) → { 1,1,1,2,2,3,4,5,7,…,2q-1} so that induced map f_+ : V(G) → { 0, 1, …,p+1} defined by $f_+(x)$ = |f(u)-f(v)| where the vertex x is incident with other vertex y and makes all the edges distinct. In this article, the padavon gracefulness of some path related graphs graphs are obtained.

Keywords Padavon sequence, vertex labeling, edge labeling, graceful, padavon graceful

1. Introduction

In this paper we consider only finite, undirected non-trivial graphs *G = (V(G),E(G))* with the vertex set *V(G)* and the edge set *E(G)*.We refer to Gallian for all detailed survey of graph labeling. For standard terminology and notations we follow Harary. Graph labeling is a strong communication between number theory and structure of graphs. The study of graceful graphs and graceful labeling methods was introduced by Rosa .Rosa defined a β-valuation of a graph G with q edges an injection from the vertices of G to the set {0,1, …,q} such that when each edge uv is assigned the label |f(u)-f(v)|, the resulting edges are distinct. β – valuation is a function that produces graceful labeling. However the term graceful labeling was not used until Golomb studied such labeling several years later. The Graph labeling is an assignment of numbers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices(edges) then the labeling is called a vertex labeling(edge labeling).

2. Definitions

Definition 2.1

A walk *W* in a graph *G* is an alternating sequence of vertices and edges $v_0, e_1, v_1, e_2, v_2, e_3,\ldots, v_{n-1}, e_n, v_n$ such that $e_i = v_{i-1}v_i$ is an edge of *G*, $1 \leq i \leq n$. The number of edges in $v_0 - v_n$ walk is the length of the walk. It is also denoted by $v_0\, v_1\, v_2 \ldots v_{n-1}\, v_n$. If $v_0 = v_n$, then *W* is called a closed walk. If $v_0 \neq v_n$, then *W* is called a open walk. If all the edges of *W* are distinct, then it is called a trail.

Definition 2.2

If all the vertices in a walk are distinct, then it is called a path. A path of length *n* is denoted by P_n and it contains *n*+1 vertices. A path in a graph is a sequence of vertices such that from each of its vertices there is an edge to the next vertex in the sequence. The first vertex is called the start vertex and the last vertex is called the end or terminal vertices of the path and the other vertices in the path are internal vertices.

Definition 2.3

If every two vertices of a graph *G* are adjacent, then *G* is called a complete graph. The complete graph on *n* vertices is denoted by K_n.

Definition 2.4

A closed path is called a cycle. A cycle of length *n* is denoted by C_n.

Definition 2.5

The Graph C_m @ P_n is obtained by attaching a pendent vertex of P_n with a vertex of C_m.

Definition 2.6

A function f of a graph G is called a graceful labeling with m edges, if f is an injection from the vertex set of G to the set {0,1,…,m} such that when each edge uv is assigned the label |f(u)-f(v)| and the resulting labels are distinct. Then the graph G is graceful.

Definition 2.7

The padavon sequence is the sequence of integers P(n) defined by
The initial values P(0) = P(1) = P(2) = 1,

and the recurrence relation P(n) = P(n-2) + P(n-3).

The first few values of P(n) are

1, 1, 1, 2, 2, 3, 4, 5, 7, 9, 12, 16, 21, 28, 37, 49, 65, 86, 114, 151, 200, 265, 351, 465, 616,…

Definition 2.8

Let G be a (p, q) graph. An injective function f: $V(G) \rightarrow \{1,2,3, \ldots, N \geq q\}$ is said to be a padavon graceful labeling if an induced edge labeling f^* defined by $f^*(uv) = |f(u) - f(v)|$ is a bijection from $E(G)$ onto the set $\{p_1, p_2, p_3, \ldots, p_q\}$ where p_i is the i^{th} padavon number. Then G is called a padavon graceful graph if it admits a padavon graceful labeling.

3. Padavon graceful labeling for complete graphs

Theorem 3.1

The complete graph K_n ($n \geq 2$) is padavon graceful if and only if $n \leq 4$.

Proof:

It is easy to observe that K_n is padavon graceful if $2 \leq n \leq 4$.

Let us assume that $n \geq 5$. Suppose, to the contrary, that K_n is graceful. Then there exists a graceful labeling of the vertices of K_n from an n – element subset of $\{0, 1\ldots q\}$, where $q = \binom{n}{2}$. As every graceful labeling of a graph of size q requires 0 and q to be vertex labels, some edge of K_n must be labeled $q - 1$, some vertex of K_n must be labeled 1 or q - 1.

Without loss of generality, we may assume that a vertex of K_n is labeled 1. Otherwise we may use the complementary labeling.

