ON #REGULAR GENERALIZED OPEN SETS IN BITOPOLOGICAL SPACE

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ABSTRACT

In this paper we introduce and study $\tau_1\tau_2$#regular generalized open (briefly, $\tau_1\tau_2$#rg-open) sets in bitopological space and obtain some of their properties. Also, we introduce $\tau_1\tau_2$#rg-neighbourhood (shortly $\tau_1\tau_2$#rg-nbhd) in bitopological spaces by using the notion of $\tau_1\tau_2$#rg-open sets. Applying $\tau_1\tau_2$# rg-closed sets and discuss some basic properties of this.

KEYWORDS

$\tau_1\tau_2$rg-open sets, $\tau_1\tau_2$rg-nbhd.

1. INTRODUCTION:

A triple (X, τ_1, τ_2), where X is a non empty set and τ_1 and τ_2 are topologies on (X, τ_1, τ_2) is called a bitopological space. In 1963, Kelly[22] initiated the study of bitopological spaces. In 1985, Fukutake[5] introduce the concept of g-closed sets in bitopological spaces and after that several authors turned their attention towards generalizations of various concepts of topology by considering bitopological spaces. The aim of this paper is to extend the same concept in bitopological spaces.We introduce a new class of sets called $\tau_1\tau_2$#regular generalized open which is properly placed in between the class of open sets and the class of $\tau_1\tau_2$rg-open sets.

Throughout this paper (X,τ_1, τ_2) represents a bitopological space on which no separation axiom is assumed unless otherwise mentioned. For a subset A of a bitopological space (X, τ_1, τ_2), $\tau_2$cl(A) and $\tau_1$int(A) denote the closure of A and the interior of A respectively. X\A or A^c denotes the complement of A in X. We recall the following definitions and results.

1.1. Definition

A subset A of a space (X,τ_1, τ_2) is called

1) a $\tau_1\tau_2$preopen set[9] if A ⊆ τ_1 int$\tau_2$cl (A) and a $\tau_1\tau_2$preclosed set if $\tau_2$cl$\tau_1$int (A)⊆A.

2) a $\tau_1\tau_2$semiopen set[1] if A ⊆ τ_2 cl$\tau_1$int (A) and a $\tau_1\tau_2$semiclosed set if $\tau_1$int$\tau_2$cl (A) ⊆A.

3) a $\tau_1\tau_2$regular open set[14]if A = $\tau_1$int$\tau_2$cl (A) and a $\tau_1\tau_2$regular closed set if A = $\tau_2$cl$\tau_1$int (A).

4) a $\tau_1\tau_2\pi$ - open set if A is a finite union of $\tau_1\tau_2$regular open sets.

5) $\tau_1\tau_2$regular semi open if there is a $\tau_1$regular open U such U ⊆A ⊆ $\tau_2$cl(U).

1.2. Definition

A subset A of (X,τ_1, τ_2) is called

1) $\tau_1\tau_2$generalized closed set (briefly, $\tau_1\tau_2$g-closed)[5] if $\tau_2$cl (A)⊆ U whenever A⊆U and U is open in X.

2) $\tau_1\tau_2$regular generalized closed set (briefly, $\tau_1\tau_2$rg-closed) if

$\tau_2$cl (A)⊆U whenever A⊆U and U is $\tau_1$regular open in X.

3) $\tau_1\tau_2$generalized preregular closed set (briefly, $\tau_1\tau_2$gpr-closed) if $\tau_2$pcl (A)⊆U whenever A ⊆ U and U is $\tau_1$regular open in X.

4) $\tau_1\tau_2$weakely generalized closed set (briefly, $\tau_1\tau_2$wg-closed) if $\tau_2$cl$\tau_1$int (A) ⊆U whenever A⊆ U and U is $\tau_1$open in X .

5) $\tau_1\tau_2\pi$-generalized closed set (briefly, $\tau_1\tau_2\pi$g-closed) if $\tau_2$cl(A)⊆U whenever A⊆U and U is $\tau_1\pi$-open in X.

6) $\tau_1\tau_2$weakely closed set (briefly, $\tau_1\tau_2$w-closed) if $\tau_2$cl(A) ⊆U whenever A⊆U and U is $\tau_1$semi open in X.

7) $\tau_1\tau_2$regular weakly generalized closed set (briefly, $\tau_1\tau_2$rwg-closed) if $\tau_2$cl$\tau_1$int(A)⊆U whenever A ⊆U and U is $\tau_1$regular open in X.

8) $\tau_1\tau_2$rw-closed if $\tau_2$cl(A)⊆U whenever A⊆U and U is $\tau_1$regular semi open.

9) $\tau_1\tau_2$*g-closed if $\tau_2$cl(A)⊆U whenever A⊆U and U is τ_1w-open.

