ON COMMUTATIVITY OF RA-SEMIGROUPS

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ABSTRACT:

An algebraic structure midway between a groupoid and commutative semigroup appeared in1972. M.A.Kazim and MD.Naseerudin introduced left almost semigroups as a generalisation of commutative semigroups. They have introduced the braces on the left of the ternary commutative law $abc = cba$ to get a new pseudo associative law, i.e., $(ab)c = (cb)a$. It is since than called left invertive law. A groupoid satisifying the left invertive law is called a left almost semigroup and is abbreviated as LA- semigroup. Similarly , groupoid satisfying the right invertive law,$a(bc) = c(ba)$ is called a rightt almost semigroup and is abbreviated as RA- semigroup

In this paper we will prove some of the properties of RA-semigroups. . In this paper in section 2 we define RA-semigroup with examples and prove some of the basic results. In section 3 we study the properties of commutative and bi-commutative and transitively commutative RA-semigroups. In 4 we prove some of the properties of the anti-commutative RA-semigroups.

Keywords: *RA-semigroup, Commutative RA-semigroups, Bi-commutative RA-semigroup, Anti-commutative RA-semigroup.*

1 INTRODUCTION :

The algebraic object encountered in this chapter is a set G with a binary operation ' . ' satisfying right invertive law is same as the algebraic structure " Right almost semigroup i.e., RA-semigroup" defind by MD.Naseeruddin in his Ph.D theses with the title "Some studies on almost semigroups and flocks" . He defined RA-semigroup as a groupoid satisfying right invertive law. i.e.,

$$a(bc) = c(ba) \ \forall a, b ,c, \in G$$

Definition : Let G be a non empty set and ' · ' be a binary operation from GxG →G. Then (G,.) is called an RA-semigroup if it satisfies,

$$a(bc) = c(ba) \quad \forall\ a, b ,c, \in G$$

The following multiplication table shows the existence of an RA-semigroup.

.	x	y	z
x	x	z	y
y	y	x	z
z	z	y	x

Definition : An RA-semigroup is called a commutative RA-semigroup if $ab = ba \ \forall\ a, b \in G$

Definition : Bi-Commutative RA-semigroup (BC- RA-semigroup) :

An RA-semigroup G is called right commutative RA-semigroup (RC- RA-semigroup) if $a(bc) = a(cb)$, for all $\forall\ a, b ,c, \in G$.

An RA-semigroup G is called a left commutative RA-semigroup (LC- RA-semigroup) if $(ab)c = (ba)c$, for all $\forall\ a, b ,c, \in G$

An RA-semigroup G is called a bi-commutative RA-semigroup (BC- RA-semigroup) if it is both LC- RA-semigroup and RC- RA-semigroup.

Definition : Anti-commutative RA-semigroup**:** An RA-semigroup G is called anti-commutative RA-semigroup if the identity , $ab = ba \implies a = b$ holds $\forall\ a, b \in G$

Alternative RA-semigroup : An RA-semigroup G is called left alternative RA-semigroup if it satisfies the identity , (aa)b = a(ab). for all a,b ∈ G

An RA-semigroup is called right alternative RA-semigroup G if it satisfies the identity, $(ab)b = a(bb)$, for all $a,b \in$ G

Self-Dual RA-semigroup : An RA-semigroup which satisfies left invertive law $(ab)c = c(ba)$ *for all* $a,b,c \in$ G is called Self -Dual RA-semigroup.