To produce an edge labeled $q - 2$, we must have adjacent vertices labeled 0, q - 2 or 1, $q - 1$ or 2, q. If a vertex is labeled 2 or $q - 1$, then we have two edges labeled which is impossible. Thus, some vertex of K_n must be labeled $q - 2$.

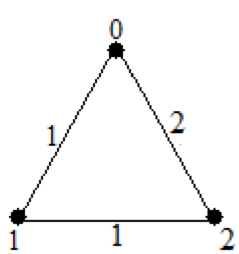
Since we now have vertices labeled 0, 1, $q - 2$ and q, we have edges labeled 1, 2, q - 3, $q - 2$, $q - 1$ and q. To have an edge labeled $q - 4$, we must have a vertex labeled 4 for all other choices result in two edges with the same label.

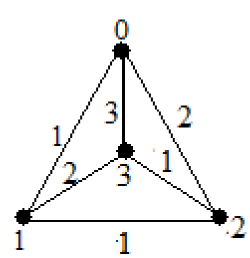
Now we have vertices labeled 0, 1, 4, $q - 2$ and q, which results in edges labeled 1, 2, 3, 4, q – 6, $q - 4$, $q - 2$, $q - 1$ and q. However, it is quickly seen that there is no vertex label that will produce the edge label $q - 5$ without also producing a duplicate edge label. Hence, no graceful labeling of K_n exists for $n \geq 5$

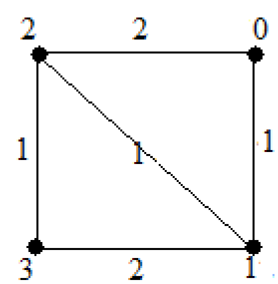
Example 3.2

The graphs K_3, K_4, $K_4 - e$ are padavon graceful graphs.

Here the vertex labels are placed within the vertices and the induced edge labels are placed near the relevant edges.







4. Padavon graceful labeling for some cycle related graphs

In this section, it is shown that C_m @ P_n; C_m @ $2P_n$ and C_n @ $K_{1, 2}$ are padavon graceful graphs.

Theorem 4.1

C_m @ P_n is a padavon graceful graph

Proof:

Let $G = C_m$ @ P_n

Let $u_1, u_2, u_3, \dots, u_m$ be the vertices of a cycle C_m and $v_1, v_2, v_3, \dots, v_n, v_{n+1}$ be the vertices of a path P_n. The sub graphs are C_m and P_n

Here $E(G) = \{u_i u_{i+1} : 1 \le i \le m-1\} \cup \{u_m u_1\} \cup \{v_j v_{j+1} : 1 \le j \le n\}$

Then $|V(G)| = |E(G)| = m + n$

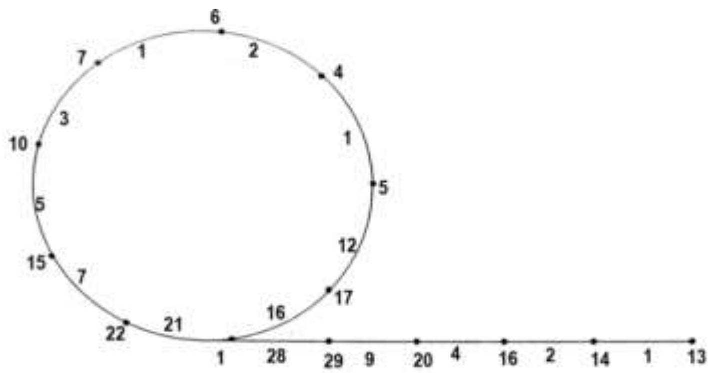
Define f: $V(G) \rightarrow \{0, 1, 2, 3, \dots, N\}$ by $f(u_1) = f(v_1) = 1$

The following example reveals that it is possible to assign suitable numbers for the values so as the edges receive distinct padavon numbers as it is shown in the following examples.

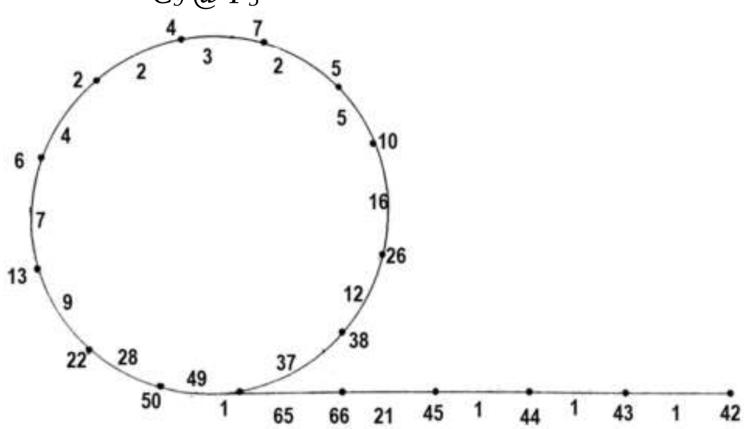
The edges of G receive the distinct labels. Therefore f is a padavon graceful labeling of G. Hence C_m @ P_n is a padavon graceful graph