10)$\tau_1\tau_2$#rg-closed if $\tau_2$cl(A)⊆U whenever A⊆U and U is $\tau_1$rw-open.

2. $\tau_1\tau_2$#REGULAR GENERALIZED OPEN SETS AND $\tau_1\tau_2$#REGULAR GENERALIZED NEIGHBOURHOODS.

2.1. Definition

A subset A of a space (X,τ_1,τ_2) is called $\tau_1\tau_2$#regular generalized open (briefly $\tau_1\tau_2$#rg-open) set if its complement is $\tau_1\tau_2$#rg-closed. The family of all $\tau_1\tau_2$#rg– open sets in X is denoted by $\tau_1\tau_2$#RGO(X).

2.2. Example

Let X= {a, b, c} be with topologies τ_1={ ϕ, X, {a}, {a, c}} and τ_2={ ϕ, X, {a}, {b, c}} then $\tau_1\tau_2$#rg-open sets are { ϕ, X, {a}, {a,c}, {b, c}}.

2.3. Remark

$\tau_2$cl(X\A) = X\$\tau_1$int(A).

2.4. Theorem

A subset A of (X,τ_1,τ_2) is $\tau_1\tau_2$#rg-open if and only if F⊆ $\tau_1$int(A) whenever F is $\tau_1\tau_2$rw-closed and F⊆A.

Proof

(*Necessity*). Let A be $\tau_1\tau_2$#rg-open . Let F be $\tau_1\tau_2$rw-closed and F⊆A then X\A⊆X\F, whenever X\F is $\tau_1\tau_2$rw-open. Since X\A is $\tau_1\tau_2$#rg-closed, $\tau_2$cl(X\A)⊆X\F. By Remark 2.2, X\$\tau_1$int(A)⊆ X\F. That is F⊆ $\tau_1$int(A).

(*Sufficiency*). Suppose F is $\tau_1\tau_2$rw-closed and F⊆A implies F⊆ $\tau_1$int(A). Let X\A⊆U where U is $\tau_1\tau_2$rw-open. Then X\U⊆A, where X\U is rw-closed. By hypothesis X\U⊆ $\tau_1$int(A). That is X\$\tau_1$int(A)⊆U. By remark 2.2 $\tau_2$cl(X\A)⊆U, implies, X\A is $\tau_1\tau_2$#rg-closed and A is $\tau_1\tau_2$#rg-open.

2.5. Theorem

If $\tau_1$int(A) ⊆ B ⊆ A and A is $\tau_1\tau_2$#rg-open, then B is $\tau_1\tau_2$#rg-open.

Proof

Let A be $\tau_1\tau_2$#rg-open set and $\tau_1$int(A) ⊆B⊆A. Now $\tau_1$int(A)⊆B ⊆ A implies X\A ⊆ X\B ⊆ X\$\tau_1$int(A). That is X\A ⊆ X\B ⊆ $\tau_2$cl(X\A). Since X\A is $\tau_1\tau_2$#rg-closed, X\B is $\tau_1\tau_2$#rg-closed and B is $\tau_1\tau_2$#rg-open.

2.6. Remark

For any A ⊆X, $\tau_1$int($\tau_2$cl(A)\A) = ϕ.

2.7. Theorem

If A⊆X is $\tau_1\tau_2$#rg-closed then $\tau_2$cl(A)\A is $\tau_1\tau_2$#rg-open.

Proof

Let A be $\tau_1\tau_2$#rg-closed. Let F be $\tau_1\tau_2$rw-closed set such that F⊆ $\tau_2$cl(A)\A. Then by theorem 2.12[23] F=ϕ. So, F ⊆ $\tau_1$int($\tau_2$cl(A)\A). This shows $\tau_2$cl(A)\A is $\tau_1\tau_2$#rg-open.

2.8. Theorem

Every open set in X is $\tau_1\tau_2$#rg-open but not conversely.

Proof

Let A be an open set in a space (X,τ_1,τ_2) . Then X\A is closed set. By theorem 2.3[23] X\A is $\tau_1\tau_2$#rg-closed. Therefore A is $\tau_1\tau_2$#rg-open set in X.

The converse of the theorem need not be true, as seen from the following example.

2.9. Example

Let X= {a, b, c} be with topologies τ_1={ ϕ, X, {a}, {b}, {a, b}} and τ_2={ ϕ, X, {a}, {b, c}}, τ_2 open sets are { ϕ, X, {a}, {b, c}} and $\tau_1\tau_2$#rg-open sets are { ϕ, X, {a}, {b}, {b, c}} then {b} is $\tau_1\tau_2$#rg-open sets but it is not τ_2 open.