Nuclear square : An RA-semigroup G is called left nuclear square if $\forall a,b,c, \in$ G, $a^2(bc) = (a^2b)c$. Similarly S is called right nuclear square if $\forall a,b,c, \in$ G, $(ab)c^2 = a(bc^2)$ and middle **nuclear square if** $\forall a,b,c, \in$ **G,** $(ab^2)c = a(b^2c)$**.**

Right transitive RA-semigroup : An RA-semigroup G is called right transitive if , $ab.cb = ac$ for all $a,b, c \in$ G

Left transitive RA-semigroup : An RA-semigroup G is called left transitive if , $ab.ac = bc$ for all $a,b, c \in$ G

Locally associative: An RA-semiroup G is said be locally associative if, $a^2a = aa^2 \quad \forall \quad a \in G$

Cancellative RA-semigroup : Let G be an RA-semigroup. If for all $a, b ,c, \in$ G, $ab = ac \implies b = c$ then we say that G is a left cancellative RA-semigroup

Let G be an RA-semigroup. If for all $a, b ,c, \in$ G, $ba = ca \implies b = c$ then we say that G is a right cancellative RA-semigroup

Let G be an RA-semigroup. If for all $a, b ,c, \in$ G, $ab = ac \implies b = c$, and also $ba = ca \implies b = c$, then we say that G is a cancellative RA-semigroup

Let G be an RA-semigroup. If for all $a, b ,c, \in$ G, $ab = ca \implies b = c$, and also $ba = ac \implies b = c$, then we say that G is a cross-cancellative RA-semigroup

2: SOME BASIC RESULTS ON RA-SEMIGROUPS.

2.1 Lemma : An RA-semigroup G satisfies medial law.

i.e., $(ab)(cd) = (ac)(bd) \quad \forall a, b, c \in$ G.

Proof : Using right invertive law,

$(ab)(cd) = d(c(ab)) = d(b(ac) = (ac)(bd)$

2.2 Lemma : If right identity 'e' exists in RA-semigroup then it is unique.

Proof : If possible there exists another right identity say f, then

$f=fe$ and $ef=e$ and $f=fe=f(ee)=e(ef)=ee=e \implies f = e$

2.3 Lemma : Ia an RA-semigroup with right identity, paramedial law holds.

i.e., $(ab)(cd) = (db)(ca)$

Proof : $(ab)(cd) = (ab)((cd)e) =e((cd)(ab)) =e(b(a(cd))) = e(b(d(ca))) = e((ca)(db))$

$=(db)((ca)e)= (db)(ca).$

2.4 Lemma : In an RA-semigroup with right identity e ,

$ab=cd \iff ba=dc$

Proof : (i) $ab = cd \implies ba = d$

$$ba = b(ae) = e(ab) = e(cd) = d(ce) = dc.$$

Similarly we can show that $(ba) = (dc) \implies (ab) = (cd)$

2.5 Lemma : If an RA-semigroup G contains a right identity the following law holds $(ab)c = (ac)b, . \forall a, b, c \in$ G.

Proof : $(ab)c = (ab)(ce) = e\big(c(ab)\big) = e\big(b(ac)\big) = (ac)(be) = (ac)b.$

3: ON COMMUTATIVITY OF RA-SEMIGROUPS

3.1 Theorem : An RA-semigroup G is a commutative semigroup iff the following law holds $\forall a, b, c \in$ G.

$a(bc)=(ba)c$ ---- (a)

Proof : (i) Let the given condition (a) holds in G,

and $a(bc) = c(ba)$ ---- (b) (right invertive law)

from (a) and (b), we have

$(ba)c = c(ba) \implies$ G is commutative.

and $a(bc) = (ba)c = (ab)c \implies$ Gis associative.

$\therefore$ G is a commutative semigroup.

(ii) Let G be a commutative semigroup.

then, $a(bc) = (ab) = c(ab) = c(ba) = (ba)c \implies a(bc) = (ba)c$

3.2 Theorem : An RA-semigroup with left identity is a commutative semigroup.

Proof : Let G is an RA-semigroup with left identity e .

then, ea = a

ab = a(eb) = b(ea) = ba $\implies$ ab =ba$\implies$ G is commutative.

$a(bc) = c(ba) = c(ab) = (ab)c$ (Since G is commutative)

$a(bc) = (ab)c \implies$ G is associative

$\therefore$ G is a commutative semigroup.