Example 4.2

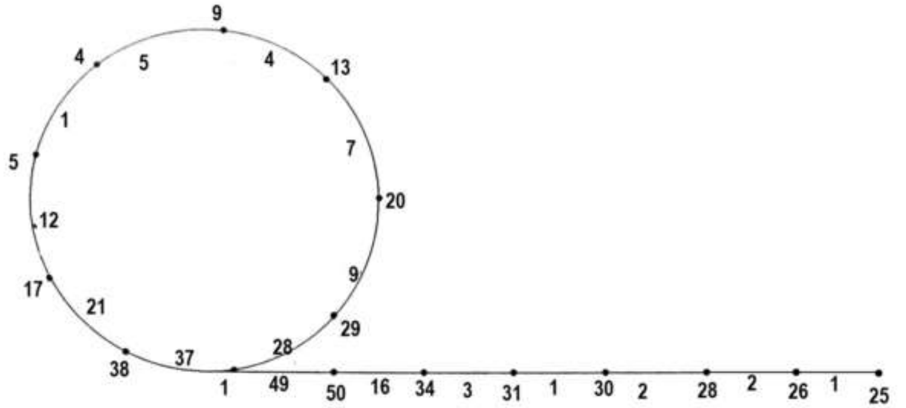
The following are some of the examples of Padavon graceful labeling for some cycle related graphs.



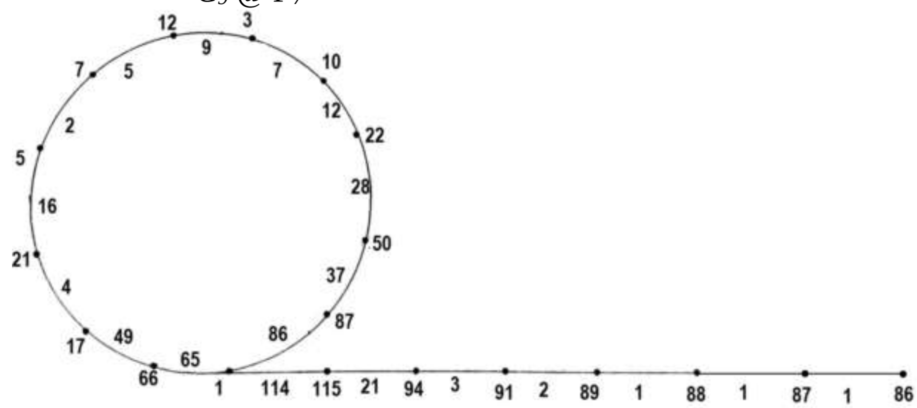
C_9 @ P_5



C_{12} @ P_5



C_9 @ P_7



C_{12} @ P_7

Theorem 4.3

C_m @ $2P_n$ is a padavon graceful graph

Proof:

Let $G = C_m$@ $2P_n$

Let $V(G) = \{w_i : 1 \le i \le m\} \cup \{u_i : 1 \le i \le n\} \cup \{v_i : 1 \le i \le n\}$ be the vertex set of G and the vertices w_2 and w_3 are identified with v_1 and u_1 of two paths of length n respectively.

Let $E(G) = \{w_i w_{i+1} : 1 \le i \le m-1\} \cup \{u_i u_{i+1}, v_i v_{i+1} : 1 \le i \le n\}$

Then $|V(G)| = 2n+m$ and $|E(G)| = 2n+m$

Define f: $V(G) \to \{0, 1, 2, 3, \ldots, N\}$ and

$$\text{and } E(G) = \bigcup_{i=1}^{n} \{|f^*(u_i u_{i+1})\}$$

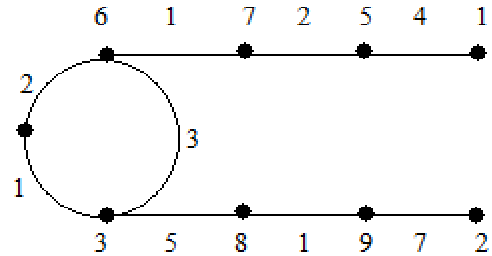
$$= \bigcup_{i=1}^{n} \{|f(u_i) - f(u_{i+1})|\}$$

So the edges of G receives the distinct labels and f is a padavon graceful labeling of G