2.10.Theorem

Every $\tau_1\tau_2$#rg-open sets in X is $\tau_1\tau_2$rg-open set in X, but not conversely..

Proof

Let A be $\tau_1\tau_2$#rg-open set in space X. Then X\A is $\tau_1\tau_2$# rg-closed set in X. By theorem 2.5[23], X\A is $\tau_1\tau_2$rg-closed set in X. Therefore A is $\tau_1\tau_2$rg-open in X.

The converse of the above theorem need not be true as seen from the following example.

2.11. Example

Let X= {a, b, c} be with topologies τ_1={ ϕ, X, {a}, {b}, {a, b}} and τ_2={ ϕ, X, {a}, {b, c}} then {c} is $\tau_1\tau_2$ rg-open but not $\tau_1\tau_2$ #rg-open sets .

2.12. Theorem

Every $\tau_1\tau_2$#rg-open set in X is $\tau_1\tau_2$*g –open set in X .

Proof

Let A be $\tau_1\tau_2$#rg-open set in space X. Then X\A is $\tau_1\tau_2$# rg-closed set in X. X\A is $\tau_1\tau_2$*g-closed set in X. Therefore A is $\tau_1\tau_2$*g-open in X.

2.13. Theorem

Every $\tau_1\tau_2$#rg-open set in X is $\tau_1\tau_2$g-open, but not conversely.

Proof.

Let A be $\tau_1\tau_2$#rg-open set in X. Then A^c is $\tau_1\tau_2$#rg-closed set in X. By theorem 2.7[23] A^c is $\tau_1\tau_2$g-closed set in X. Hence A is $\tau_1\tau_2$g-open in X.

The converse of the above theorem need not be true as seen from the following example.

2.14. Example.

Let X= {a, b, c, d} be with topologies τ_1={ ϕ, X, {a}, {b}, {a, b}, {a, b, c}} and τ_2={ ϕ, X, {a}, {b}, {a, b}} , then the set A={a,b,c} is $\tau_1\tau_2$ g-open but not $\tau_1\tau_2$#rg-open set in X.

2.15. Theorem

If a subset A of a space X is $\tau_1\tau_2$#rg-open then it is $\tau_1\tau_2$ πg-open set in X.

Proof

Let A be $\tau_1\tau_2$#rg-open set in space X. Then X\A is $\tau_1\tau_2$# rg-closed set in X. X\A is $\tau_1\tau_2$ π g-closed set in X. Therefore A is $\tau_1\tau_2$ πg-open in X.

The converse of the above theorem need not be true as seen from the following example.

2.16. Example

Let X={a,b,c,d} be with topologies τ_1={X, ϕ, {a}, {b}, {a.b}, {a,b,c}} and τ_2={ ϕ, X, {a}, {b}, {a, b}, {a, b}} then the set A={a,b,d} is $\tau_1\tau_2$ πg-open but not $\tau_1\tau_2$#rg-open set in X.

2.17. Theorem

If A and B are $\tau_1\tau_2$#rg-open set in a space X. Then A∩B is also $\tau_1\tau_2$#rg-open set in X.

Proof

If A and B are $\tau_1\tau_2$#rg-open sets in a space X. Then X\A and X\A are $\tau_1\tau_2$#rg-closed sets in a space X. (X\A)∪ (X\B) is also $\tau_1\tau_2$#rg-closed sets in X. Therefore A∩B is $\tau_1\tau_2$#rg-open set in X.

2.18. Remark

The union of two $\tau_1\tau_2$#rg-open sets in X is generally not a $\tau_1\tau_2$#rg-open set in X.

2.19. Example

Let X= {a, b, c, d} be with topologies τ_1={ ϕ, X, {a}, {b}, {a, b}, {a, b, c}} and τ_2={ ϕ, X, {a}, {b}, {a, b}} , $\tau_1\tau_2$#rg-open sets are { ϕ, X, {a}, {b},{c},{d}, {b, d}, {b, c}, {a, d}, {a, c}, {a, b}, } then {a, d}∩{a, c}={a, c, d} but it is not $\tau_1\tau_2$#rg-open set.

2.20.Theorem

If a subset A of a bitopological space X is both $\tau_1\tau_2$rw-closed and $\tau_1\tau_2$#rg-open then it is open.

Proof.

Let A be $\tau_1\tau_2$rw-closed and $\tau_1\tau_2$#rg-open set in X. Now A ⊆A. By theorem 2.4 A⊆ $\tau_1$int(A). Hence A is open.

2.21.Theorem

If a set A is $\tau_1\tau_2$#rg-open in X, then G=X, whenever G is $\tau_1\tau_2$rw-open and ($\tau_1$int(A) ∪ (X\A)) ⊆G.