3.3 Theorem: Let G be a RA-semigroup with right identity. If G is left alternative then G is a commutative semigroup.

Proof: G is an RA-semigroup

Let G is left alternative then,

$aa.b = a.ab \quad \forall a, b \in$ G.

$aa.b = a.ab = b.aa \implies a^2 b = b.a^2$

Now replace a by e we have, $e^2 a = a.e^2 \implies a.e = e.a$

Since e is the right identity in G we have $a.e = e.a = a \implies e$ is the identity in G

Now by right invertive law and identity in G we have,

$ab = a(be) = e(ba) = ba \implies$ G is commutative

and by commutativity and right invertive law we have,

$a(bc) = c(ba) = c(ab) = (ab)c \implies$ G is associative

$\therefore$ G is a commutative semigroup

3.4 Theorem : A commutative RA-semigroup is,

(i) Associative

(ii) Permutable

(iii Self-Dual

(iv) Bi-commutative

(v) Alternative

(vi) Paramedial

(vii) Nuclear square

Proof: Let G is a commutative RA-semigroup

(i) Consider $a(bc) = c(ba) = c(ab) = (ab)c \implies a(bc) = (ab)c \implies$ G is associative

(ii) Let G be a commutative RA-semigroup, then

Consider $a(bc) = c(ba) = c(ab) = b(ac) = (ac)b \implies a(bc) = b(ac) \implies$ Gis right permutable

similarly $a(bc) = c(ba) = c(ab) = b(ac)$ i.e., Gis left permutable.

(iii) Consider $a(bc) = c(ba)$

By using commutativity on both sides we get,

$$a(bc) = c(ba) \Leftrightarrow (bc)a = c(ab) \Leftrightarrow (cb)a = (ab)c$$

Right invertive law $\Leftrightarrow$ Left invertive law. $\implies$G is Self-Dual RA-semigroup.

(iv) Consider $a(bc)$ & $(ab)c$

Since G is commutative ,we have

$a(bc) = a(cb)$ (commutativity)

$(ab)c = (ba)c$ (commutativity)

$\implies$ G is Bi-commutative.

(v) $a(ab) = b(aa) = (aa)b \implies$ G is left alternative.

$a(bb) = b(ba) = b(ab) = (ab)b \implies$ G is right alternative

$\implies$ G is an alternative RA-semigroup.

(vi) Consider $(ab)(cd) = (cd)(ab)$ (commutativity)

$= (dc)(ba)$ (commutativity)

$= (db)(ca)$ (medial law)

$\implies$ G is paramedial

(vii) since Commutativity $\implies$ Associativity we have,

$a^2(bc) = (a^2b)c$, $(ab)c^2 = a(bc^2)$, $(ab^2)c = a(b^2c)$. $\forall a, b, c, \in G \implies$G is a nuclear square.

3.5 Theorem : Let G be an RA-semigroup. Then G is a commutative semigroup if G satisfies any one of the following.

(i) G is slim RA-semigroup

(ii) G is left alternative

(iii) G is right alternative satisfying cross-cancellation

(iv) Idempotent and paramedial

(v) Left commutative with right cancellation

(vi) Right commutative with left cancellation

(vii) Self - dual with right identity

(viii) Left transitive

(ix) Right transitive

Proof: Let G be an RA-semigroup and let $a, b, c, \in$ G

(i) Let G be a slim RA-semigroup. Then, $a(bc) = ac$

Consider $ab = a(bb) = b(ba) = ab \Longrightarrow ab = ba \Longrightarrow$ G is commutative.

(ii) Let G be a left alternative RA-semigroup

By left alternativity in G we have , $(aa)b = a(ab) \quad \forall a, b \in G$.

Using right invertive law on the right we get $(aa)b = b(aa) \quad \forall a, b \in G$.

$\Longrightarrow$ G is commutative.

(iii) Let G is right alternative RA-semigroup satisfying cross-cancellation

Since G is right alternative we have , $(ab)b = a(bb) \quad \forall a, b \in G$.

