



Advanced Wavelet Image Compression

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ABSTRACT - Wavelets are mathematical functions that cut up data into different frequency components, and then study each component with a resolution matched to its scale. They have advantages over traditional Fourier methods in analyzing physical situations where the signal contains discontinuities and sharp spikes. Wavelets were developed independently in the fields of mathematics, data communication, quantum physics, electrical engineering, and seismic geology. Interchanges between these fields during the last ten years have led to many new wavelet applications such as image compression, turbulence, human vision, radar, and earthquake prediction. This paper introduces wavelets to the interested technical person outside of the wavelets are used in image compression. I describe the history of wavelets beginning with Fourier, compare wavelet transforms with Fourier transforms, state properties and other special aspects of wavelets, and finish with some interesting applications such as image compression.

I. INTRODUCTION

Wavelets are functions which allow data analysis of signals or images, according to scales or resolutions. The processing of signals by wavelet algorithms in fact works much the same way the human eye does; or the way a digital camera processes visual scales of resolutions, and intermediate details. But the same principle also captures cell phone signals, and even digitized color images used in medicine. Wavelets are of real use in these areas, for example in approximating data with sharp discontinuities such as choppy signals, or pictures with lots of edges. While wavelets is perhaps a chapter in function theory, we show that the algorithms that result are key to the processing of numbers, or more precisely of digitized information, signals, time series, still-images, movies, color images, etc.

Thus, applications of the wavelet idea include big parts of signal and image processing, data compression, fingerprint encoding, and many other fields of science and engineering. This paper focuses on the processing of color images with the use of custom designed wavelet algorithms, and mathematical threshold filters. Although there have been a number of recent papers on the operator theory of wavelets, there is a need for a tutorial which explains some applied trends from scratch to operator theorists.

II. OVERVIEW

A. FOURIER ANALYSIS

Fourier's representation of functions as a superposition of sine's and cosines has become ubiquitous. He asserted that any 2π periodic function $f(x)$ is the sum

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right) \quad (1.1)$$

of its Fourier series. The coefficients a_0 , a_n and b_n are calculated by

$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} f(x) dx \quad (1.2)$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos(nx) dx \quad (1.3)$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin(nx) dx \quad (1.4)$$

both the analytic and numerical solution of differential equations and for the analysis and treatment of communication signals. Fourier and wavelet analysis have some very strong links.

The Fourier transform's utility lies in its ability to analyze a signal in the time domain for its frequency content. The transform works by first translating a function in the time domain into a function in the frequency domain. The signal can then be analyzed for its frequency content because the Fourier coefficients of the transformed function represent the contribution of each sine and cosine function at each frequency. An inverse Fourier transform does just what you'd expected transform data from the frequency domain into the time domain.

B. WINDOW FOURIER TRANSFORMS

If $f(t)$ is a nonperiodic signal, the summation of the periodic functions, sine and cosine, does not accurately represent the signal. You could artificially extend the signal to make it periodic but it would require additional continuity at the endpoints. The windowed Fourier transform (WFT) is one solution to the problem of better representing the nonperiodic signal. With the WFT, the input signal $f(t)$ is chopped up into

sections, and each section is analyzed for its frequency content separately. If the signal has sharp transitions we window the input data so that the sections converge to zero at the endpoints. This windowing is accomplished via a weight function that places less emphasis near the interval's endpoints than in the middle. The effect of the window is to localize the signal in time.

C. FAST FOURIER TRANSFORMS

To approximate a function by samples, and to approximate the Fourier integral by the discrete Fourier transform, requires applying a matrix whose order is the number sample points n : Since multiplying an $n \times n$ matrix by a vector costs on the order of n^2 arithmetic operations, the problem gets quickly worse as the number of sample points increases. However, if the samples are uniformly spaced, then the Fourier matrix can be factored into a product of just a few sparse matrices, and the resulting factors can be applied to a vector in a total of order $n \log_2 n$ arithmetic operations. This is the so-called fast Fourier transform or FFT.

D. WAVELET TRANSFORMS VERSUS FOURIER TRANSFORMS

The fast Fourier transform (FFT) and the discrete wavelet transform (DWT) are both linear operations that generate a data structure that contains $\log_2 n$ segments of various lengths, usually filling and transforming it into a different data vector of length $2n$. The mathematical properties of the matrices involved in the transforms are similar as well. The inverse transform matrix for both the FFT and the DWT is the transpose of the original. As a result, both transforms can be viewed as a rotation in function space to a different domain. For the FFT, this new domain contains basis functions that are sines and cosines. For the wavelet transform, this new domain contains more complicated basis functions called wavelets, mother wavelets, or analyzing wavelets.

E. DISSIMILARITIES BETWEEN FOURIER AND WAVELET TRANSFORMS

The most interesting dissimilarity between these two kinds of transforms is that individual wavelet functions are localized in space. Fourier sine and cosine functions are not. This localization feature, along with wavelets' localization of frequency, makes many functions and operators using wavelets sparse, when transformed into the wavelet domain. This sparseness, in turn, results in a number of useful applications such as data compression, detecting features in images, and removing noise from time series.