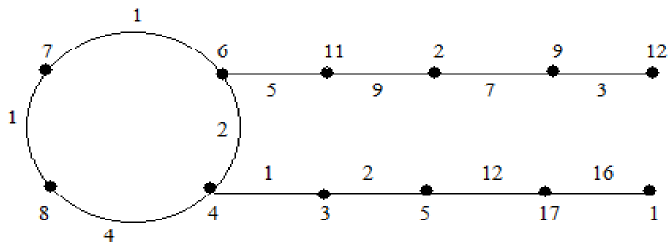
Hence C_m @ $2P_n$ is a padavon graceful graph

Example 4.4

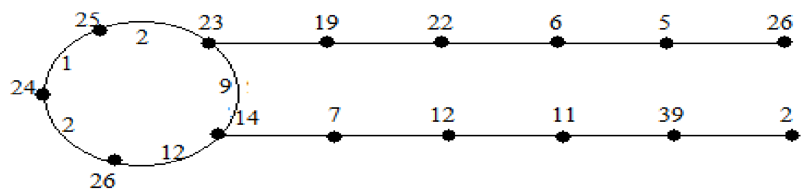
The following graphs admit padavon graceful labeling.



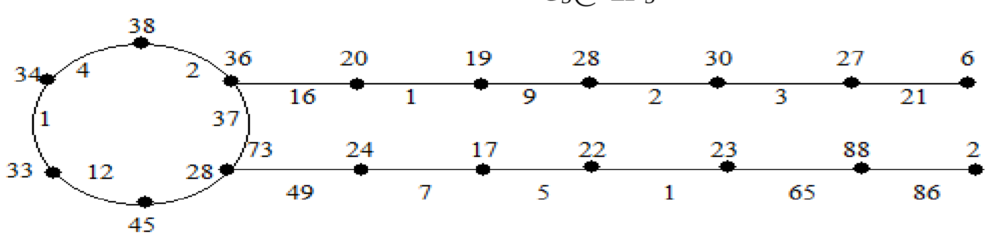
$C_3@\ 2P_3$



$C_4@\ 2P_4$



$C_5@\ 2P_5$



$C_6@\ 2P_6$

Theorem 4.5

$C_n@\ K_{1,\,2}$ is a padavon graceful graph

Proof:

Let $G = C_n@\ K_{1,\,2}$

Let $V(G) = \{u_i : 1 \leq i \leq n\} \cup \{v_1, v_2\}$

and $E(G) = \{u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{u_n u_1, u_v v_n, u_2 v_2\}$

Define f: $V(G) \rightarrow \{ p_1,\ p_2\ , p_3, \ldots p_{n+2}\}$ in such a way that

$f^{*}(e_i) = |\, f(u_i) - f(u_{i+1})|$

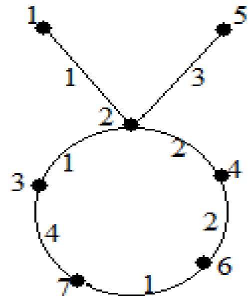
$= p_i$ where p_i is the i^{th} padavon number.

The edges of G receives the distinct labels and f is a padavon graceful labeling of G

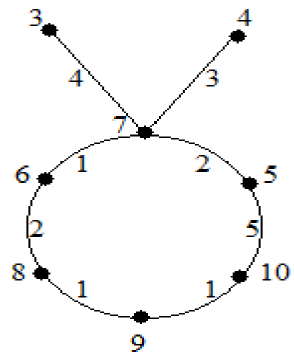
Hence $C_n@\ K_{1,\,2}$ is a padavon graceful graph

Example 4.6

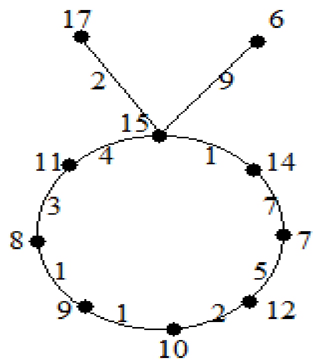
The following graphs admit padavon graceful labeling.



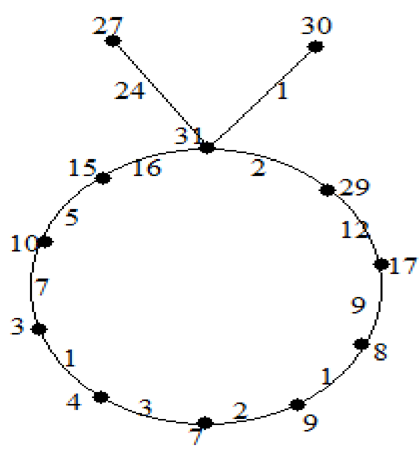
C_5@ $K_{1,2}$



C_6@ $K_{1,2}$



C_8@ $K_{1,2}$



C_{10}@ $K_{1,2}$

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