Using right invertive law on the right we get $(ab)b = b(ba)$

using cross-cancellativity in G we have $ab = ba \quad \forall a, b \in G \Longrightarrow$ G is commutative.

(iv) Let G be an idempotent RA-semigroup. Then $a^2 = a \quad \forall a, \in G$.

Let G be paramedial then, $(ab)(cd) = (db)(ca)$

Consider $ab = (ab)^2 = (ab)(ab) = (bb)(aa) = b^2a^2 = ba \Longrightarrow ab = ba \Longrightarrow$ G is commutative

(v) Let G be a left commutative Ra-semigroup with right cancellativity

G is left commutative we have , $(ab)c = (ba)c \quad \forall a, b, c \in G$.

cancellativity in G we have $ab = ba \quad \forall a, b \in G \Longrightarrow$ G is commutative

(vi) Let G be a right commutative using right RA-semigroup.

Since G is right commutative we have , $a(bc) = a(cb) \quad \forall a, b, c \in G$.

using left cancellativity in G we have $bc = cb \Longrightarrow$ G is commutative.

A(vii) Let G be an RA-semigroup with right identity. Then $ae = a \quad \forall a, \in G$.

Let G be self-dual then, $(ab)c = (cb)a$

Consider $ea = (ee)a = (ae)e = ae = a \Longrightarrow eaa \Longrightarrow e$ is the left identity $\Longrightarrow e$ is the identity

Now $ab = a(be) = e(ba) = ba \Longrightarrow ab = ba \Longrightarrow$ G is commutative

(viii) Let G be a left transitive RA-semigroup

By left transitive condition in G we have , $bc = bb.bc$

$$bb.bc = bb(bb.bc) \qquad (\text{Since } bc = bb.bc).$$

Using right invertive law on the right we get

$bc = bb(bb.bc) = bc(bb.bb) = bc.bb = cb \quad$ (*by left transitivity in G*)

Thus $bc = cb \Longrightarrow$ G is commutative.

(ix) Let G be a right transitive RA-semigroup

By right transitive condition in G we have , $ac = ac.aa = (a.ac)(a.aa)$

Using right invertive law on the right we get $ac = (c.aa)(a.aa)$ (*by right transitivity in G*)

$$= ca \qquad (\text{by right transitivity in G})$$

Thus $ac = ca \Longrightarrow$ G is commutative.

In RA-semigroups always commutativity implies associarivity.

Hence in all the above cases G is associative and hence G is a commutative semigroup.

3.6 Theorem: Let G be a left-cancellative RA-semigroup. Then G is transitively commutative.

Proof: Let G be an RA-semigroup. Let $a, b, c \in G$ such that $ab = ba$ and $bc = cb$.

By right invertive law in G we have,

$$a(bc) = c(ba)$$

Since $ab = ba$ and $bc = cb$ we have,

$$a(cb) = c(ab) \Longrightarrow b(ca) = b(ac)$$

using left-cancellation we get, $ca = ac$

$$ab = ba \text{ and } bc = cb \Longrightarrow ca = ac \Longrightarrow \text{G is transitively commutative}$$

3.7 Theorem: Let G be an RA- semigroup. Then G is transitively commutative if ,

(i) G is left transitive

(ii) G is right transitive

Proof:

(i) Let G be a left transitive RA-semigroup $\Longrightarrow ac = ba.bc \quad \forall\, a, b, c \in$ G

Let $ab = ba$ and $bc = cb$.

Now we use the above assumptions and right invertive law to show that $ac = ca$

Consider $ac = ba.bc$

$= ba.cb$ $(bc=cb)$

$= b(c.ba)$ (right invertive law)

$= b(a.bc)$) (right invertive law)

$= bc.ab$) (right invertive law)

$=bc.ba$ $(ab = ba)$

$=ca$ (G is left transitive)

$ab =ba \;\&\; bc =cb \Longrightarrow ac = ca \quad \forall\, a, b, c \in$ G

$\Longrightarrow$ G is transitively commutative

(ii) Let G be a right transitive RA-semigroup $\Longrightarrow ac = ab.cb \quad \forall\, a, b, c \in$ G

Let $ab = ba$ and $bc = cb$.