Because a single window is used for all frequencies in the WFT, the resolution of the analysis is the same at all locations in the time-frequency plane. An advantage of wavelet transforms is that the windows vary. This happy medium is exactly what you get with wavelet. One thing to remember is that wavelet transforms do not have a single set of basis functions like the Fourier transform. Instead, wavelet transforms have an infinite set of possible basis functions. Thus

wavelet analysis provides immediate access to information that can be obscured by other time-frequency methods such as Fourier analysis.

III. WHAT DO SOME WAVELETS LOOK LIKE?

Wavelet transforms comprise an infinite set. The different wavelet families make different trade of between how compactly the basis functions are localized in space and how smooth they are.

Some of the wavelet bases have fractal structure. The Daubechies wavelet family is one example (see Figure 1).

Within each family of wavelets (such as the Daubechies family) are wavelet subclasses distinguished by the number of coefficients and by the level of iteration.(see figure 2)

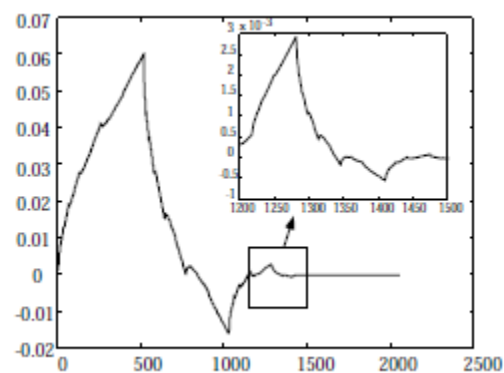


Fig. 1. The fractal self-similarity of the Daubechies mother wavelet. This figure was generated using the Matlab.

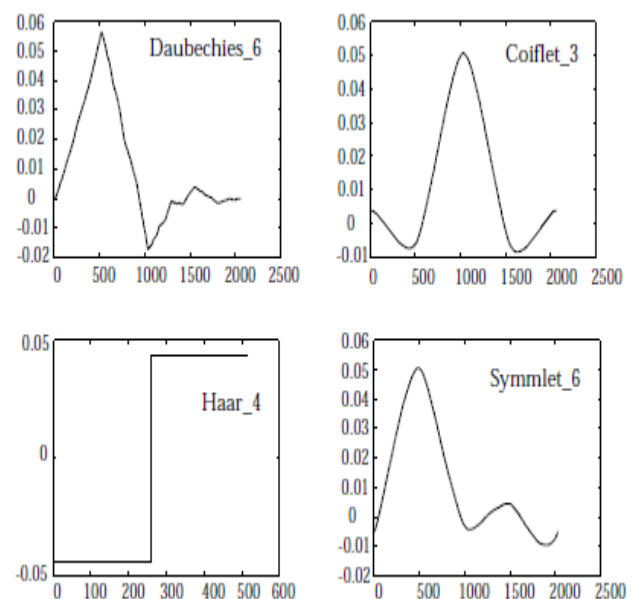


Fig. 2. Several different families of wavelets. The number next to the wavelet name represents the number of vanishing moments (A stringent mathematical definition related to the number of wavelet coefficients) for the subclass of wavelet. These figures were generated using Matlab.

Relationship for the coefficients that must be satisfied, and is directly related to the number of coefficient.

IV. Wavelet Color Image Compression

A. Methods- The whole process of wavelet image compression is performed as follows: An input image is taken by the computer, forward wavelet transform is performed on the digital image, thresholding is one on the digital image, entropy coding is done on the image where necessary, thus the compression of image is done on the computer. Then with the compressed image, reconstruction of wavelet transformed image is done, then inverse wavelet transform is performed on the image, thus image is reconstructed.

B. Forward Wavelet Transform- Various wavelet transforms are used in this step. Namely Daubechies wavelets, Coiflets, biorthogonal wavelets, and Symlets. These various transforms are implemented to observe how various mathematical properties such as symmetry, number of vanishing moments and orthogonality differ the result of compressed image. Advantages short support is that it preserves locality. The Daubechies wavelets used are orthogonal, so do Coiflets. Symlets have the property of being close to symmetric. The biorthogonal wavelets are not orthogonal but not having to be orthogonal gives more options to a variety of filters such as symmetric filters thus allowing them to possess the symmetric property.

C. Wavelets- Compactly supported wavelets are functions defined over a finite interval and having an average value of zero. The basic idea of the wavelet transform is to represent any arbitrary function $f(x)$ as a superposition of a set of such wavelets or basis functions. These basis functions are obtained from a single prototype wavelet called the mother wavelet $\psi(x)$, by dilations or scaling and translations. Wavelet bases are very good at efficiently representing functions that are smooth

except for a small set of discontinuities.