Proof

Suppose that A is $\tau_1\tau_2$#rg-open in X. Let G is $\tau_1\tau_2$rw-open and ($\tau_1$int(A) ∪(X\A)) ⊆G. Thus G^c ⊆ ($\tau_1$int(A) ∪A^c)^c =($\tau_1$int(A))^c ∩A. That is G^c ⊆ ($\tau_1$int(A))^c\A^c. Since ($\tau_1$int(A))^c = $\tau_2$cl(A^c), G^c ⊆ $\tau_2$cl(A^c)\ A^c. Now, G^c is $\tau_1\tau_2$rw-closed and A^c is $\tau_1\tau_2$#rg-closed, G^c = ∅. Hence G=X.

2.22. Theorem

Every singleton point set in a space is either $\tau_1\tau_2$#rg-open (or) $\tau_1\tau_2$rw-closed.

Proof

It is follows from theorem 2.16[23].

2.23. Definition

Let (X,τ_1,τ_2) be a bitopological space and let x ∈ X. A subset N of X is said to be a $\tau_1\tau_2$#rg-nbhd of x iff there exists a $\tau_1\tau_2$#rg-open set U such that x ∈ U⊆N.

2.24. Definition

A subset N of space (X,τ_1,τ_2), is called a $\tau_1\tau_2$#rg-nbhd of A ⊆X iff there exists a $\tau_1\tau_2$#rg-open set U such that A⊆U⊆ N.

2.25. Theorem

Every nbhd N of x ∈X is a $\tau_1\tau_2$#rg-nbhd of X , but not conversely.

Proof

Let N be a nbhd of point x ∈X. Then there exists an open set U such that x ∈ U ⊆N. Since every open set is $\tau_1\tau_2$#rg-open set, U is a $\tau_1\tau_2$#rg-open set such that x ∈U ⊆ N. This implies N is $\tau_1\tau_2$#rg-nbhd of x.

The converse of the above theorem need not be true as seen from the following example.

2.26. Example

Let X = {a, b, c, d} with topologies τ_1={ ϕ, X, {a}, {b}, {a, b}, {a, b, c}} and τ_2={ ϕ, X, {a}, {b}, {a, b}} ,. Then $\tau_1\tau_2$#RGO (X) = {X, ϕ,{a},{b},{c}, {d}, {a,b}, {a,c}, {a, d},{b,c}, {b,d}}. The set {c,d} is $\tau_1\tau_2$#rg-nbhd of the point c, since the $\tau_1\tau_2$#rg-open sets {c} is such that c ∈{c}⊂ {c,d}. However, the set {c,d} is not a nbhd of the point c, since no open set U exists such that c ∈{c}⊂ {c,d}.

2.27. Theorem

Every $\tau_1\tau_2$#rg-open set is $\tau_1\tau_2$#rg-nbhd of each of its points, but not conversely.

Proof

Suppose N is $\tau_1\tau_2$#rg-open. Let x ∈ N. For N is a $\tau_1\tau_2$#rg-open set such that x ∈ N⊆ N. Since x is an arbitrary point of N, it follows that N is a $\tau_1\tau_2$#rg-nbhd of each of its points.

The converse of the above theorem is not true in general as seen from the following example.

2.28. Example

Let X = {a, b, c, d} with topologies τ_1={ ϕ, X, {a}, {b}, {a, b}, {a, b, c}} and τ_2={ ϕ, X, {a}, {b}, {a, b}} ,. Then $\tau_1\tau_2$#RGO (X) = {X, ϕ,{a},{b},{c}, {d}, {a,b}, {a,c}, {a, d},{b,c}, {b,d}}. The set {b, d} is a $\tau_1\tau_2$#rg-nbhd of the point b, since the $\tau_1\tau_2$#rg-open set {b} is such that c ∈{b}⊆ {b,d}. Also the set {b, d} is a $\tau_1\tau_2$#rg-nbhd of the point {d}, Since the $\tau_1\tau_2$#rg-open set {d} is such that d ∈ {d}⊆{c, d}. Hence {b, d} is a $\tau_1\tau_2$#rg-nbhd of each of its points, but the set { b, d} is not a $\tau_1\tau_2$#rg-open set in X.

2.29. Theorem

If F is a $\tau_1\tau_2$#rg-closed subset of X, and x∈ F^c then there exists a $\tau_1\tau_2$#rg-nbhd N of x such that N∩F =∅.

Proof

Let F be $\tau_1\tau_2$#rg-closed subset of X and x∈F^c. Then Fc is $\tau_1\tau_2$#rg-open set of X. So by theorem 2.27. F^c contains a $\tau_1\tau_2$#rg-nbhd of each of its points. Hence there exists a $\tau_1\tau_2$#rg-nbhd N of x such that N∩F^c. Hence N ∩ F =∅.

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