Now we use the above assumptions and right invertive law to show that $ac = ca$

Consider $ac = ab.cb$

$= ba.cb$ $(bc=cb)$

$= b(c.ba)$ (right invertive law)

$= b(a.bc)$) (right invertive law)

$= b(a.cb)$ $(bc = cb)$

$=cb.ab$ (right invertive law)

$=ca$ (G is left transitive)

$ab =ba \;\&\; bc =cb \Longrightarrow ac = ca \quad \forall\, a, b, c \in$ G $\Longrightarrow$ G is transitively commutative

3.8 Theorem: Let G be a RA- semigroup with the condition $a(bc)=ac$ for all a, b, c in G. Then G is,

(i) Left commutative

(ii) Right commutative

(iii) Transitively commutative

Proof: Let G be an RA-semigroup

And let $a, b, c \in G$ such that $a(bc)=ac$

(i) To show that G is left commutative , we have to show $(ab)c = (ba)c$

For this consider $(ab)c$

Using right invertive law, medial law and slim groupoid property we show that G is left commutative.

$(ab)c = (ab)(bb(c)) = c(bb.ab) = c(ba.bb) = c(b(b.ba) = c(b(ba)) = (ba)(bc) = (ba)c$

$(ab)c = (ba)c\ ) \implies$ G is a left commutative RA-semigroup

(ii) To show that G is right commutative , we have to show $a(bc) = a(cb)$

Using right invertive law, medial law and slim groupoid property we show that G is right commutative.

$a(bc) = a(cc(bc)) = a(cb.cc) = a(c(c.cb)) = a(c(cb)) = a(cb)$

$a(bc) = a\ (cb) \implies$ G is a right commutative RA-semigroup

(iii) Let $a, b, c \in G$ such that $a(bc)=ac$

and let $ab = ba\ \&\ bc = cb$

Consider $ac = a(bc) = a(b(bc)) = a(c(bb)) = a(bb) = ab$ ----(1)

And $ab = ba = b(ca) = b(c(ca)) = b(a(cc) = b(cc) = bc$ ------(2)

Again $bc = cb = c(ab) = c(a(ab) = c(b(aa)) = c(aa) = ca$ ------(3)

From (1), (2) & (3) $ac = ab = ba = bc = cb = ca \implies ac = ca$

$\implies$ G is transitively commutative RA-semigroup

3.9 Theorem: Let G be an RA-semigroup with the identity $a(bc) = (ac)b\ \ \forall a, b, c \in$ G.Then G is left commutative

Proof: G is an RA-semigroup with the identity $a(bc) = (ac)b\ \ \forall a, b, c \in$ G.

Consider $(ab)c = a(cb)$ (by the assumption $a(bc) = (ac)b$)

$= b(ca)$ (right invertive law)

$= (ba)c$ (by the assumption $a(bc) = (ac)b$)

$(ab)c = (ba)c \quad \forall a, b, c \in$ G $\implies$ G is left commutative

3.10 Theorem: Let G be (right commutative) RC-RA-semigroup. Then G is a commutative semigroup if,

(i) G has right identity

(ii) G is cancellative

Proof: G is a right commutative RA-semigroup

(i) Let e be the right identity in G

Consider $ab = a(be) = a(eb) = b(ea) = b(ae) = ba \implies$ G is commutative

G is commutative $\implies$ G is associative $\implies$ G is commutative semigroup.

(ii) G is right commutative $\implies a(bc) = a(cb)\ \ \forall a, b, c \in$ G.

by left cancellativity we have , $bc = cb \implies$ G is commutative

G is commutative $\implies$ G is associative $\implies$ G is commutative semigroup.