For each $n, k \in \mathbb{Z}$, define $\psi_{n,k}(x)$ by

$$\psi_{n,k}(x) = 2^{n/2} \psi(2^n x - k) \quad (4.1)$$

Constructing the function $\psi(x)$, L^2 on $\mathbb{R}$, such that $\{\psi_{n,k}(x)\}_{n,k \in \mathbb{Z}}$ is an orthonormal basis on $\mathbb{R}$. As mentioned before $\psi(x)$ is a wavelet and the collection $\{\psi_{n,k}(x)\}_{n,k \in \mathbb{Z}}$ is a wavelet orthonormal basis on $\mathbb{R}$; this framework for constructing wavelets involves the concept of a multiresolution analysis or MRA.

D. Multiresolution Analysis- Multiresolution analysis is a device for computation of basis coefficients in $L^2(\mathbb{R})$: $f = \sum \sum c_{n,k} \psi_{n,k}$. It is defined as follows, see [Kei04]: Define

$$V_n = \{f(x) | f(x) = 2^{n/2} g(2^n x), g(x) \in V_0\}, \quad (4.2)$$

where

$$f(x) = \sum_{n \in \mathbb{Z}} \varphi(-n) i \varphi(x - n) \quad (4.3)$$

E. Thresholding- Since the whole purpose of this project was to compare the performance of each image compression using different wavelets, fixed thresholds were used. Soft threshold was used in this project in the hope that the drastic differences in gradient in the image would be noted less apparently. The soft and hard thresholding T_{soft} , T_{hard} are defined as follows:

$$T_{\text{soft}}(x) = \begin{cases} 0 & \text{if } x \leq \lambda \\ x - \lambda & \text{if } x > \lambda \\ x + \lambda & \text{if } x < -\lambda \end{cases} \quad (4.4)$$

$$T_{\text{hard}}(x) = \begin{cases} 0 & \text{if } x \leq \lambda \\ x & \text{if } x > \lambda \end{cases} \quad (4.5)$$

It could be observed by looking at the definitions, the difference between them is related to how the coefficients larger than a threshold value λ in absolute values are handled. In hard thresholding, these coefficient values are left alone. Unlike in hard thresholding, the coefficient values are decreased by

λ if positive and increased by λ if negative [Waln02].

However, a fixed threshold values were used so as to have the same given condition for every wavelet transform to compare the performances of different conditions. Here, fixed thresholds 10 and 20 were used. For the lossless compression 0 is used as the threshold for an obvious reason.

IV. LOSSLESS COMPRESSION

It is not always important for digitised photo's to be stored exactly. There is already natural noise in the picture. Storing this noise is a waste of space, and adding some extra noise would not really harm anyone. It seems that lossy compression is much

Level	Paint	Kermit	Child
0	100%	100%	100%
1	89%	80%	85%
2	83%	70%	77%
3	80%	65%	72%
4	79%	63%	69%
5	78%	62%	68%
6	78%	62%	68%

more relevant than lossless compression for photo's.

Lossy compression that is not flexible is worthless. To have this freedom, the user must also have the option of lossless compression. It is one of the options you will enjoy having even if you do not use it.

The second point of this section is to show that natural images have redundancy, and that the working of lossy compression is not solely based on throwing things away. The results in this section are the real baseline for the evaluation of the lossy compression schemes, instead of the original file size.

I have investigated the use of the PLUS wavelet for lossless compression. For several monochrome images I calculated the

entropy after applying the PLUS wavelet transform 0,1,2,3,4,5 and 6 times.

One sees that the compression achieved varies from image to image, but that compression is achieved in all cases.



Figure 1. Original Image



Figure 4 Wavelet Decomposition of an Image Component. The image has been modified: the average detail has been lightened and the horizontal, vertical and diagonal details are shown as negative images with a reversal of white and black, because of constraints of the printing process.

Doing more than 5 or 6 levels is not useful, because the data resulting from 6 transforms consists mostly of detail coefficients, not of smooth coefficients. Another transform would only work on these smooth coefficients, so overall it cannot make much difference. The percents given above are not unbeatable. For the calculation of the entropy the formula of the data compression is used. This formula treats the data as a large, unordered bag of samples. The probability estimates can also be based on the already sent neighbours. This is more

complicated, but can yield better compression. Said and Pearlman do so in their paper [9]. I did not do this because things would get quite complicated.

According to David Salomon[14], the art of data compression is not to squeeze every bit out of everything, but to find a balance between programming complexity and compression ratio.

V. CONCLUSION

Wavelet Image Processing enables computers to store an image in many scales of resolutions, thus decomposing an image into various levels and types of details and approximation with different-valued resolutions. Wavelets allow one to compress the image using less storage space with more details of the image. The advantage of decomposing images to approximate and detail parts is that it enables to isolate and manipulate the data with specific properties. With this, it is possible to determine whether to preserve more specific details. For instance, keeping more vertical detail instead of keeping all the horizontal, diagonal and vertical details of an image that has more vertical aspects. This would allow the image to lose a certain amount of horizontal and diagonal details, but would not affect the image in human perception.

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