4 ANTI-COMMUTATIVITY OF RA-SEMIGROUPS.

4.1 Theorem : Let G be an anti-commutative RA-semigroup with right identity 'e ' . Then ,

(i) G is quasi-cancellative.

(ii) G is unipotent

(iii) G is RA-3-band

Proof:

Let G be an anti-commutative RA-semigroup with right identity. Then, $\forall\ a, b, c \in G$ we have

$a(bc) = c(ba)$ ------(I)

$ab = ba \implies a=b$ ------(II)

$(ab)c = (ac)b$ -------(III)

$ab = cd \implies ba = dc$ ----- (IV)

(i) Let $a^2 = ab \implies aa=ab \implies aa=ba \implies ab = ba \implies a=b$ (Since G is anti-commutative)

$a^2 = ab \implies a=b$

Similarly let, $b^2=ba \implies bb=ba \implies bb=ab \implies ba=ab \implies a = b$ (Since G is anti-commutative)

$b^2 = ba \implies a=b$

Hence G is quasi-cancellative

(ii) consider $a^2b^2 = aa.bb$

By medial law $a^2b^2 = ab.ab$

By (iv) $b^2a^2 = ab.ab = aa.bb = a^2b^2$

By anti-commutativity in G $b^2a^2 = a^2b^2 \implies a^2=b^2 \implies$ G is unipotent

(iii) Consider $a(a.aa)$

by right invertive law $a(a.aa) = aa.aa$

by (IV) $(a.aa)a = aa.aa$

again using right invertive law on the right, $(a.aa)a = a(a.aa)$

from anti-commutativity property $a.aa = a \implies$ G is RA-3-band.

4.2 Theorem : Let G be an anti-commutative RA-semigroup . Then ,

G is transitively commutative.

Proof: Let G be an anti-commutative RA-semigroup

And let $a, b, c \in$ G such that $ab = ba$ & $bc = cb$

Now we have to show that $ac = ca$

Since G is anti-commutative, $ab = ba \implies a = b$

and $bc = cb \implies b = c$

$a = b$ & $b = c \implies a = c \implies ac = ca$

$ab = ba$ & $bc = cb \implies ac = ca \implies$ G is transitively commutative.

G is anti-commutative $\Longrightarrow$ G is transitively commutative

4.3 Theorem : Let G be an anti-commutative RA-semigroup . Then G is an RA-semigroup band if and only if G is locally associative.

Proof:

(i) Let G be a RA-semigroup band

then $a^2 = a \quad \forall \; a \in G$

clearly we have $a^2a = aa^2 \Longrightarrow$ G is locally associative

(ii) Let G be locally associative

then $a^2a = aa^2 \quad \forall \; a \in G$

By anti- commutativity in G we have $a^2 = a \quad \Longrightarrow$ G is an RA-semigroup band

4.4 Theorem : Let G be an anti-commutative RA-semigroup. If G is paramedial then G is,

(i) Unipotent

(ii) Rectangular

Proof: Let G be an anti-commutative paramedial RA-semigroup

(i) Consider $a^2b^2 = aa.bb = ab.ab$ (medial law)

$= bb.aa$ (paramedial law)

$= b^2a^2$

$a^2b^2 = b^2a^2$

By anti-commutativity of G , $a^2b^2 = b^2a^2 \Longrightarrow a^2 = b^2$

$\Longrightarrow$ G is unipotent.

(ii) Consider $(ab.ad)(cb.cd) = (aa.bd)(cc.bd)$ (medial law)

$= (aa.cc)(bd.bd)$ (medial law)

$= (ca.ca)(bd.bd)$ (paramedial law)

$= (cc.aa)(bd.bd)$ (medial law)

$= (cc.bd)(aa.bd)$ (medial law)

$= (cb.cd)(ab.ad)$ (medial law)

$\Longrightarrow (ab.ad)(cb.cd) = (cb.cd)(ab.ad)$

By anti-commutativity of G $(ab.ad)(cb.cd) = (cb.cd)(ab.ad) \Longrightarrow$ G is rectangular

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